

61A Lecture 19

Wednesday, October 10

Generic Functions, Continued

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What's different? Today's generic functions apply to multiple arguments that don't share a common interface

Rational Numbers

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class Rational(object):  
  
    def __init__(self, numer, denom):  
        g = gcd(numer, denom)  
        self.numer = numer // g  
        self.denom = denom // g
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    def __repr__(self):
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    def __repr__(self):
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```
def add_rational(x, y):
```

```
    nx, dx = x.numer, x.denom
```

```
    ny, dy = y.numer, y.denom
```

```
    return Rational(nx * dy + ny * dx, dx * dy)
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    ny, dy = y.numer, y.denom
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```
    return Rational(nx * dy + ny * dx, dx * dy)
```

```
def mul_rational(x, y):
```

```
    return Rational(x.numer * y.numer, x.denom * y.denom)
```

Complex Numbers: the Rectangular Representation

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```
class ComplexRI(object):
    def __init__(self, real, imag):
        self.real = real
        self.imag = imag

    @property
    def magnitude(self):
        return (self.real ** 2 + self.imag ** 2) ** 0.5

    @property
    def angle(self):
        return atan2(self.imag, self.real)

    def __repr__(self):
        return 'ComplexRI({0}, {1})'.format(self.real,
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def add_complex(z1, z2):
    return ComplexRI(z1.real + z2.real,
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                                            self.imag)
```

```
def add_complex(z1, z2):
    return ComplexRI(z1.real + z2.real,
                    z1.imag + z2.imag)
```

Might be either ComplexMA
or ComplexRI instances

Special Methods

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Adding instances of user-defined classes with `__add__`.

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ComplexRI(3.0, 2.0)
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ComplexMA(1.0, 3.141592653589793)
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<http://getpython3.com/diveintopython3/special-method-names.html>

<http://docs.python.org/py3k/reference/datamodel.html#special-method-names>

The Independence of Data Types

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Data abstraction and class definitions keep types separate

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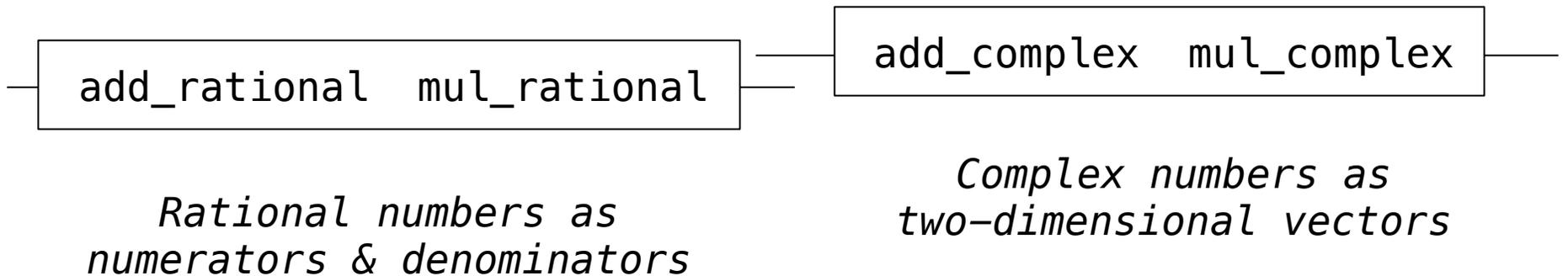
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*Rational numbers as
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*How do we add a complex number
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*Complex numbers as
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There are many different techniques for doing this!

Type Dispatching

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def add_complex_and_rational(z, r):  
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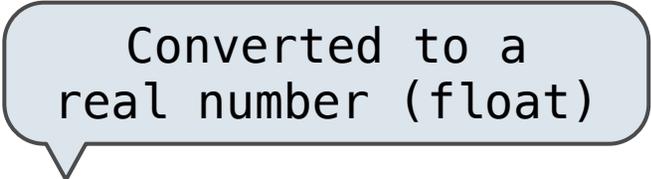
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Converted to a
real number (float)

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def type_tag(x):  
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type_tag.tags = {ComplexRI: 'com',  
                 ComplexMA: 'com',  
                 Rational:  'rat'}
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```
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add.implementations[('rat', 'rat')] = add_rational  
add.implementations[('com', 'rat')] = add_complex_and_rational  
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lambda r, z: add_complex_and_rational(z, r)

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add, subtract,
multiply, divide

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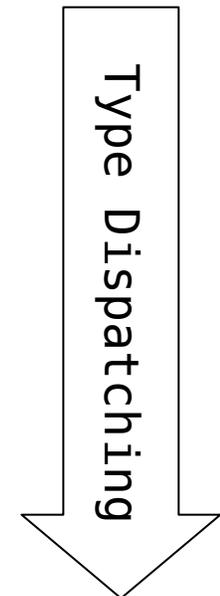
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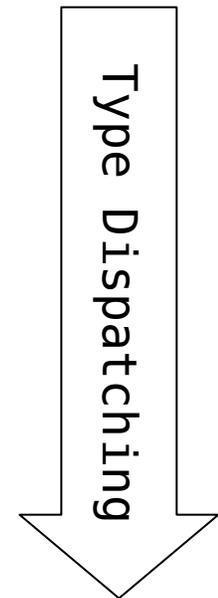


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```
def apply(operator_name, x, y):  
    tags = (type_tag(x), type_tag(y))  
    key = (operator_name, tags)  
    return apply.implementations[key](x, y)
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```

Question: Can any numeric type be coerced into any other?

Question: Have we been repeating ourselves with data-directed programming?

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Demo

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