

# Image Processing

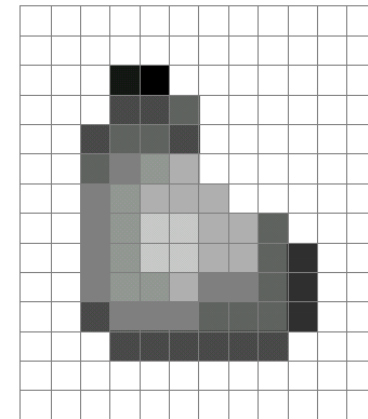
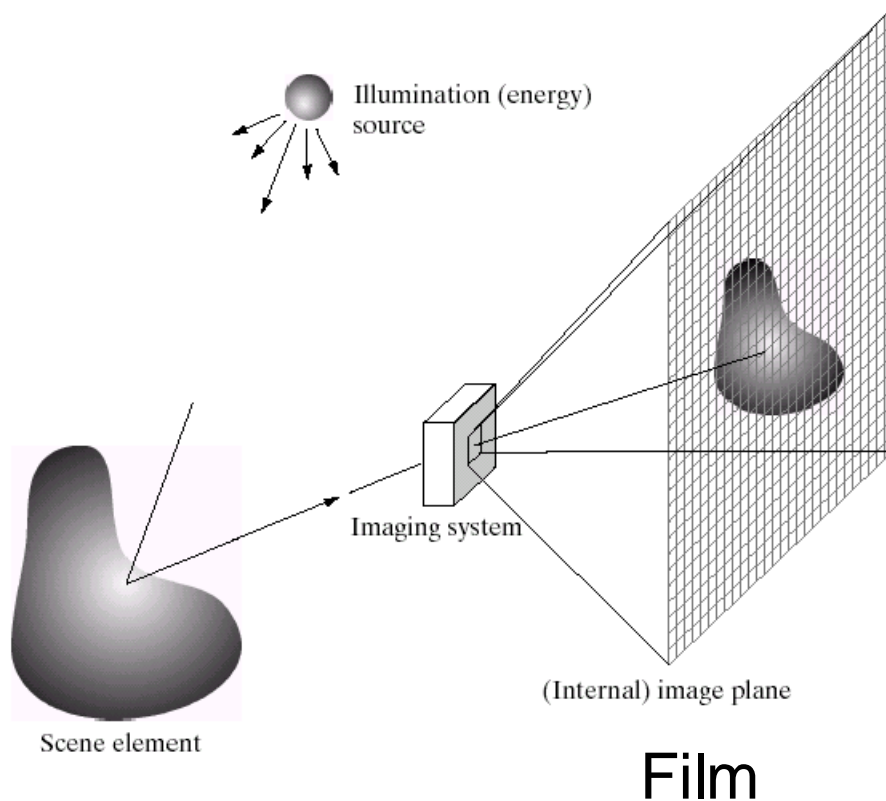
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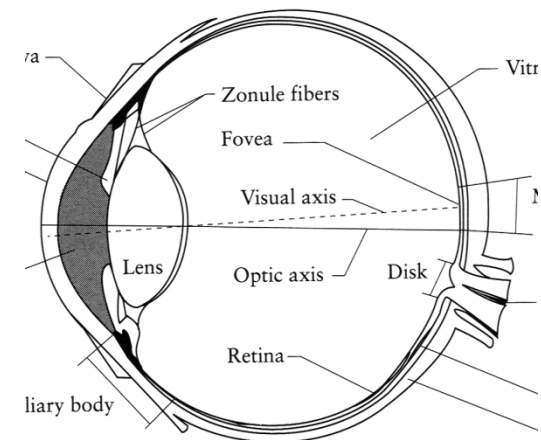
Alexei Efros, CS280, Spring 2018

# Image Formation

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Digital Camera



The Eye



# What is an image?

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We can think of an **image** as a function,  $f$ , from  $\mathbb{R}^2$  to  $\mathbb{R}$ :

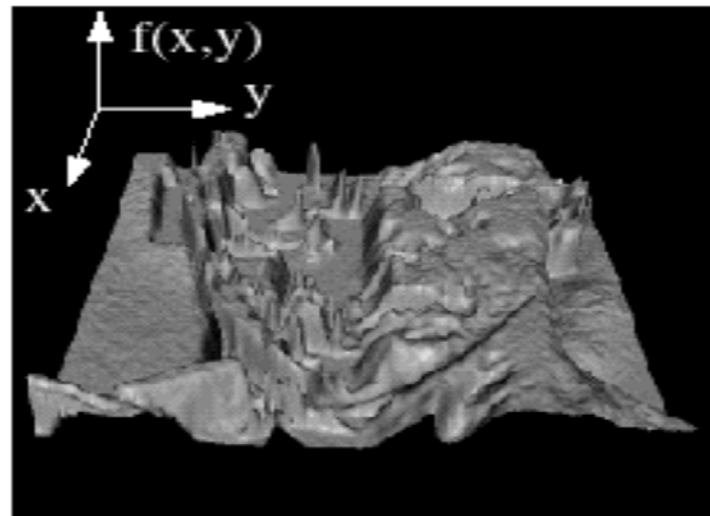
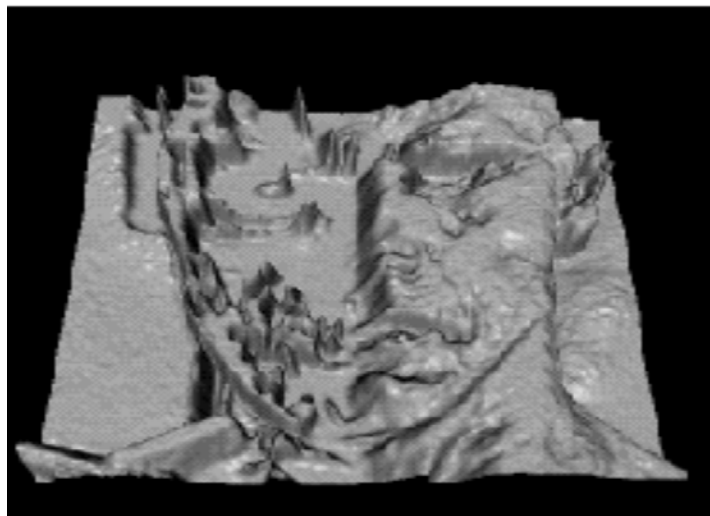
- $f(x, y)$  gives the **intensity** at position  $(x, y)$
- Realistically, we expect the image only to be defined over a rectangle, with a finite range:
  - $f: [a,b] \times [c,d] \rightarrow [0,1]$

A color image is just three functions pasted together.

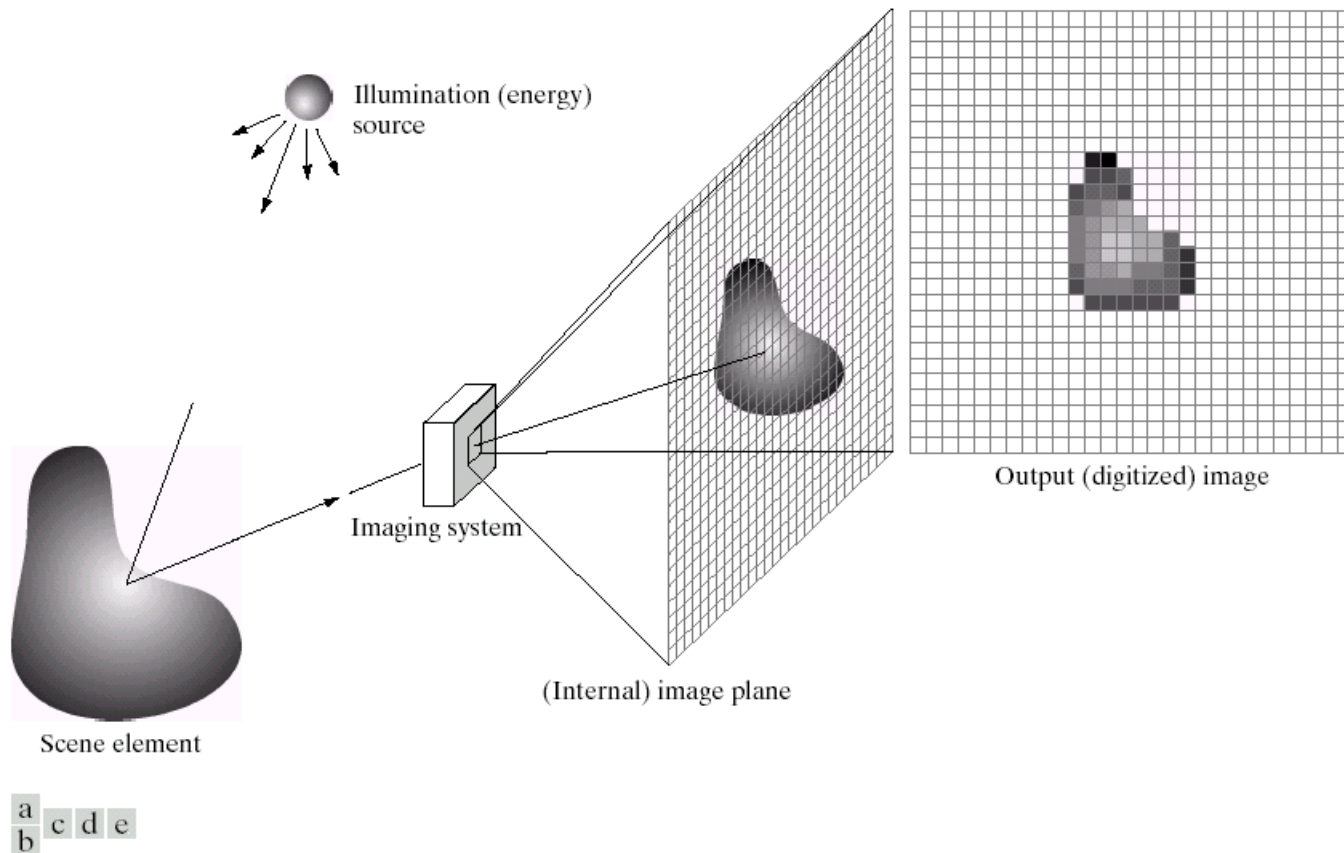
We can write this as a “vector-valued” function:

$$f(x, y) = \begin{bmatrix} r(x, y) \\ g(x, y) \\ b(x, y) \end{bmatrix}$$

# Images as functions



# What's in the “pixel intensity”?



**FIGURE 2.15** An example of the digital image acquisition process. (a) Energy (“illumination”) source. (b) An element of a scene. (c) Imaging system. (d) Projection of the scene onto the image plane. (e) Digitized image.

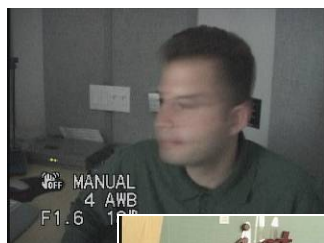
$$f(x,y) = \text{reflectance}(x,y) * \text{illumination}(x,y)$$

*Reflectance in  $[0, 1]$ , illumination in  $[0, \text{inf}]$*

# Problem: Dynamic Range

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The real world is  
High dynamic range



1



1500



25,000



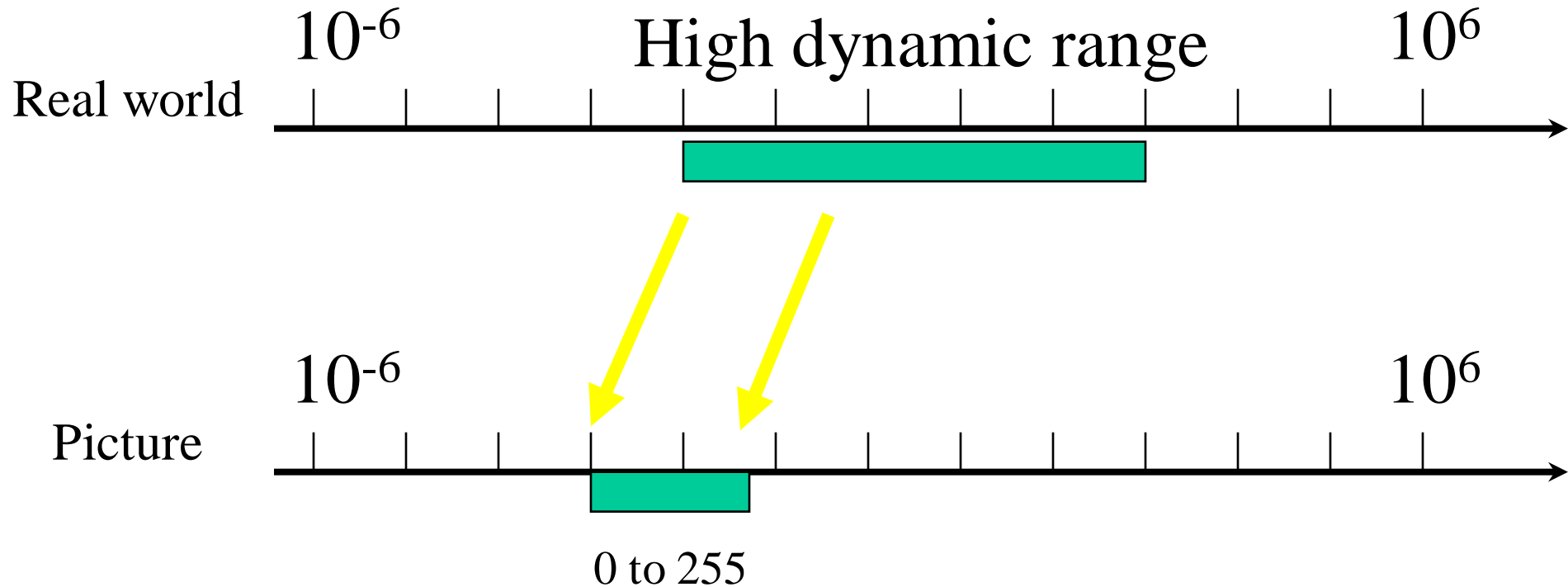
400,000



2,000,000,000

# Long Exposure

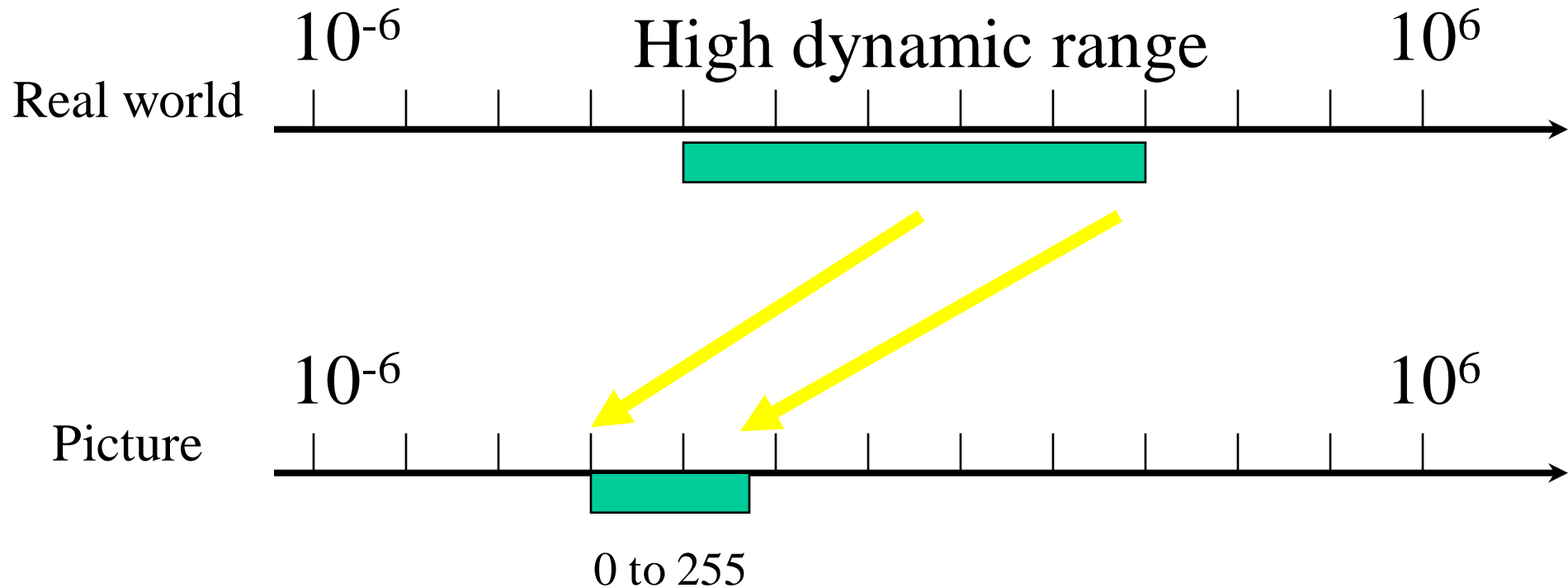
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- What does pixel value 255 mean?

# Short Exposure

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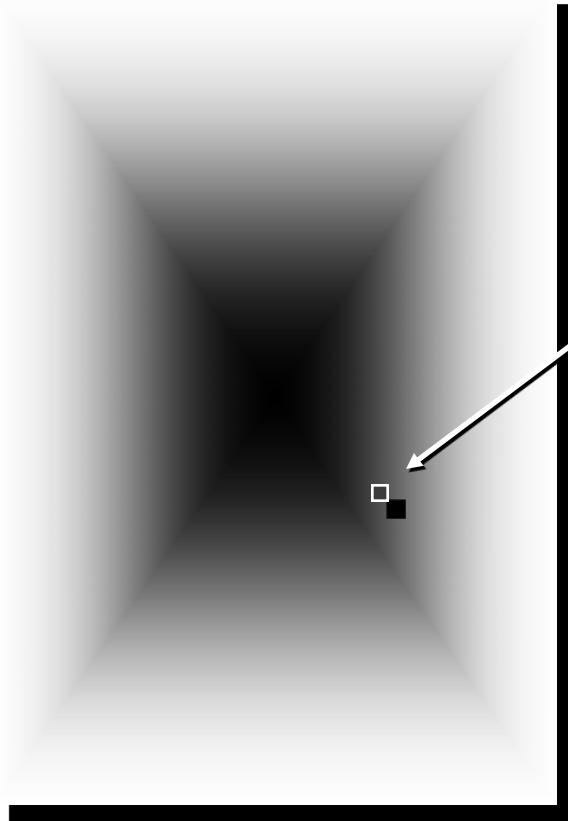
- What does pixel value 0 mean?



# Is Camera a photometer?

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Image

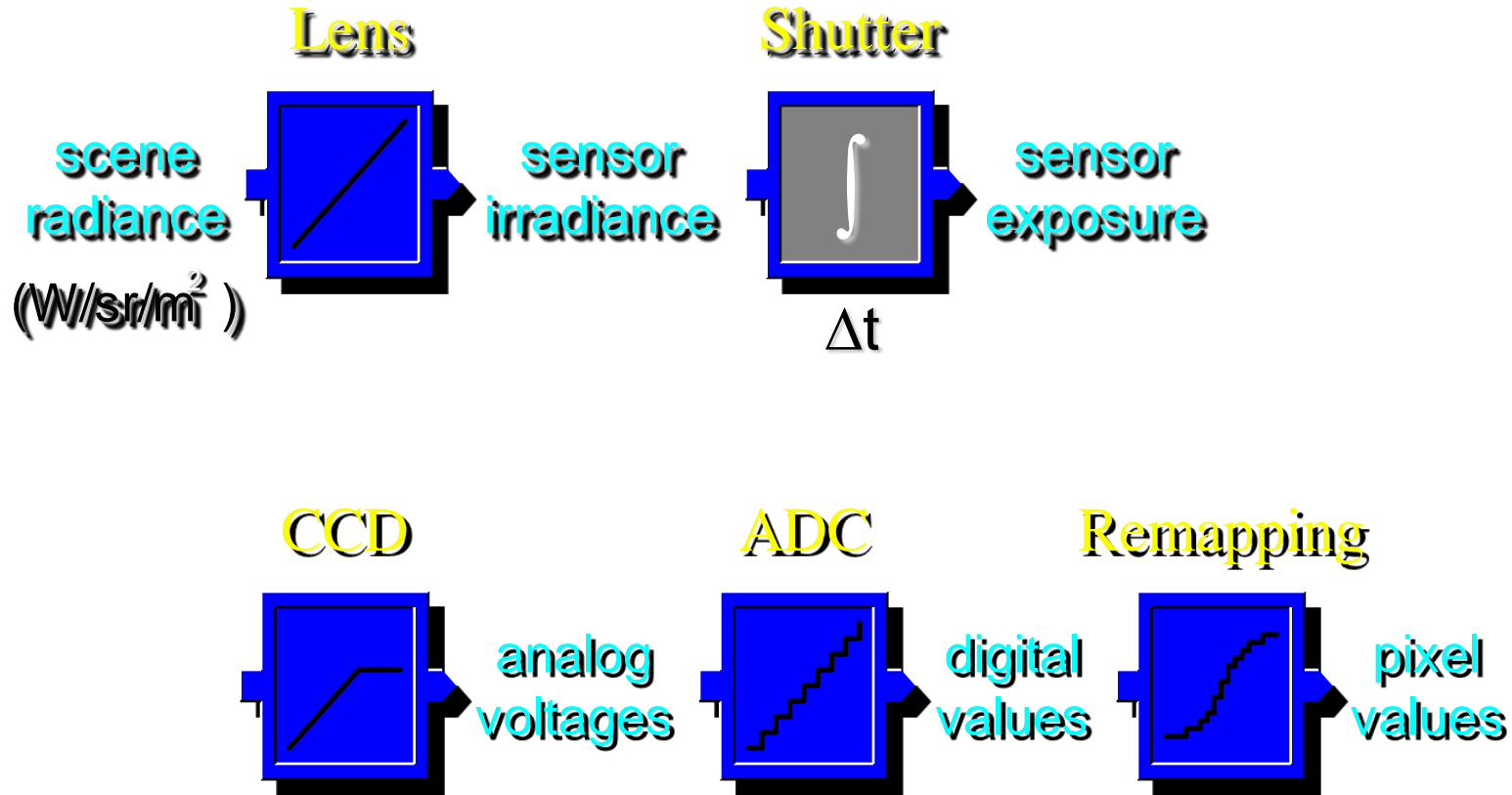


pixel (312, 284) = 42

42 photos?

# Image Acquisition Pipeline

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Camera is NOT a photometer!

# Simple Point Processing: Enhancement

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a	b
c	d

**FIGURE 3.9**

(a) Aerial image.  
(b)–(d) Results of  
applying the  
transformation in  
Eq. (3.2-3) with  
 $c = 1$  and  
 $\gamma = 3.0, 4.0,$  and  
 $5.0$ , respectively.  
(Original image  
for this example  
courtesy of  
NASA.)



# Point Processing

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The simplest kind of range transformations are these independent of position  $x,y$ :

$$g = T(f)$$

This is called point processing.

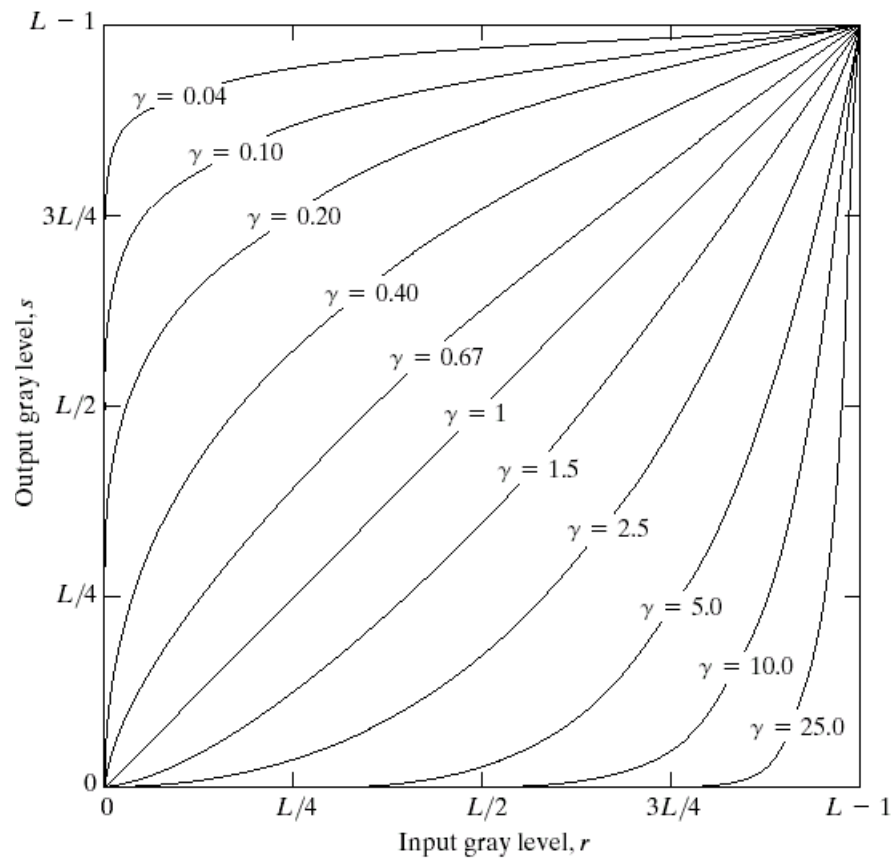
e.g. Gain and Bias transform:

$$g(x,y) = a*f(x,y) + b$$

**Important:** every pixel for himself – spatial information completely lost!

# Power-law transformations

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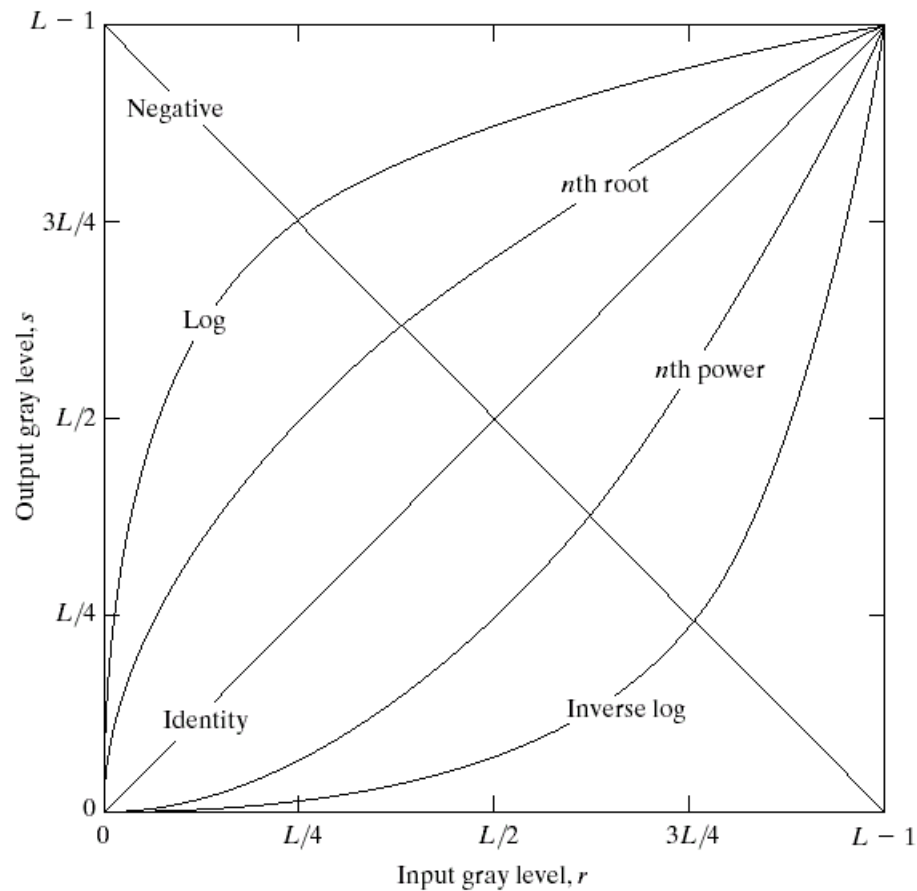
**FIGURE 3.6** Plots of the equation  $s = cr^\gamma$  for various values of  $\gamma$  ( $c = 1$  in all cases).

$$s = cr^\gamma$$

# Basic Point Processing

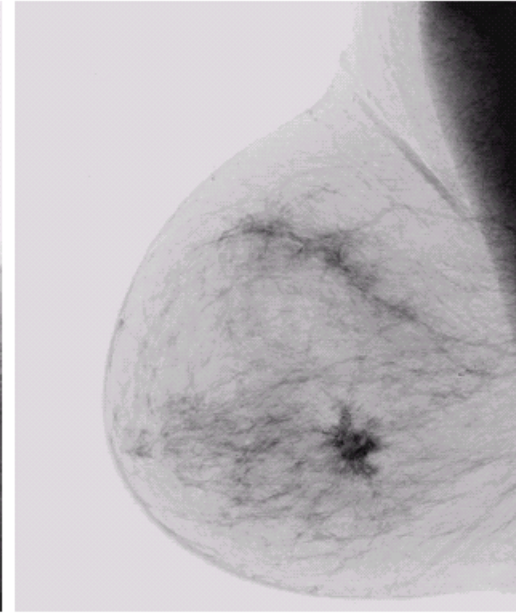
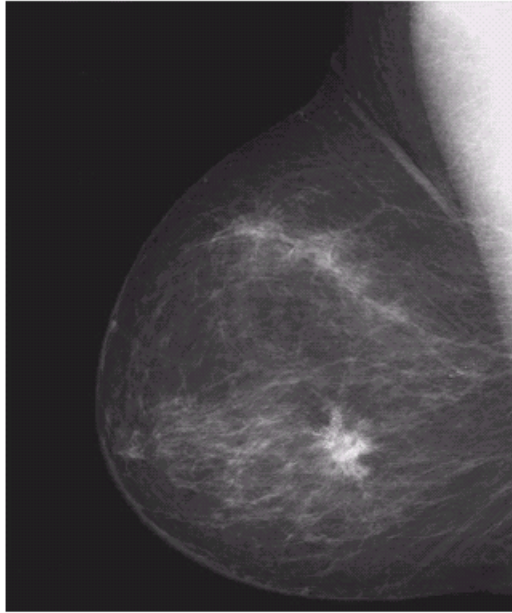
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**FIGURE 3.3** Some basic gray-level transformation functions used for image enhancement.



# Negative

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a b

**FIGURE 3.4**

(a) Original digital mammogram.

(b) Negative image obtained using the negative transformation in Eq. (3.2-1).

(Courtesy of G.E. Medical Systems.)



# Log

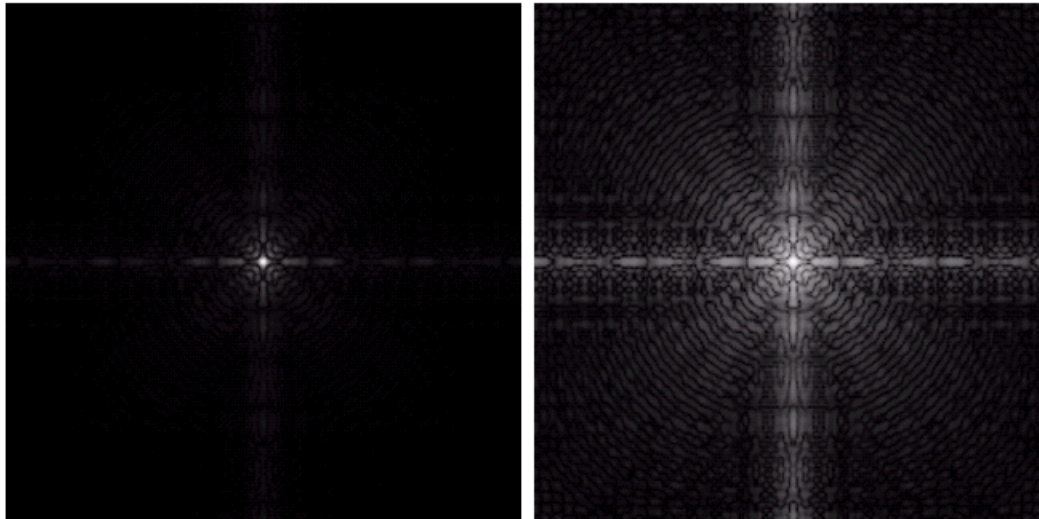
---

a b

## FIGURE 3.5

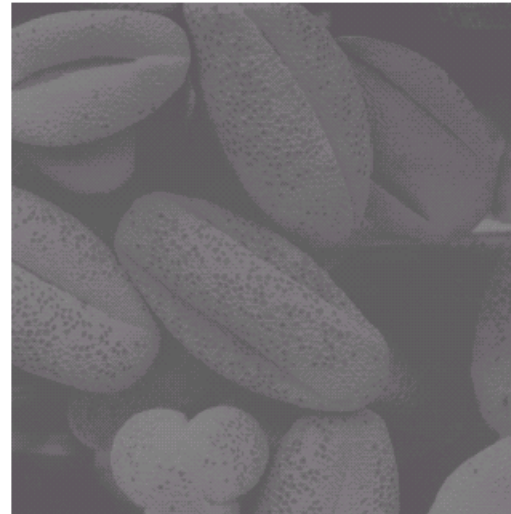
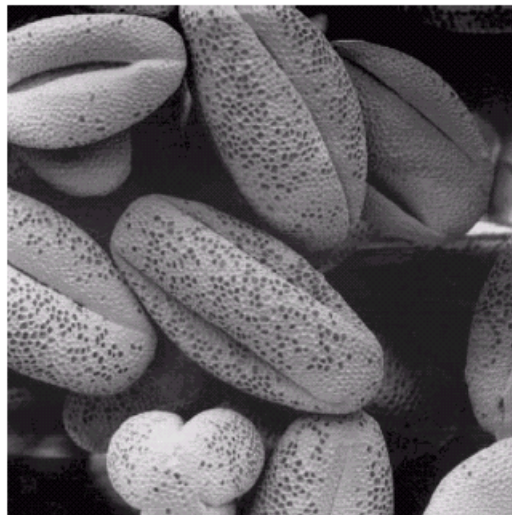
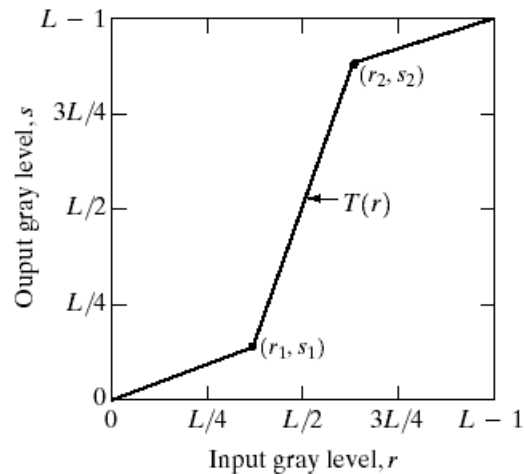
(a) Fourier spectrum.

(b) Result of applying the log transformation given in Eq. (3.2-2) with  $c = 1$ .





# Contrast Stretching

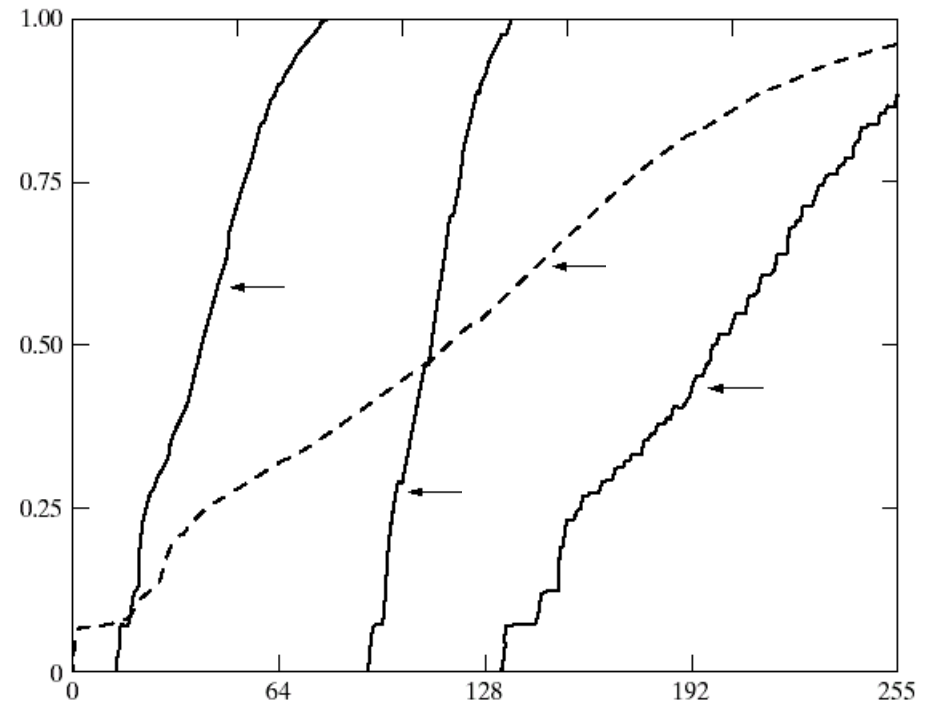
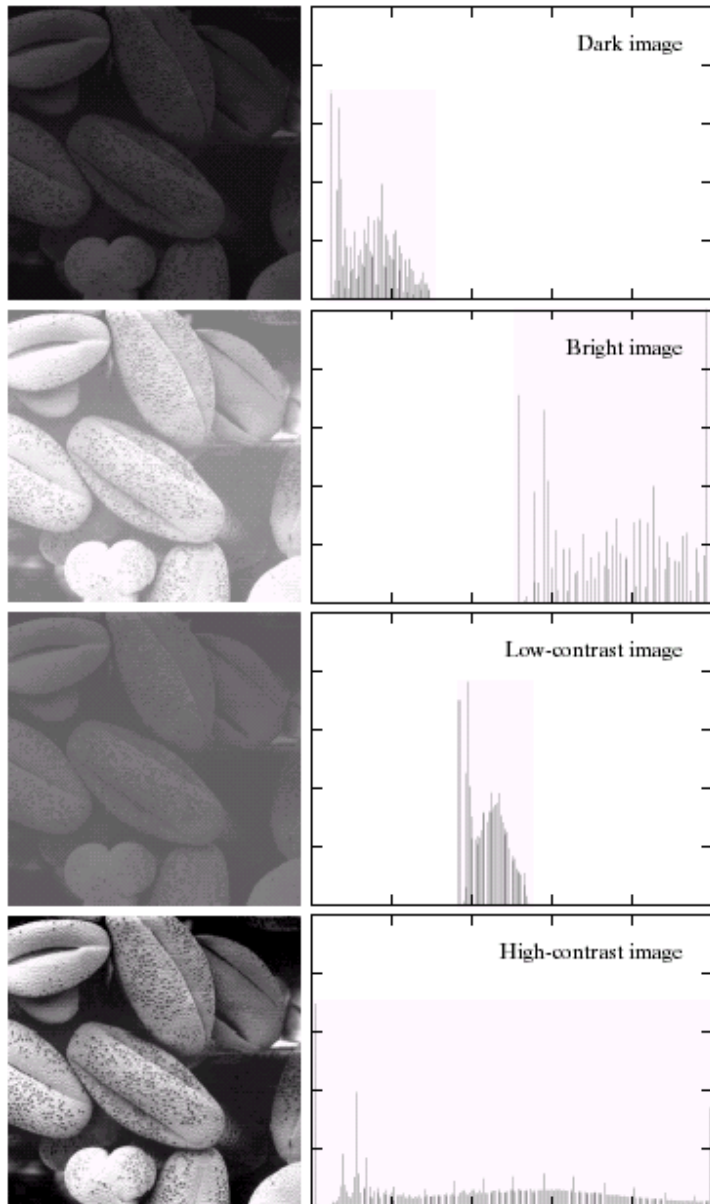


a b  
c d

**FIGURE 3.10**

Contrast stretching. (a) Form of transformation function. (b) A low-contrast image. (c) Result of contrast stretching. (d) Result of thresholding. (Original image courtesy of Dr. Roger Heady, Research School of Biological Sciences, Australian National University, Canberra, Australia.)

# Image Histograms



Cumulative Histograms

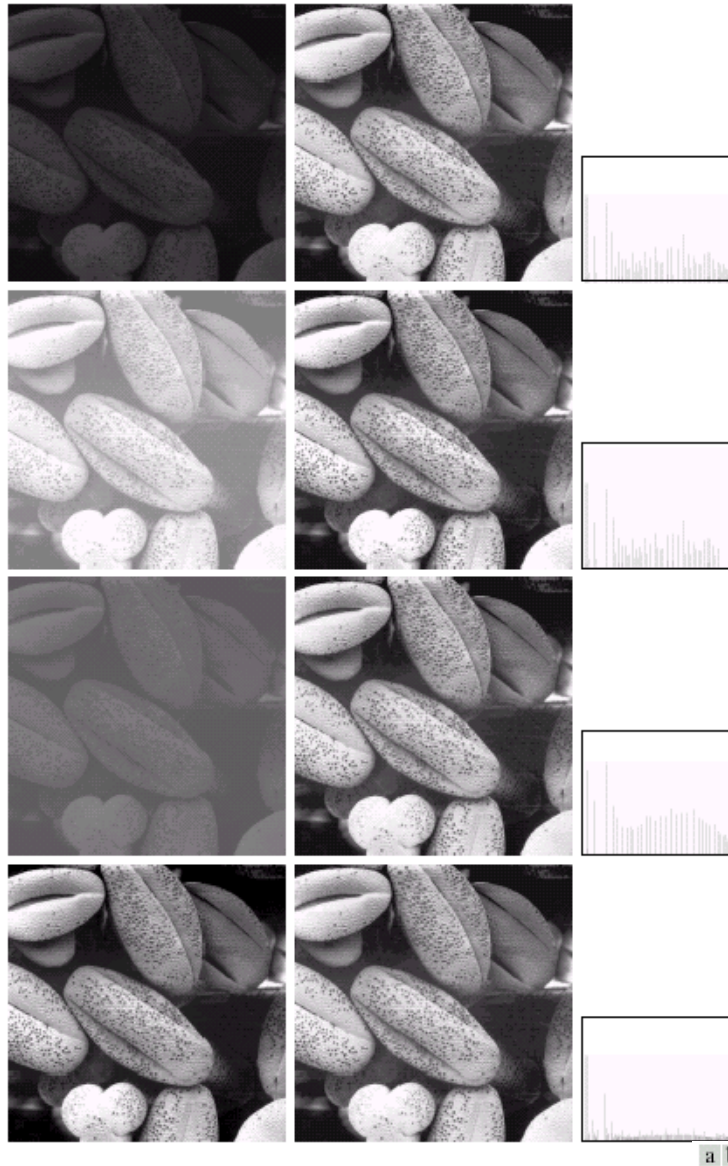
$$s = T(r)$$

a b

**FIGURE 3.15** Four basic image types: dark, light, low contrast, high contrast, and their corresponding histograms. (Original image courtesy of Dr. Roger Heady, Research School of Biological Sciences, Australian National University, Canberra, Australia.)

# Histogram Equalization

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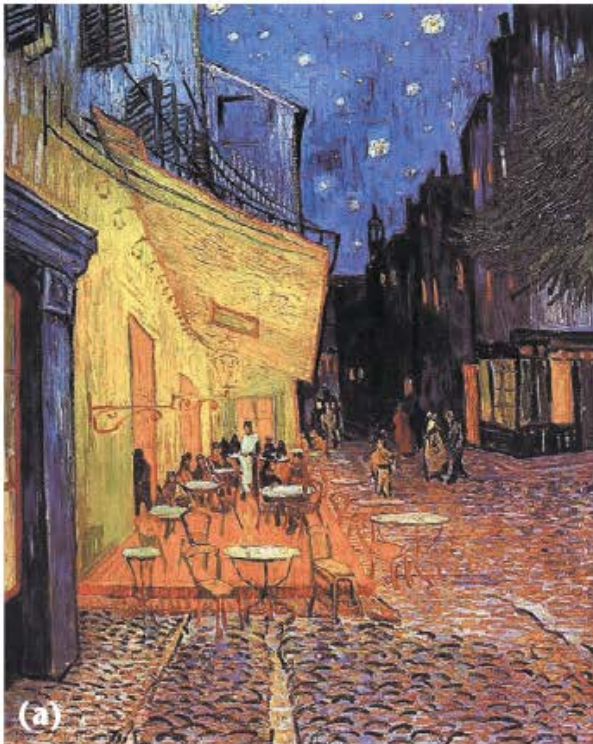


a b c

**FIGURE 3.17** (a) Images from Fig. 3.15. (b) Results of histogram equalization. (c) Corresponding histograms.

# Color Transfer [Reinhard, et al, 2001]

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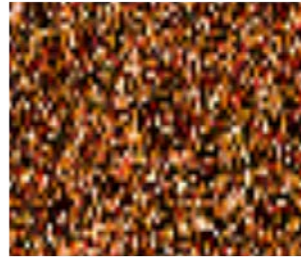
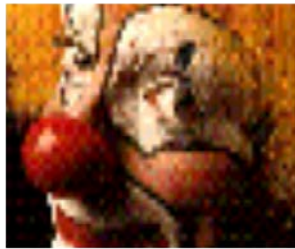
Erik Reinhard, Michael Ashikhmin, Bruce Gooch, Peter Shirley, [Color Transfer between Images](#). *IEEE Computer Graphics and Applications*, 21(5), pp. 34–41. September 2001.



# Limitations of Point Processing

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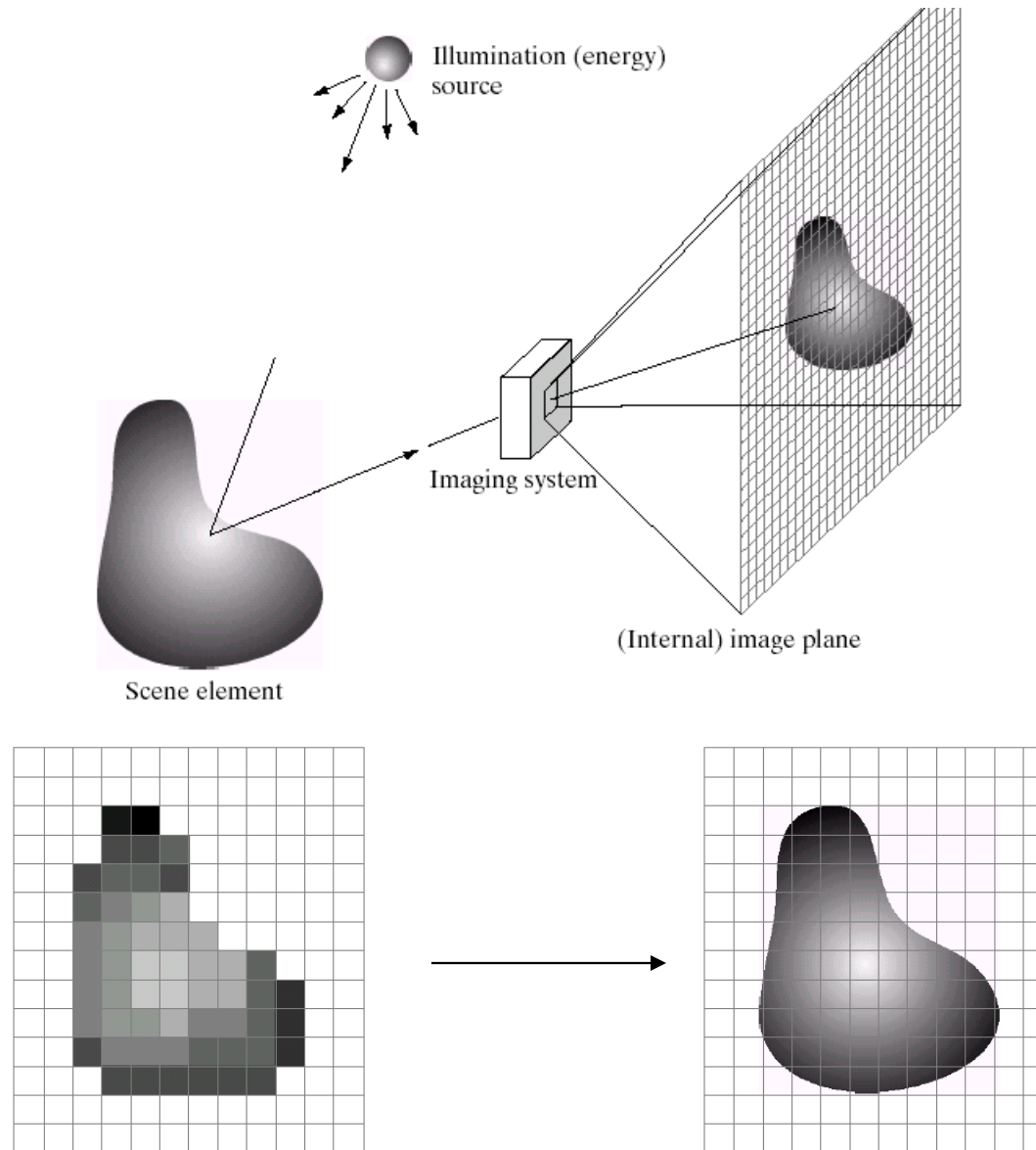
Q: What happens if I reshuffle all pixels within the image?



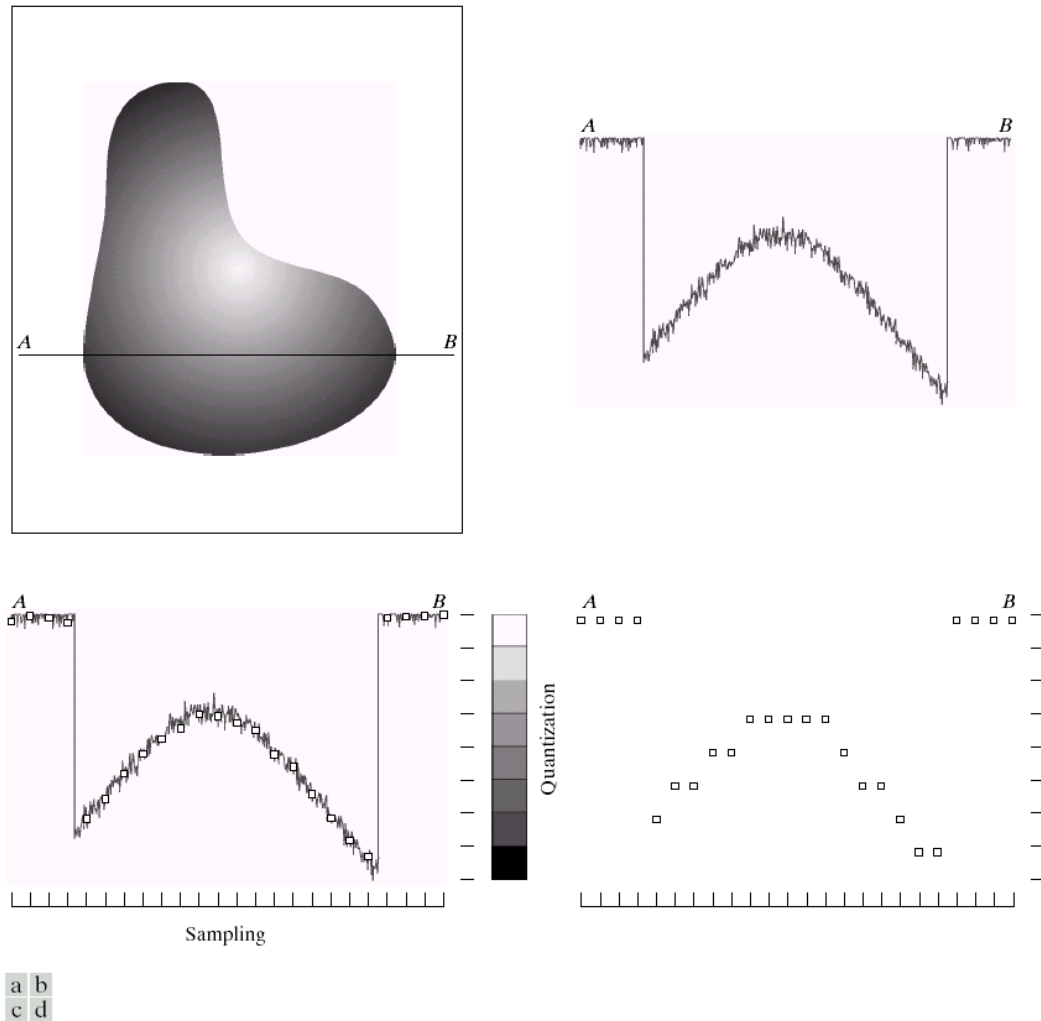
A: It's histogram won't change. No point processing will be affected...

# Sampling and Reconstruction

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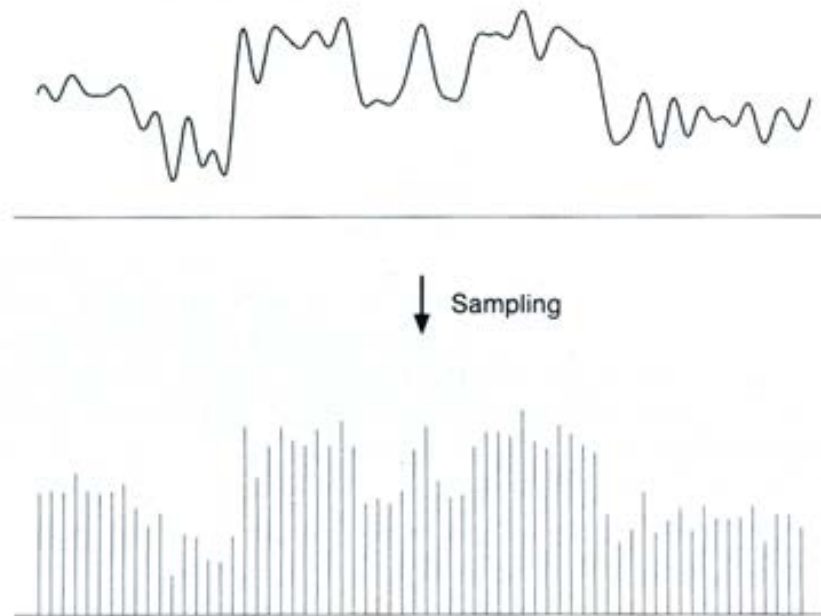
# Sampling and Quantization



**FIGURE 2.16** Generating a digital image. (a) Continuous image. (b) A scan line from *A* to *B* in the continuous image, used to illustrate the concepts of sampling and quantization. (c) Sampling and quantization. (d) Digital scan line.

# Sampled representations

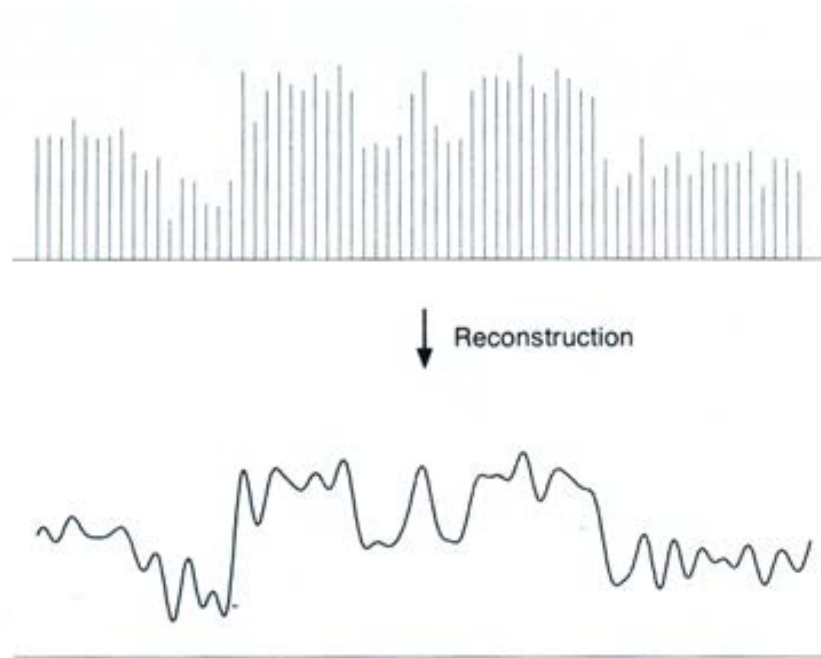
- How to store and compute with continuous functions?
- Common scheme for representation: samples
  - write down the function's values at many points



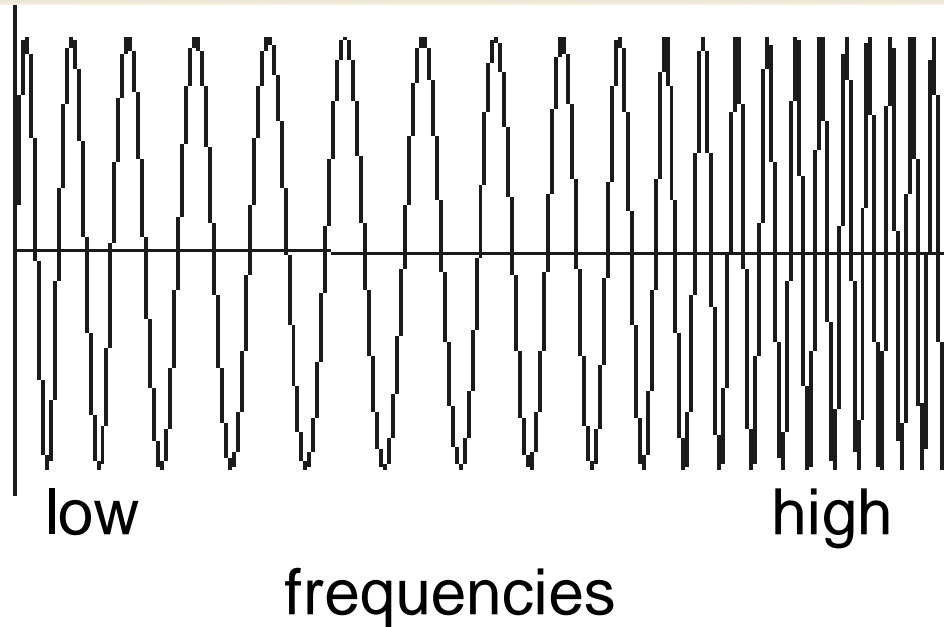
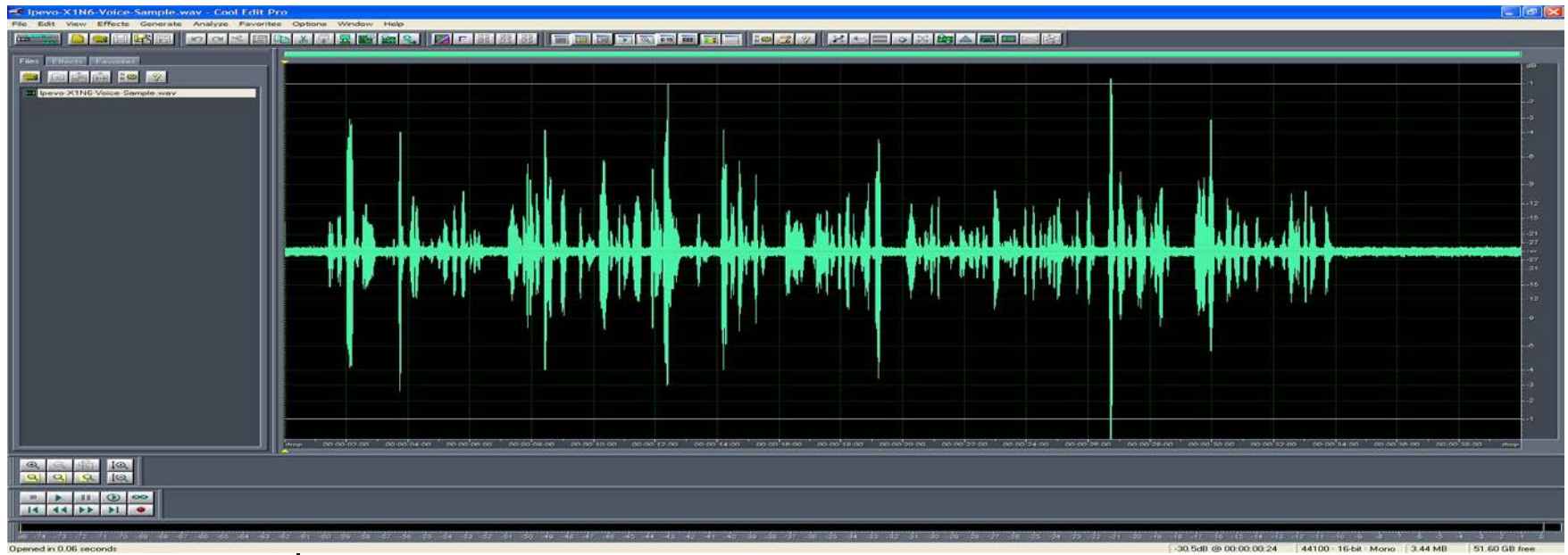


# Reconstruction

- Making samples back into a continuous function
  - for output (need realizable method)
  - for analysis or processing (need mathematical method)
  - amounts to “guessing” what the function did in between

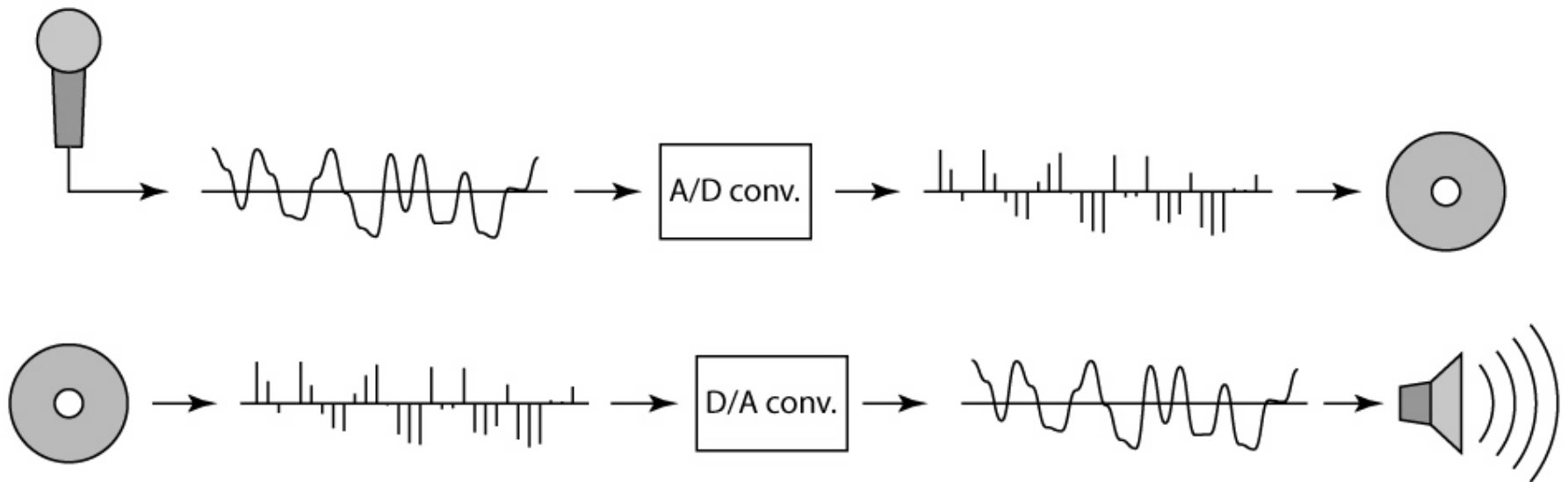


# 1D Example: Audio



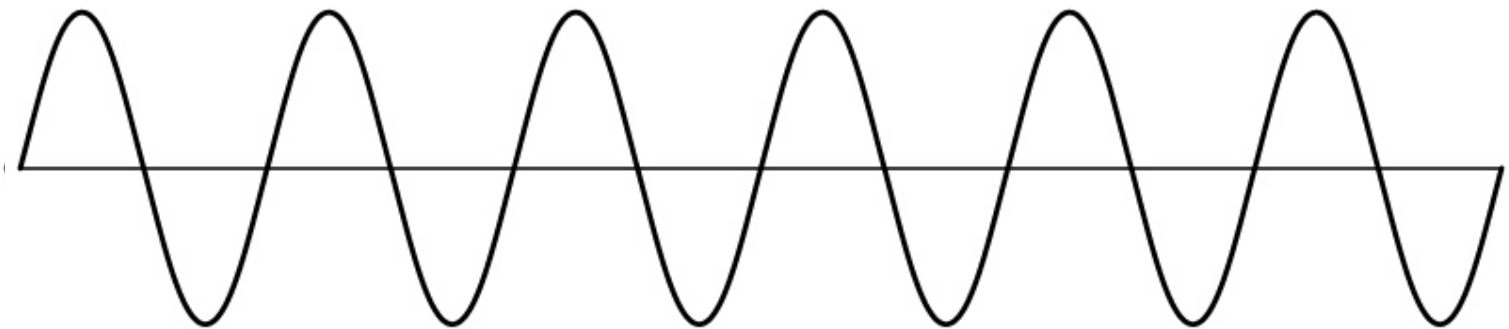
# Sampling in digital audio

- Recording: sound to analog to samples to disc
- Playback: disc to samples to analog to sound again
  - how can we be sure we are filling in the gaps correctly?



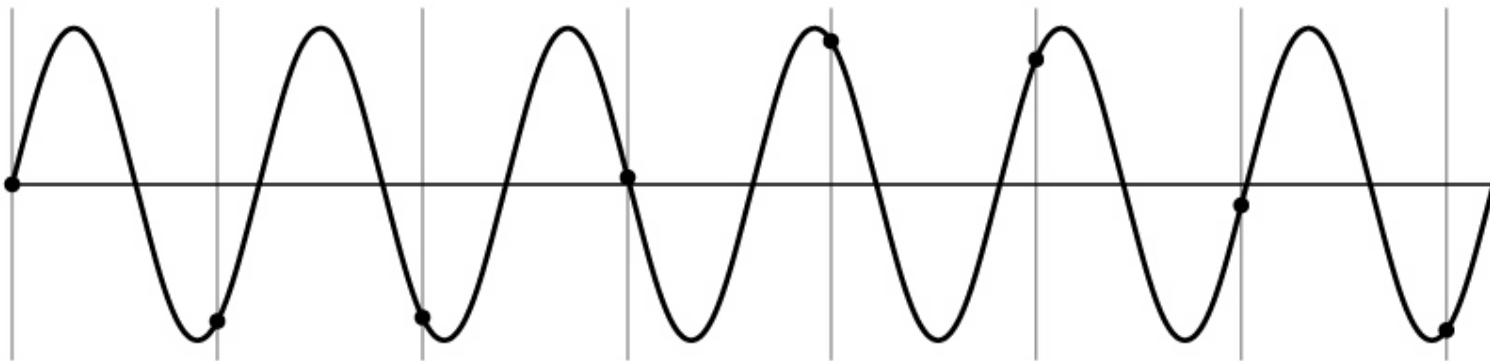
# Sampling and Reconstruction

- Simple example: a sign wave



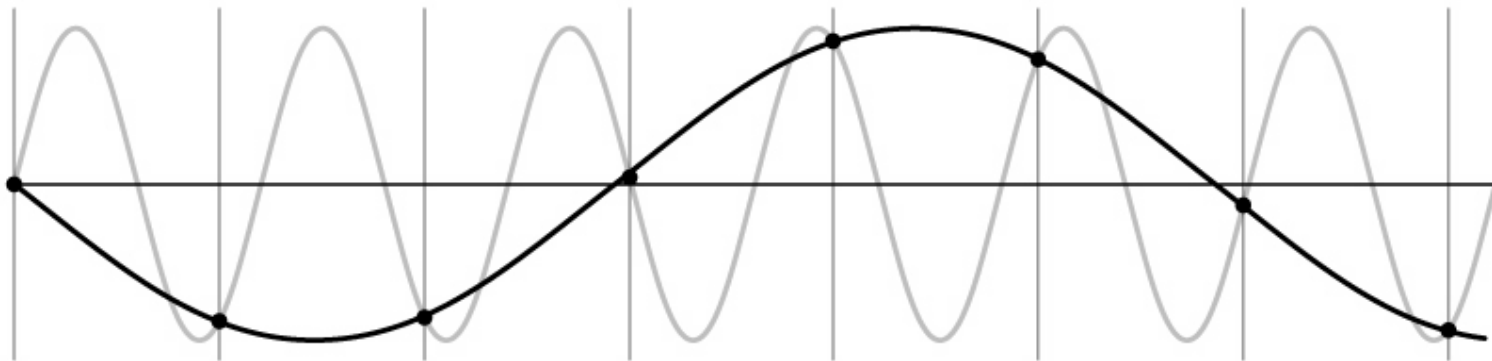
# Undersampling

- What if we “missed” things between the samples?
- Simple example: undersampling a sine wave
  - unsurprising result: information is lost



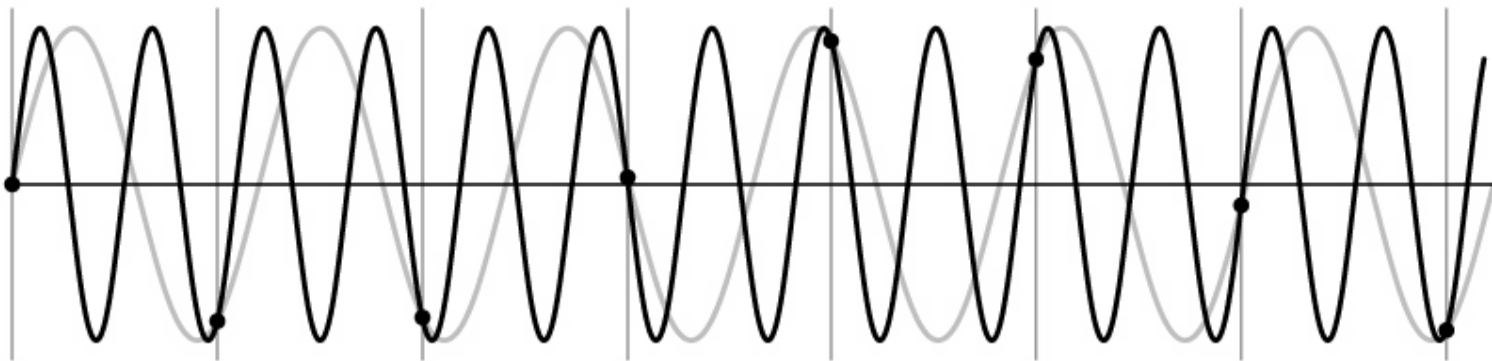
# Undersampling

- What if we “missed” things between the samples?
- Simple example: undersampling a sine wave
  - unsurprising result: information is lost
  - surprising result: indistinguishable from lower frequency



# Undersampling

- What if we “missed” things between the samples?
- Simple example: undersampling a sine wave
  - unsurprising result: information is lost
  - surprising result: indistinguishable from lower frequency
  - also, was always indistinguishable from higher frequencies
  - aliasing: signals “traveling in disguise” as other frequencies



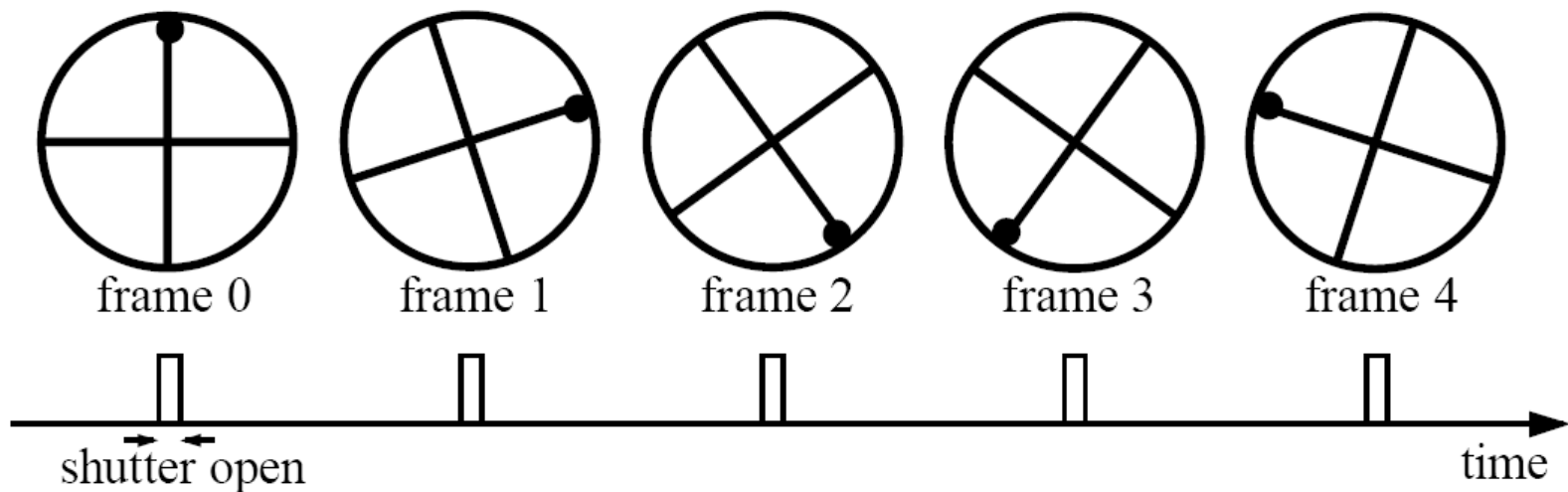
# Aliasing in video

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Imagine a spoked wheel moving to the right (rotating clockwise).

Mark wheel with dot so we can see what's happening.

If camera shutter is only open for a fraction of a frame time (frame time =  $1/30$  sec. for video,  $1/24$  sec. for film):

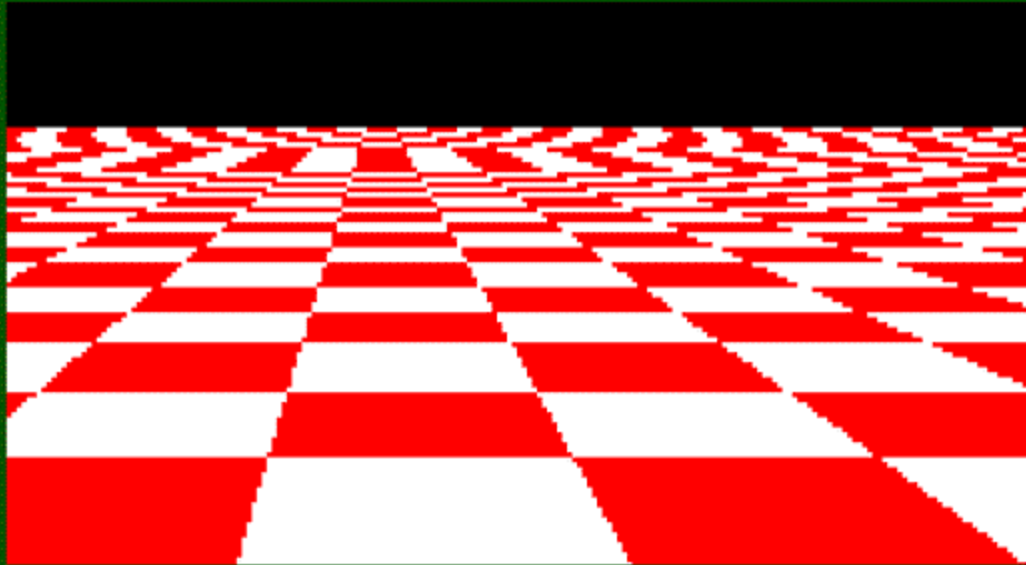


Without dot, wheel appears to be rotating slowly backwards!  
(counterclockwise)



# Aliasing in images

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**Disintegrating textures**

# Antialiasing

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What can we do about aliasing?

Sample more often

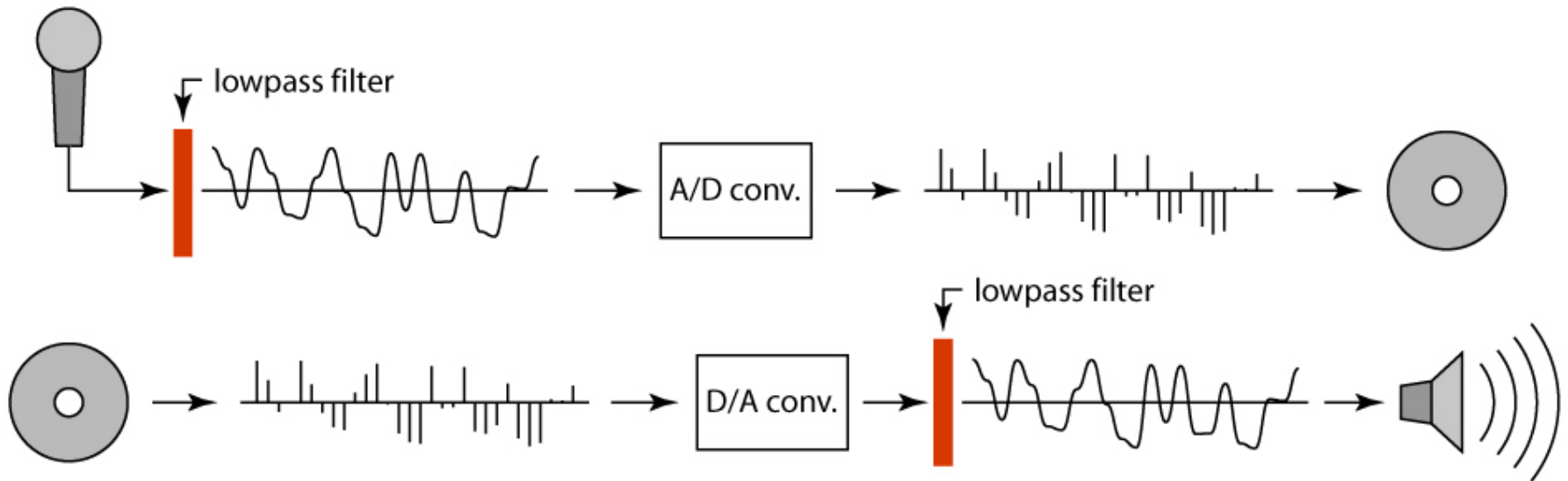
- Join the Mega-Pixel craze of the photo industry
- But this can't go on forever

Make the signal less “wiggly”

- Get rid of some high frequencies
- Will lose information
- But it's better than aliasing

# Preventing aliasing

- Introduce lowpass filters:
  - remove high frequencies leaving only safe, low frequencies
  - choose lowest frequency in reconstruction (disambiguate)

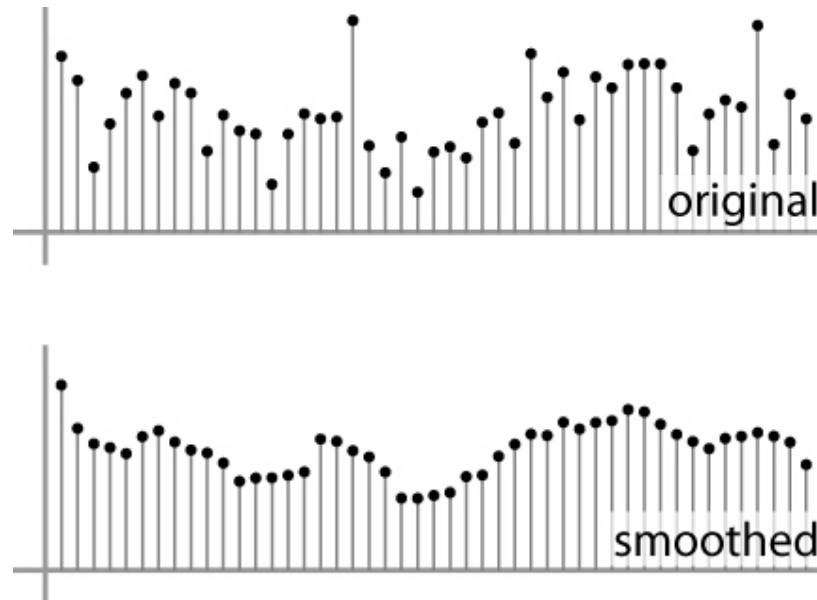


# Linear filtering: a key idea

- Transformations on signals; e.g.:
  - bass/treble controls on stereo
  - blurring/sharpening operations in image editing
  - smoothing/noise reduction in tracking
- Key properties
  - linearity:  $\text{filter}(f + g) = \text{filter}(f) + \text{filter}(g)$
  - shift invariance: behavior invariant to shifting the input
    - delaying an audio signal
    - sliding an image around
- Can be modeled mathematically by *convolution*

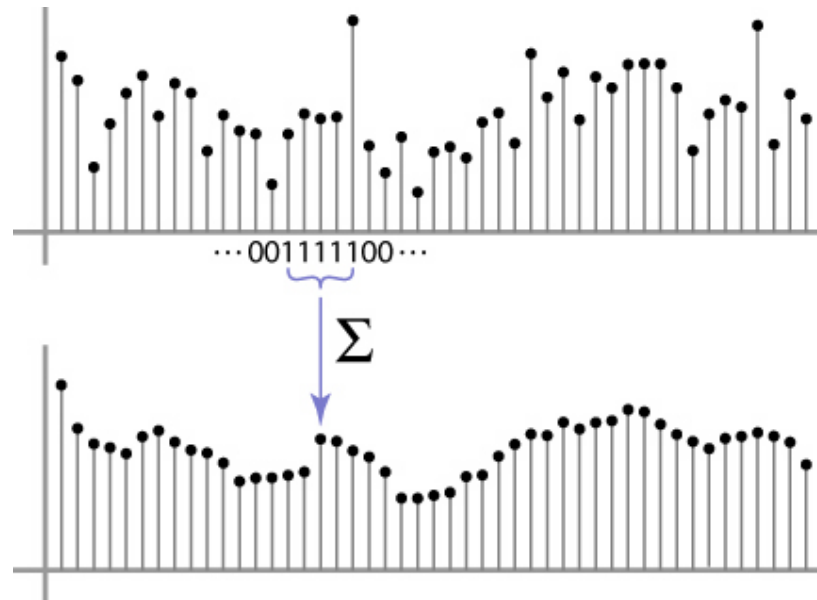
# Moving Average

- basic idea: define a new function by averaging over a sliding window
- a simple example to start off: smoothing



# Moving Average

- Can add weights to our moving average
- *Weights* [..., 0, 1, 1, 1, 1, 1, 0, ...] / 5



# In 2D: box filter

$$\frac{1}{9} h[\cdot, \cdot]$$

1	1	1
1	1	1
1	1	1



# Image filtering

$$h[\cdot, \cdot] \frac{1}{9}$$

1	1	1
1	1	1
1	1	1

$f[\cdot, \cdot]$

0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0	0
0	0	0	90	90	90	90	90	0	0	0
0	0	0	90	90	90	90	90	0	0	0
0	0	0	90	0	90	90	90	0	0	0
0	0	0	90	90	90	90	90	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0

$g[\cdot, \cdot]$


$$g[m, n] = \sum_{k, l} h[k, l] f[m + k, n + l]$$





# Image filtering

$$h[\cdot, \cdot] \frac{1}{9}$$

1	1	1
1	1	1
1	1	1

$f[\cdot, \cdot]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$g[\cdot, \cdot]$

	0	10							

$$g[m, n] = \sum_{k, l} h[k, l] f[m + k, n + l]$$

# Image filtering

$$h[\cdot, \cdot] \frac{1}{9}$$

1	1	1
1	1	1
1	1	1

$f[\cdot, \cdot]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$g[\cdot, \cdot]$

	0	10	20						

# Image filtering

$$h[\cdot, \cdot] \frac{1}{9}$$

1	1	1
1	1	1
1	1	1

$f[\cdot, \cdot]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$g[\cdot, \cdot]$

	0	10	20	30					

# Image filtering

$$h[\cdot, \cdot] \frac{1}{9}$$

1	1	1
1	1	1
1	1	1

$f[\cdot, \cdot]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$g[\cdot, \cdot]$

	0	10	20	30	30				



$$h[\cdot, \cdot]^{\frac{1}{9}}$$

$$f[\cdot, \cdot]$$
[illegible]
$$g[\cdot, \cdot]$$
[illegible]

# Image filtering

$$h[\cdot, \cdot] \frac{1}{9}$$

1	1	1
1	1	1
1	1	1

$f[\cdot, \cdot]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$g[\cdot, \cdot]$

	0	10	20	30	30				
						?			
				50					

# Image filtering

$$h[\cdot, \cdot] \quad \frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$f[\cdot, \cdot]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$g[\cdot, \cdot]$

	0	10	20	30	30	30	20	10	
	0	20	40	60	60	60	40	20	
	0	30	60	90	90	90	60	30	
	0	30	50	80	80	90	60	30	
	0	30	50	80	80	90	60	30	
	0	20	30	50	50	60	40	20	
	10	20	30	30	30	30	20	10	
	10	10	10	0	0	0	0	0	

$$g[m, n] = \sum_{k, l} h[k, l] f[m + k, n + l]$$

# Box Filter

What does it do?

- Replaces each pixel with an average of its neighborhood
- Achieve smoothing effect (remove sharp features)

$$\frac{1}{9} h[\cdot, \cdot]$$

1	1	1
1	1	1
1	1	1

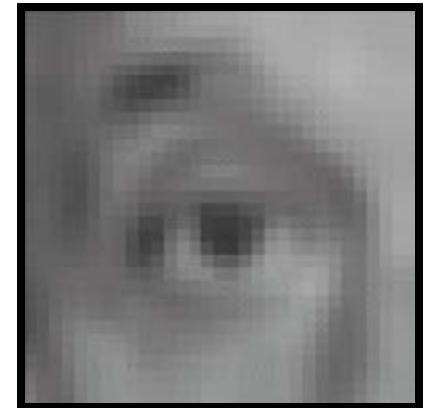


# Linear filters: examples



Original

$$\frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} =$$



Blur (with a mean filter)

# Cross-correlation

Let  $F$  be the image,  $H$  be the kernel (of size  $2k+1 \times 2k+1$ ), and  $G$  be the output image

$$G[i, j] = \sum_{u=-k}^k \sum_{v=-k}^k H[u, v] F[i + u, j + v]$$

This is called a **cross-correlation** operation:

$$G = H \otimes F$$

- Can think of as a “dot product” between local neighborhood and kernel for each pixel

# Practice with linear filters



Original

0	0	0
0	1	0
0	0	0

?

# Practice with linear filters



Original

0	0	0
0	1	0
0	0	0



Filtered  
(no change)

# Practice with linear filters



Original

0	0	0
0	0	1
0	0	0

?

# Practice with linear filters



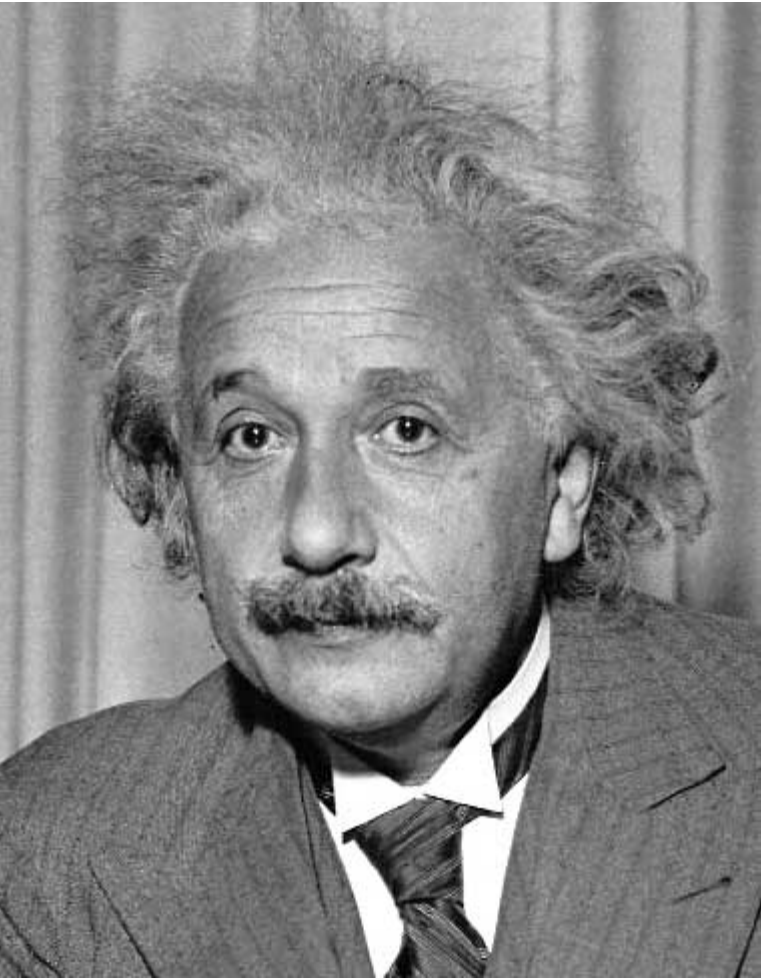
Original

0	0	0
0	0	1
0	0	0



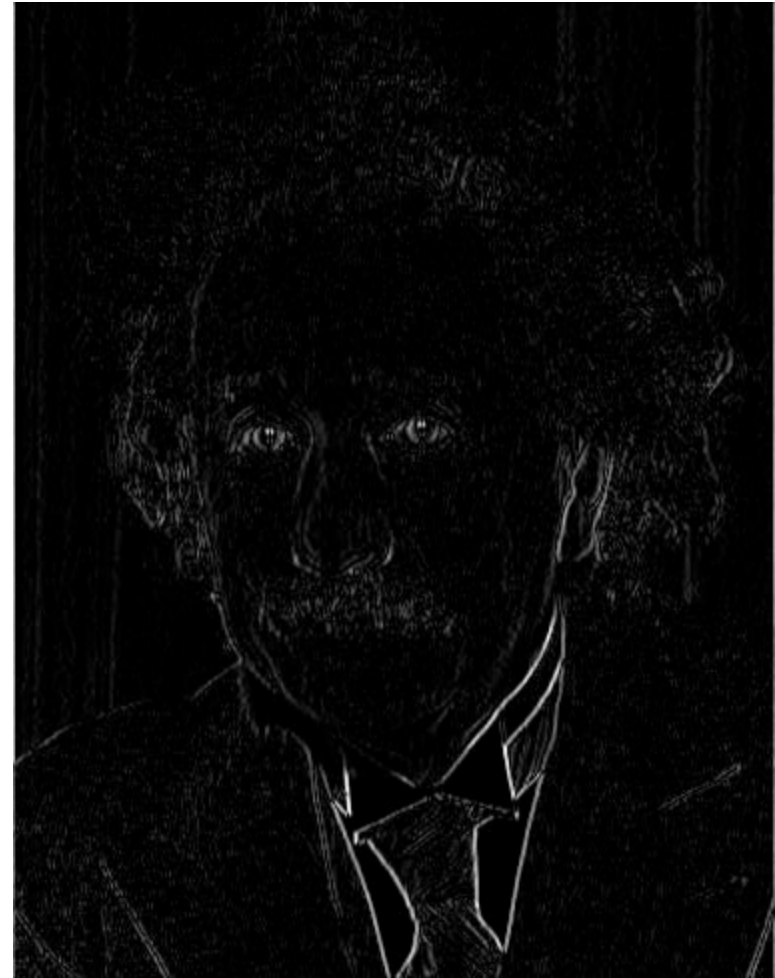
Shifted left  
By 1 pixel

# Other filters



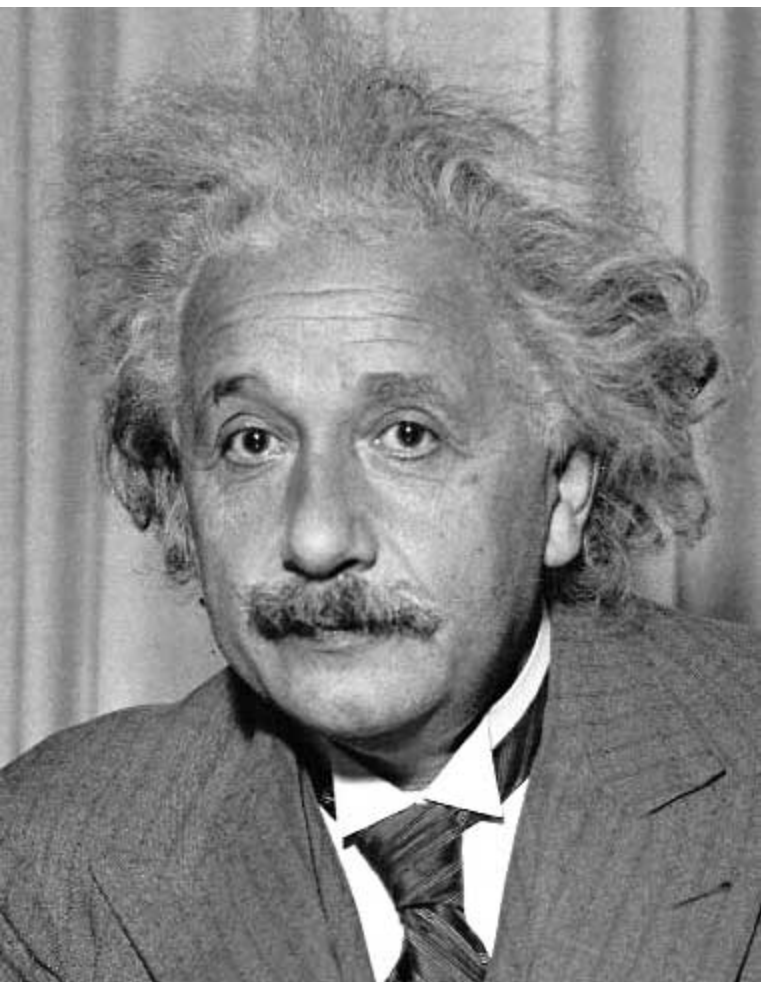
1	0	-1
2	0	-2
1	0	-1

Sobel



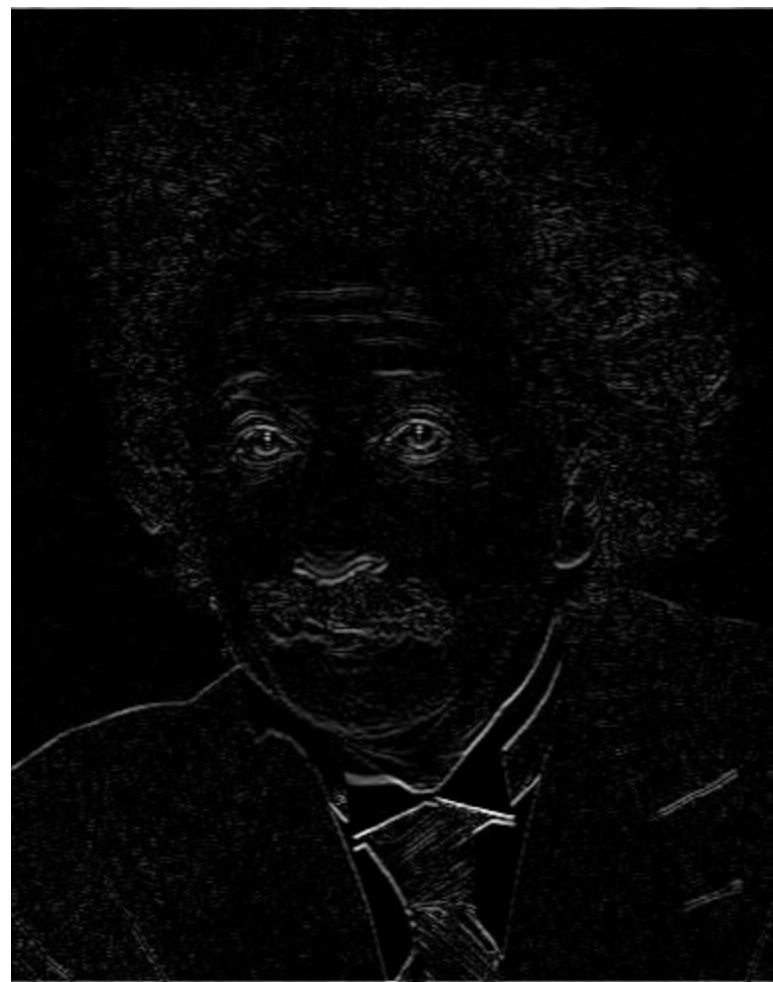
Vertical Edge  
(absolute value)

# Other filters



1	2	1
0	0	0
-1	-2	-1

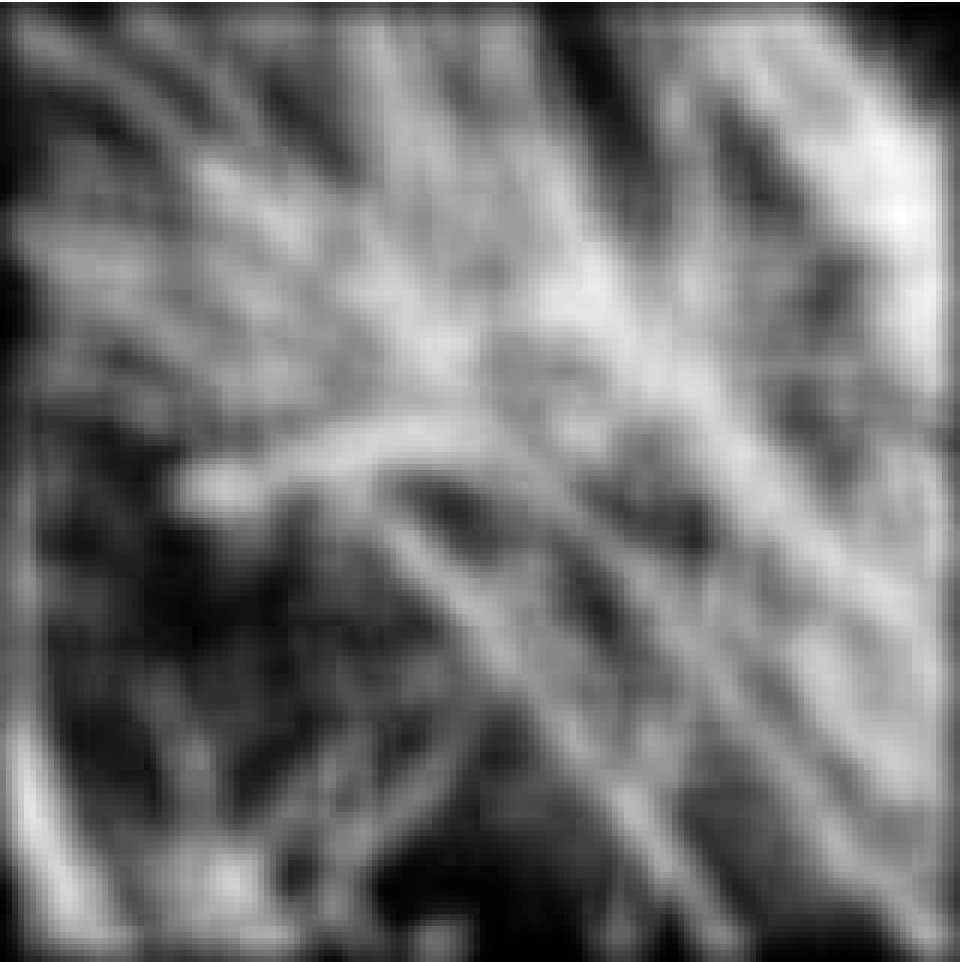
Sobel



Horizontal Edge  
(absolute value)

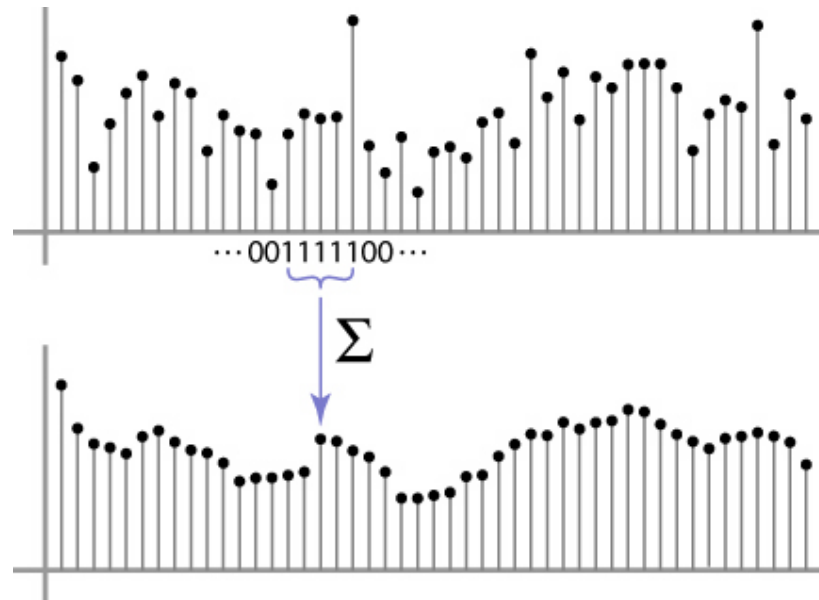


# Back to the box filter



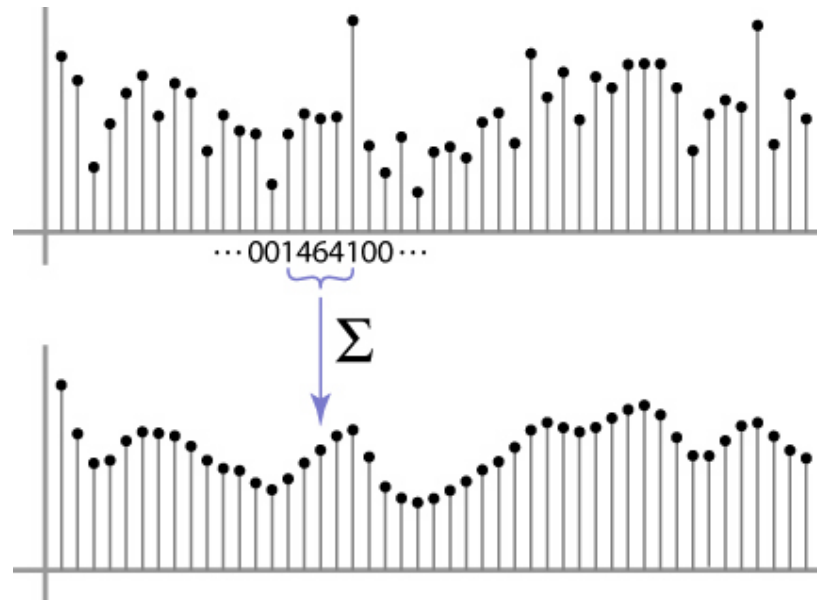
# Moving Average

- Can add weights to our moving average
- *Weights* [..., 0, 1, 1, 1, 1, 1, 0, ...] / 5



# Weighted Moving Average

- bell curve (gaussian-like) weights [..., 1, 4, 6, 4, 1, ...]





# Moving Average In 2D

What are the weights  $H$ ?

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$F[x, y]$


$H[u, v]$

# Gaussian filtering

A Gaussian kernel gives less weight to pixels further from the center of the window

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

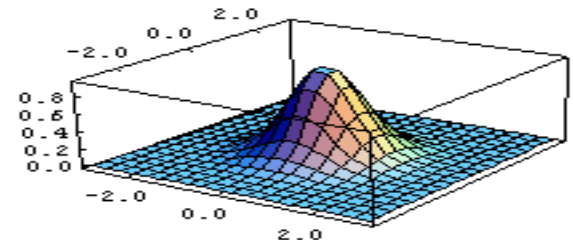
$F[x, y]$

1	2	1
2	4	2
1	2	1

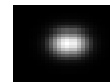
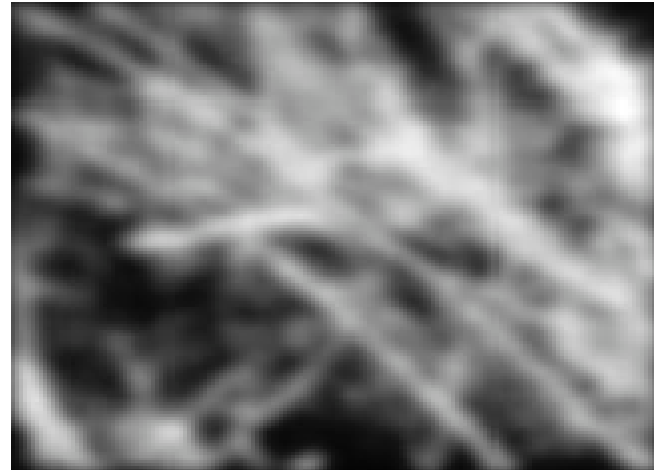
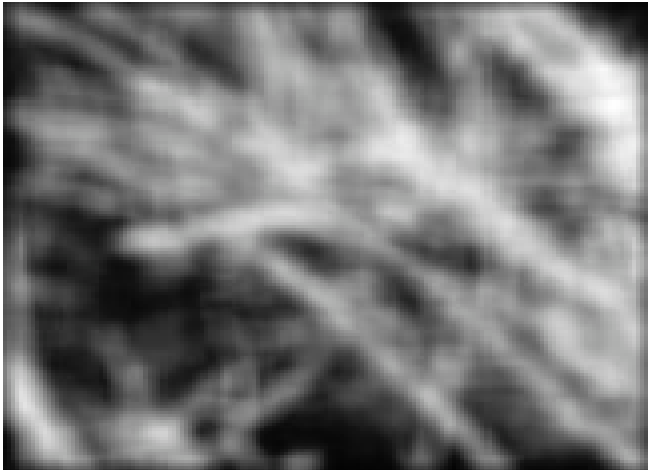
$H[u, v]$

$$h(u, v) = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{\sigma^2}}$$

This kernel is an approximation of a Gaussian function:

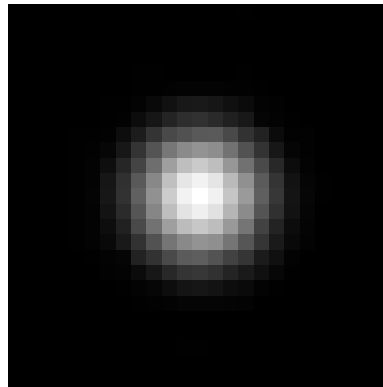
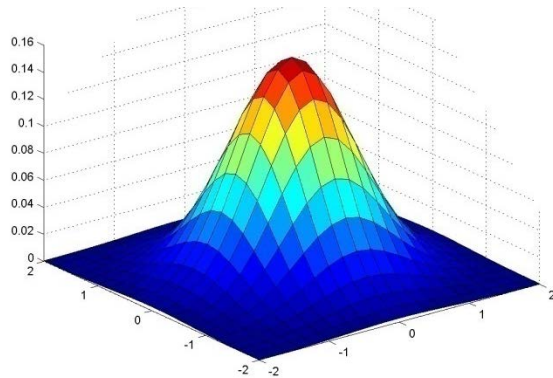


# Mean vs. Gaussian filtering



# Important filter: Gaussian

Weight contributions of neighboring pixels by nearness



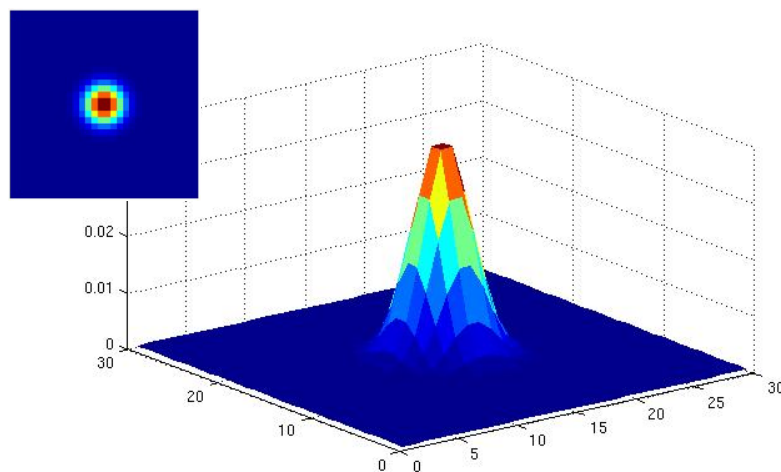
0.003	0.013	0.022	0.013	0.003
0.013	0.059	0.097	0.059	0.013
0.022	0.097	0.159	0.097	0.022
0.013	0.059	0.097	0.059	0.013
0.003	0.013	0.022	0.013	0.003

5 x 5,  $\sigma = 1$

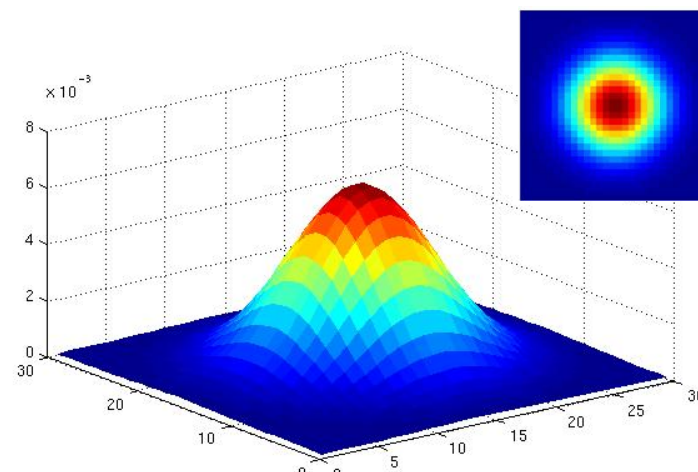
$$G_{\sigma} = \frac{1}{2\pi\sigma^2} e^{-\frac{(x^2+y^2)}{2\sigma^2}}$$

# Gaussian Kernel

$$G_{\sigma} = \frac{1}{2\pi\sigma^2} e^{-\frac{(x^2+y^2)}{2\sigma^2}}$$



$\sigma = 2$  with  $30 \times 30$   
kernel

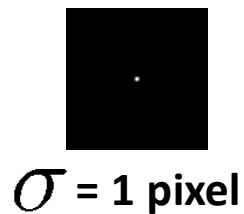


$\sigma = 5$  with  $30 \times 30$   
kernel

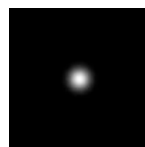
- Standard deviation  $\sigma$ : determines extent of smoothing



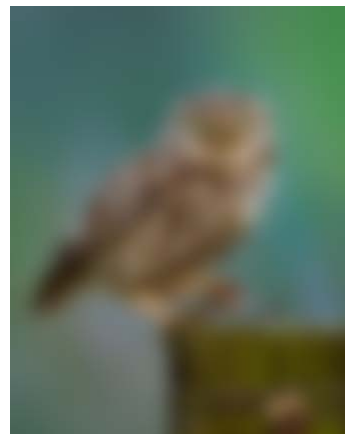
# Gaussian filters



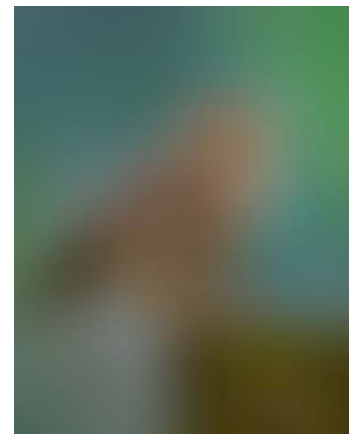
$\sigma = 1$  pixel



$\sigma = 5$  pixels



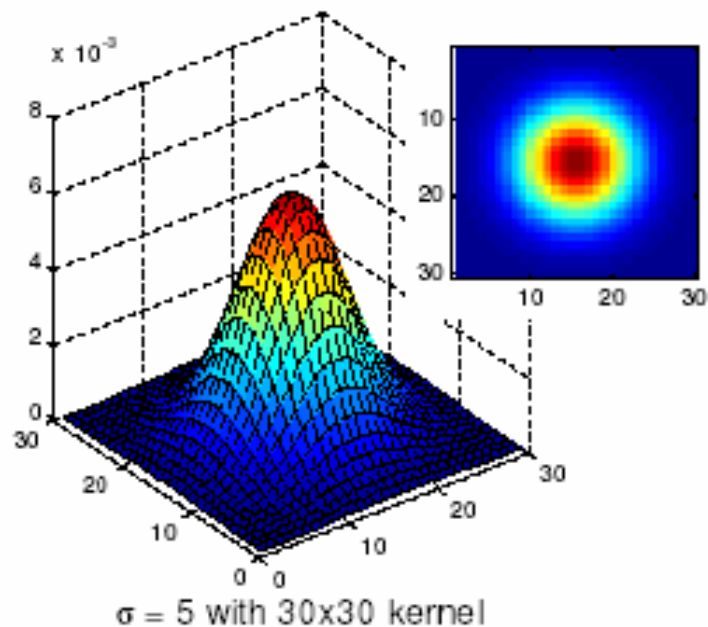
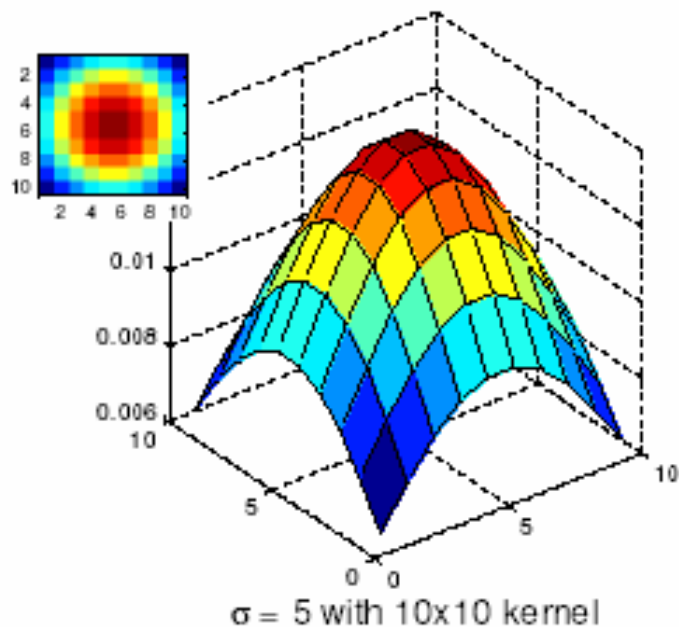
$\sigma = 10$  pixels



$\sigma = 30$  pixels

# Choosing kernel width

- The Gaussian function has infinite support, but discrete filters use finite kernels

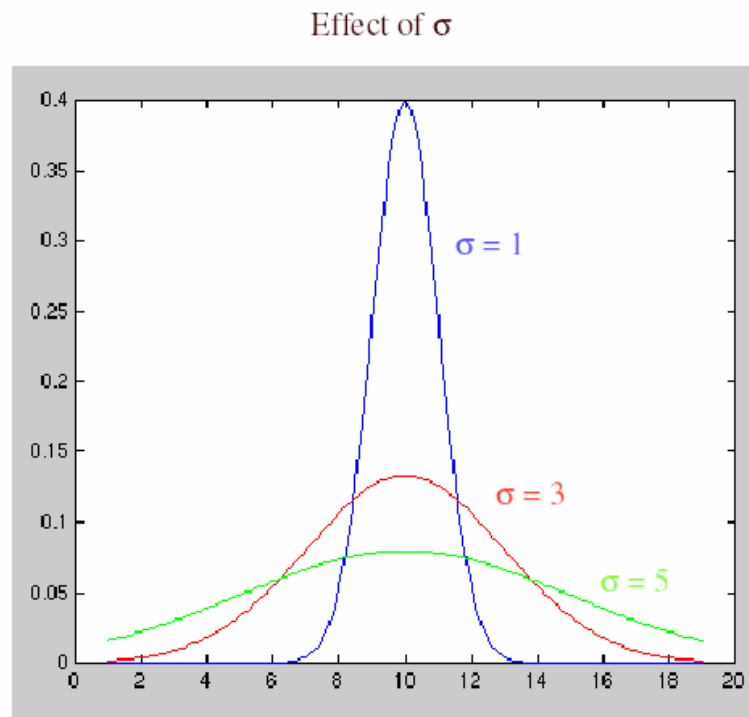


# Practical matters

## How big should the filter be?

Values at edges should be near zero

Rule of thumb for Gaussian: set filter half-width to about  $3\sigma$



# Cross-correlation vs. Convolution

---

**cross-correlation:**  $G = H \otimes F$

$$G[i, j] = \sum_{u=-k}^k \sum_{v=-k}^k H[u, v] F[i + u, j + v]$$

A **convolution** operation is a cross-correlation where the filter is flipped both horizontally and vertically before being applied to the image:

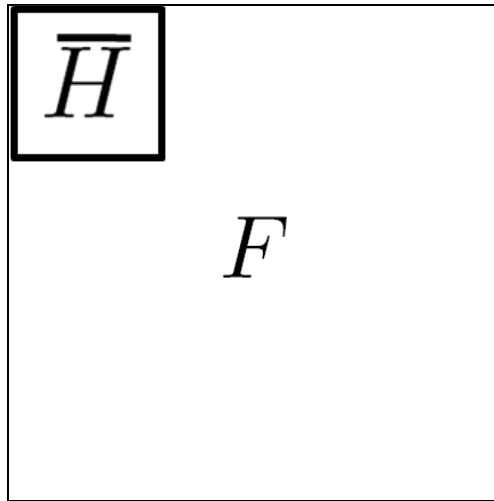
$$G[i, j] = \sum_{u=-k}^k \sum_{v=-k}^k H[u, v] F[i - u, j - v]$$

It is written:

$$G = H \star F$$

Convolution is **commutative** and **associative**

# Convolution

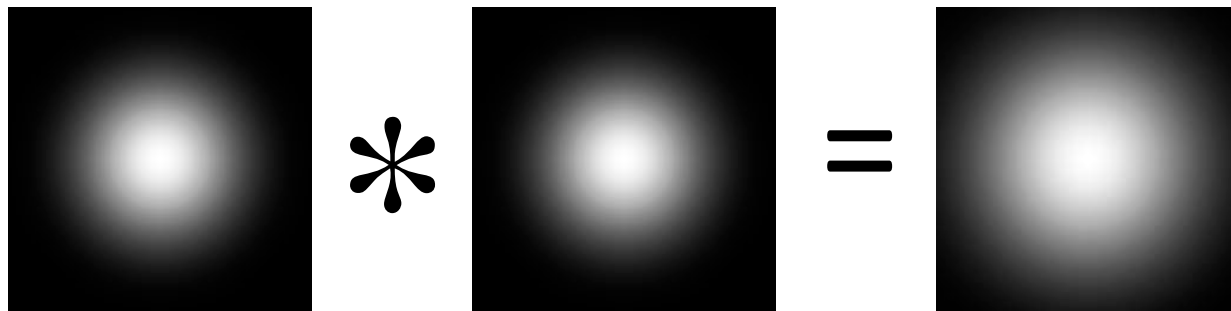


# Convolution is nice!

- Notation:  $b = c \star a$
- Convolution is a multiplication-like operation
  - commutative  $a \star b = b \star a$
  - associative  $a \star (b \star c) = (a \star b) \star c$
  - distributes over addition  $a \star (b + c) = a \star b + a \star c$
  - scalars factor out  $\alpha a \star b = a \star \alpha b = \alpha(a \star b)$
  - identity: unit impulse  $e = [\dots, 0, 0, 1, 0, 0, \dots]$ 
$$a \star e = a$$
- Conceptually no distinction between filter and signal
- Usefulness of associativity
  - often apply several filters one after another:  $((a \star b_1) \star b_2) \star b_3$
  - this is equivalent to applying one filter:  $a \star (b_1 \star b_2 \star b_3)$

# Gaussian and convolution

- Removes “high-frequency” components from the image (low-pass filter)
- Convolution with self is another Gaussian



- Convolving twice with Gaussian kernel of width  $\sigma$   
= convolving once with kernel of width  $\sigma\sqrt{2}$

# The Convolution Theorem

---

The greatest thing since sliced (banana) bread!

- The Fourier transform of the convolution of two functions is the product of their Fourier transforms

$$F[g * h] = F[g]F[h]$$

- The inverse Fourier transform of the product of two Fourier transforms is the convolution of the two inverse Fourier transforms

$$F^{-1}[gh] = F^{-1}[g] * F^{-1}[h]$$

- **Convolution** in spatial domain is equivalent to **multiplication** in frequency domain!

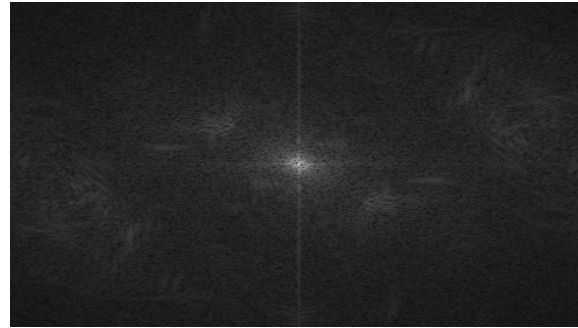


# 2D convolution theorem example

---



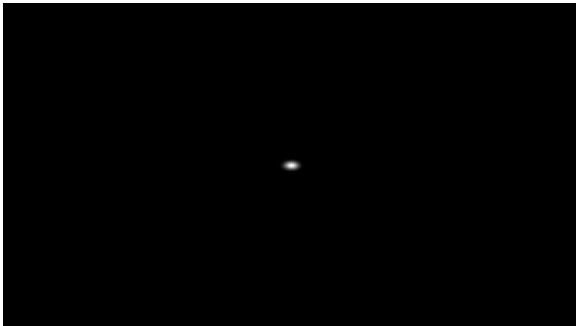
$f(x,y)$



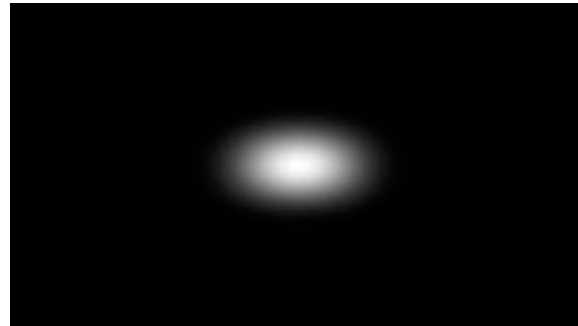
$|F(s_x, s_y)|$

\*

×



$h(x,y)$



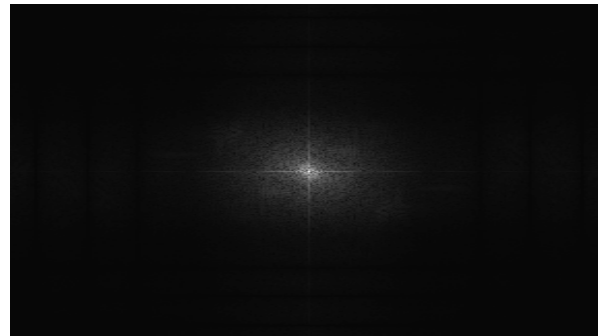
$|H(s_x, s_y)|$

⇓

⇓



$g(x,y)$

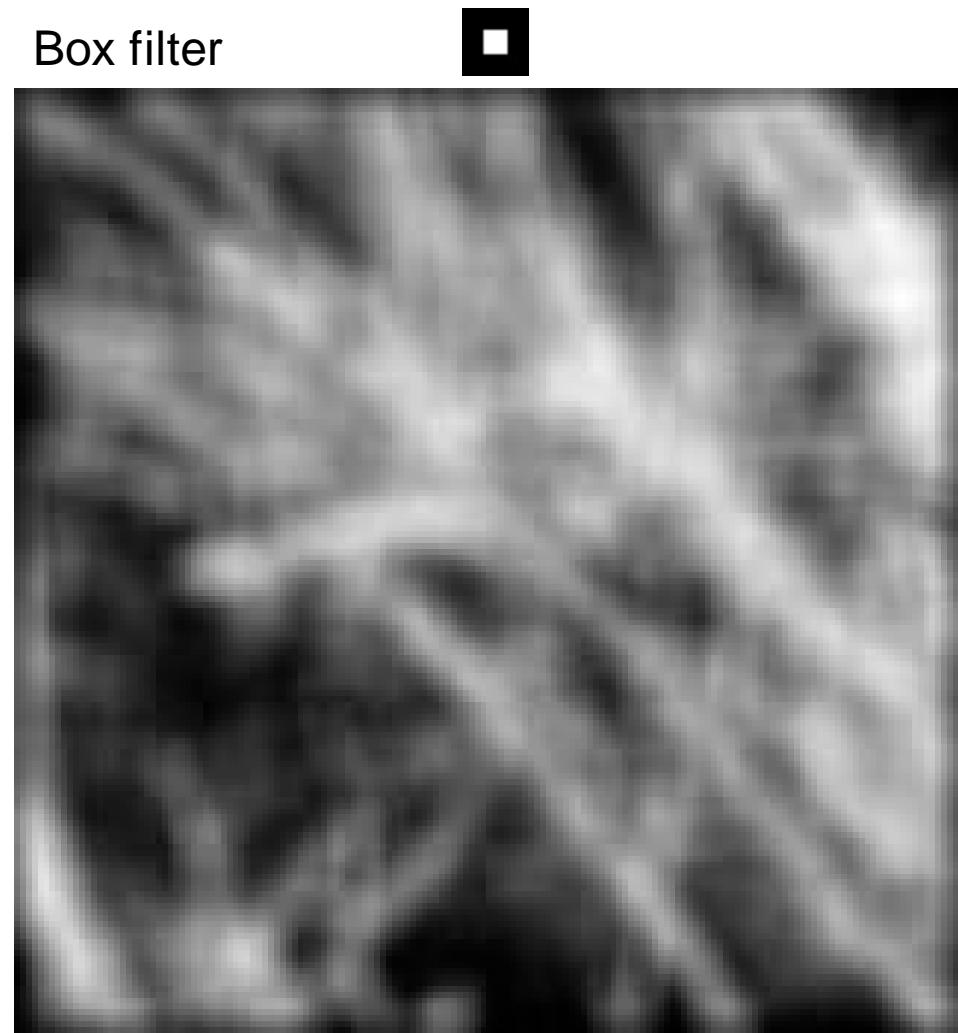
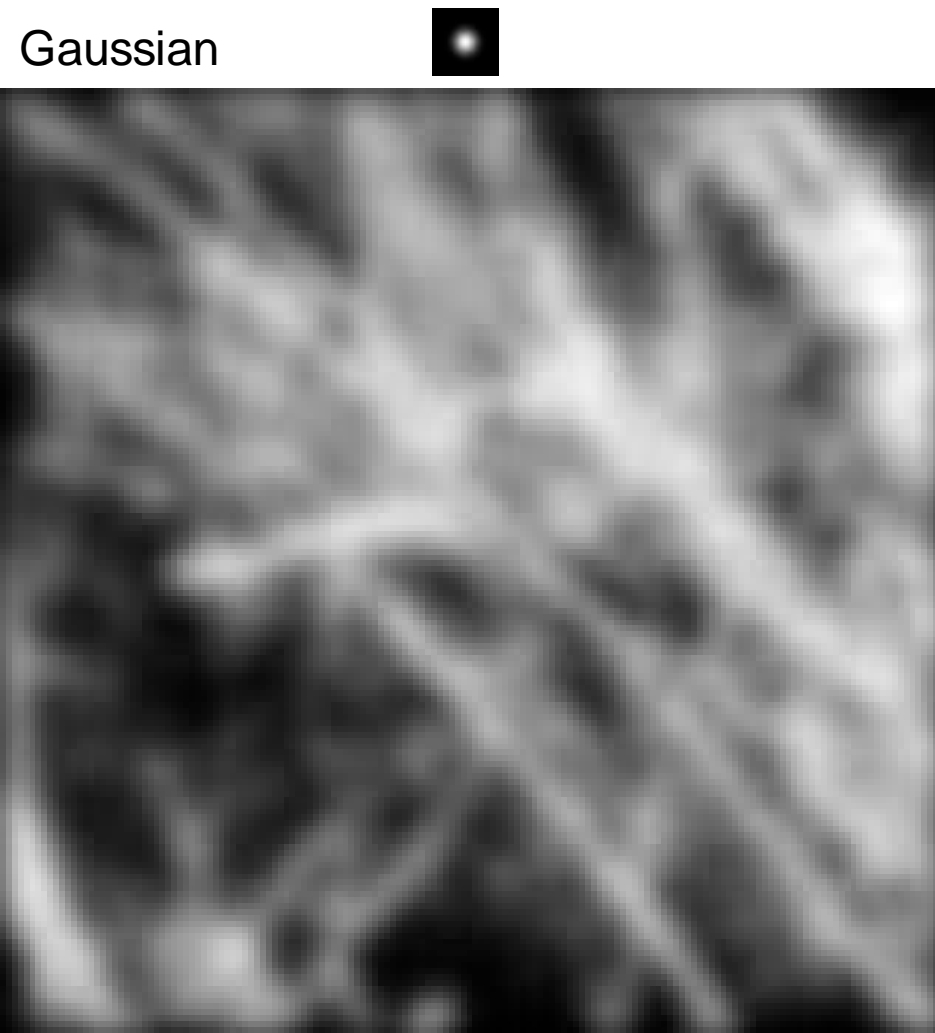


$|G(s_x, s_y)|$



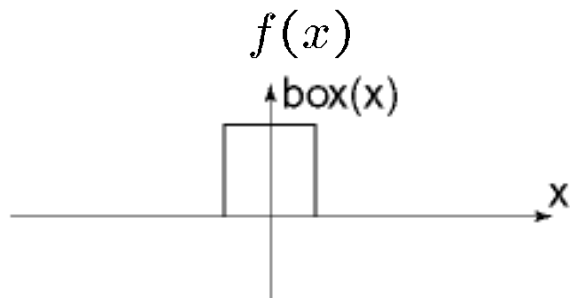
# Filtering

**Why does the Gaussian give a nice smooth image, but the square filter give edgy artifacts?**

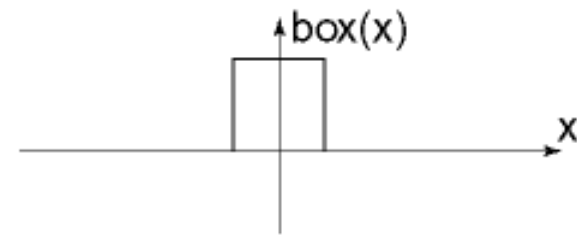
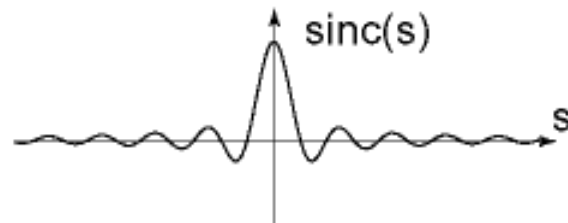
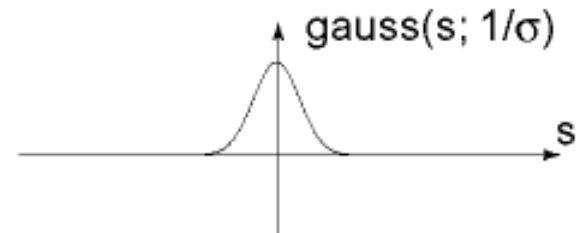
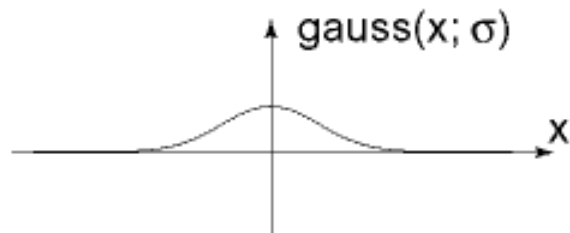
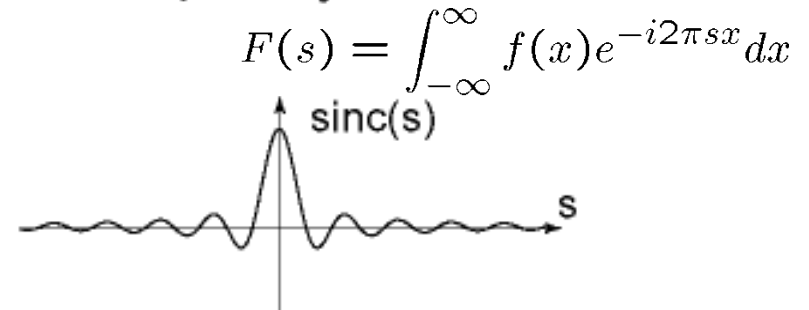


# Fourier Transform pairs

Spatial domain



Frequency domain



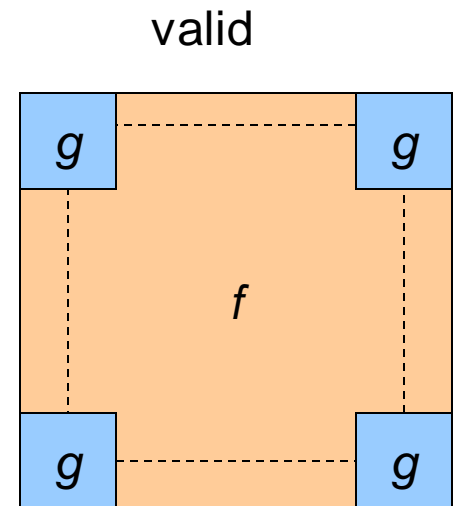
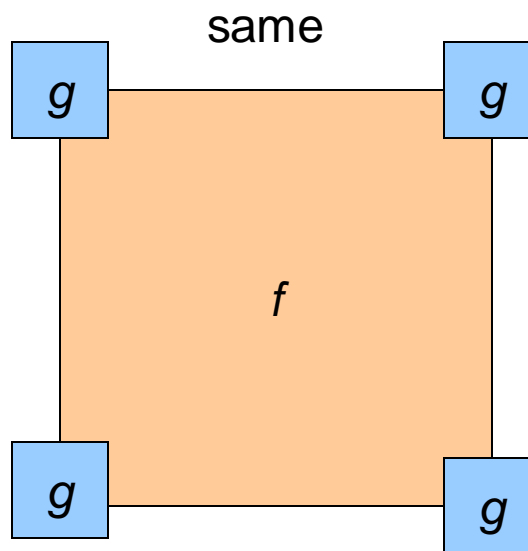
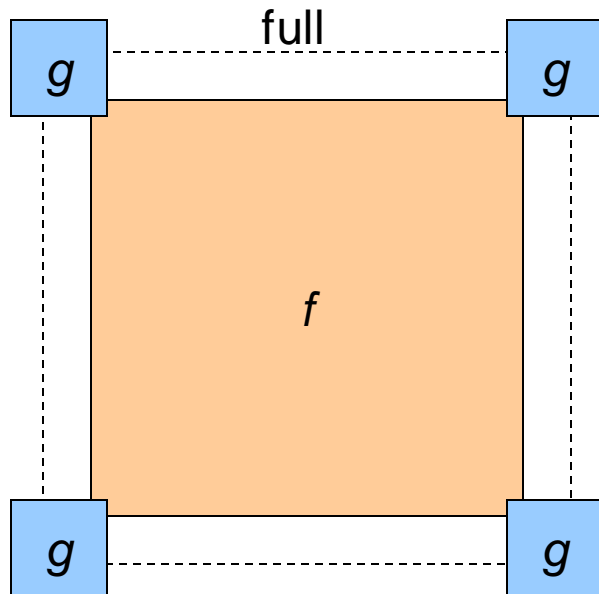
# Practical matters

---

What is the size of the output?

MATLAB: `filter2(g, f, shape)` or `conv2(g,f,shape)`

- *shape* = 'full': output size is sum of sizes of *f* and *g*
- *shape* = 'same': output size is same as *f*
- *shape* = 'valid': output size is difference of sizes of *f* and *g*

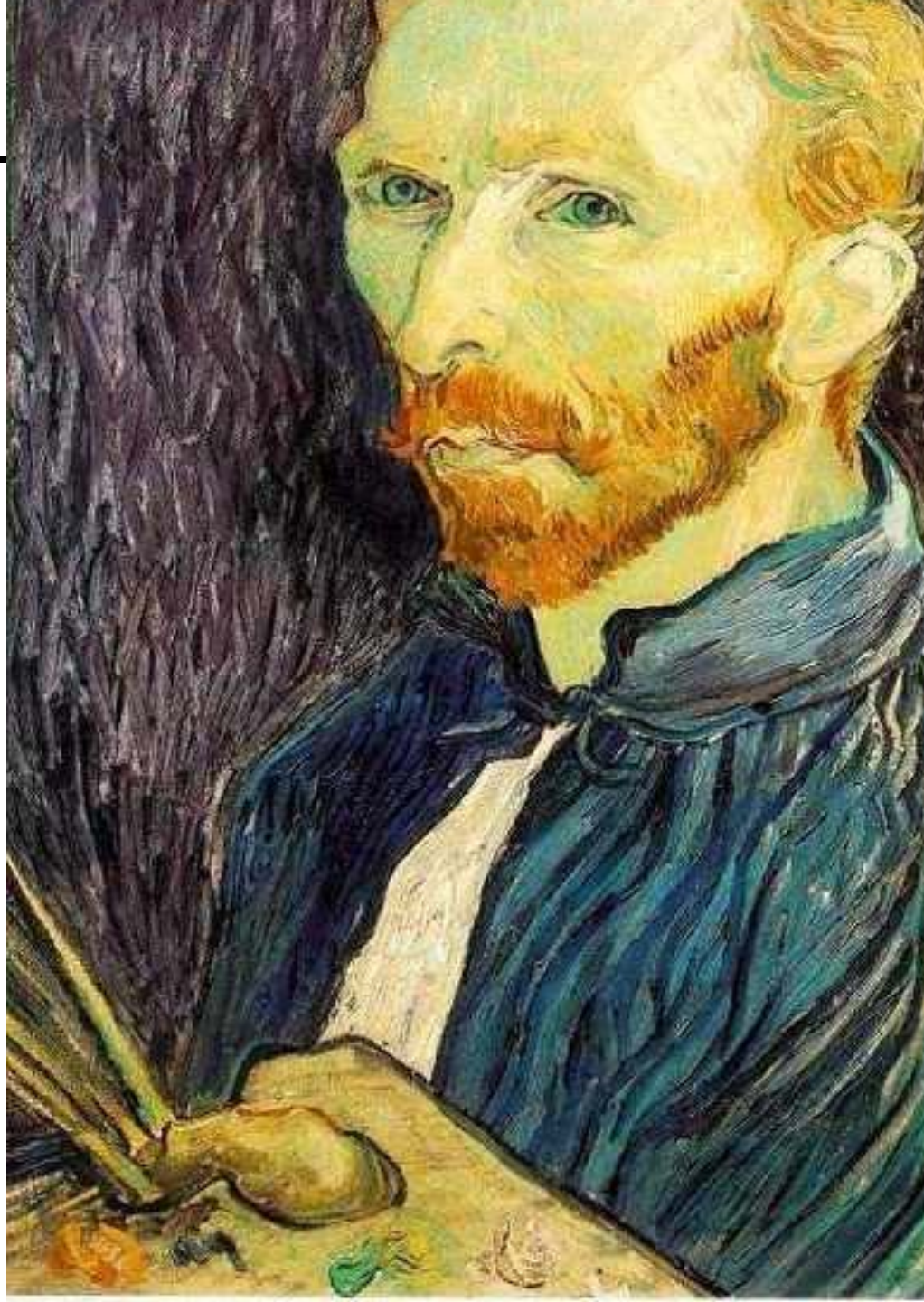


# Image half-sizing

---

This image is too big to fit on the screen. How can we reduce it?

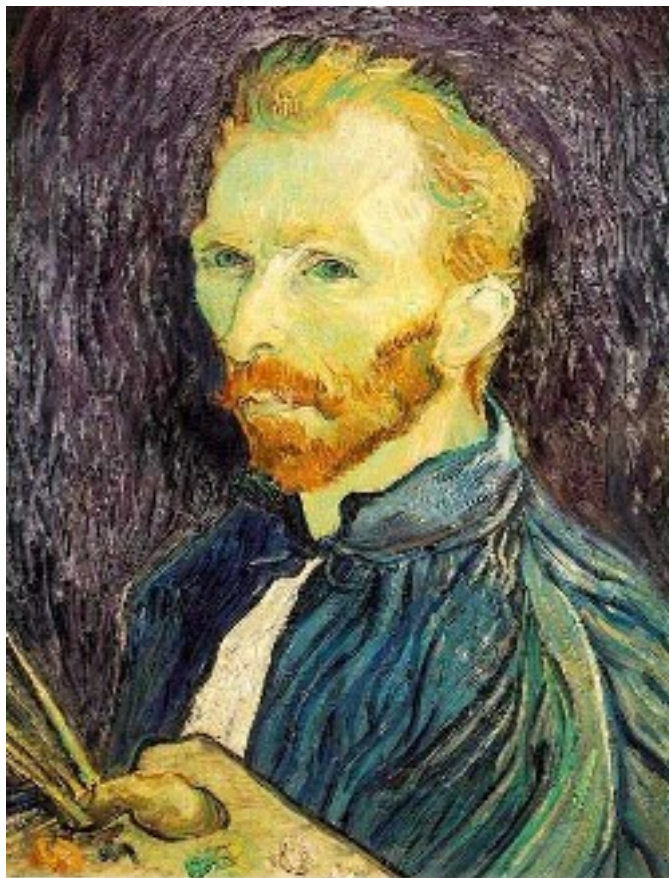
How to generate a half-sized version?





# Image sub-sampling

---



1/4

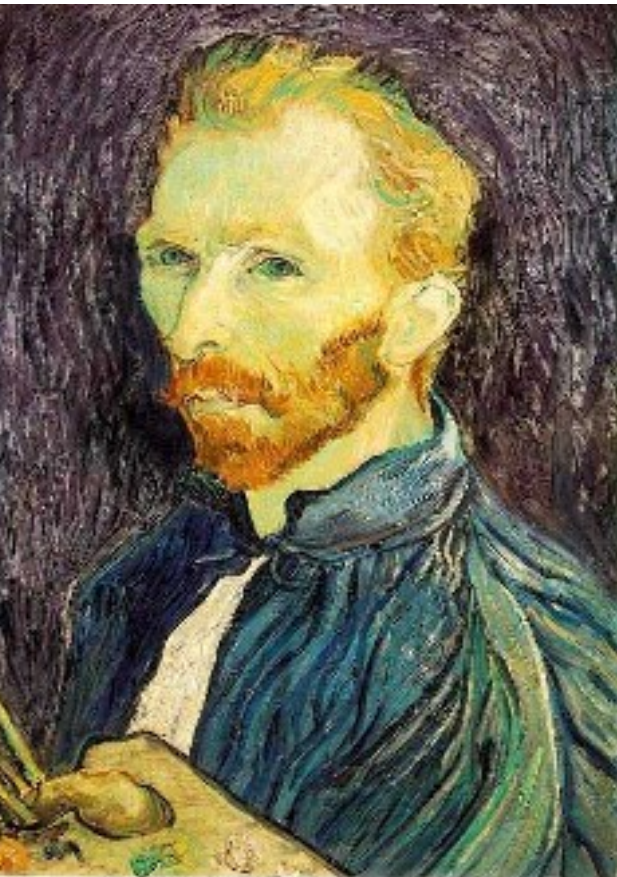


1/8

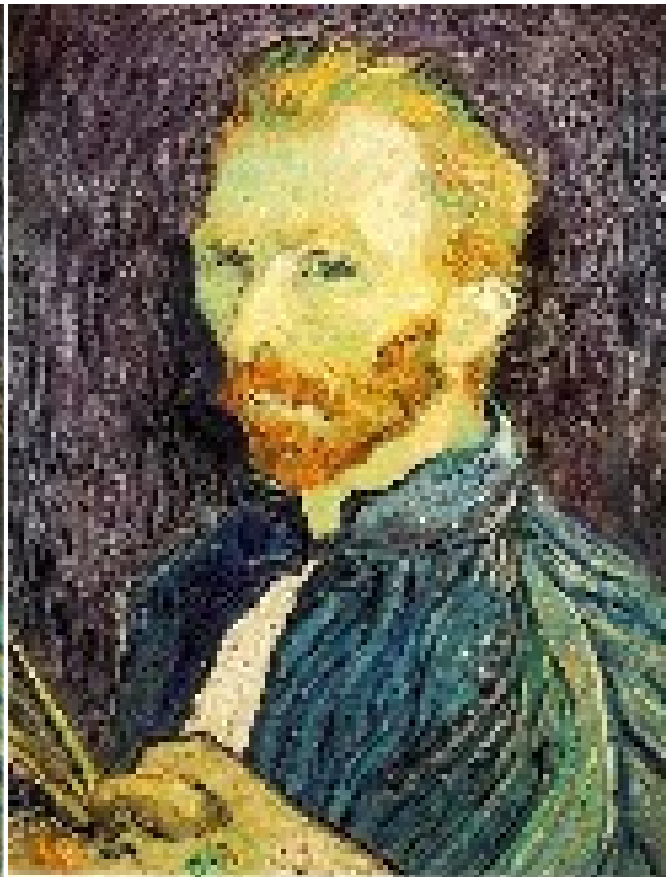
Throw away every other row and column to create a 1/2 size image  
- called *image sub-sampling*

# Image sub-sampling

---



1/2



1/4 (2x zoom)

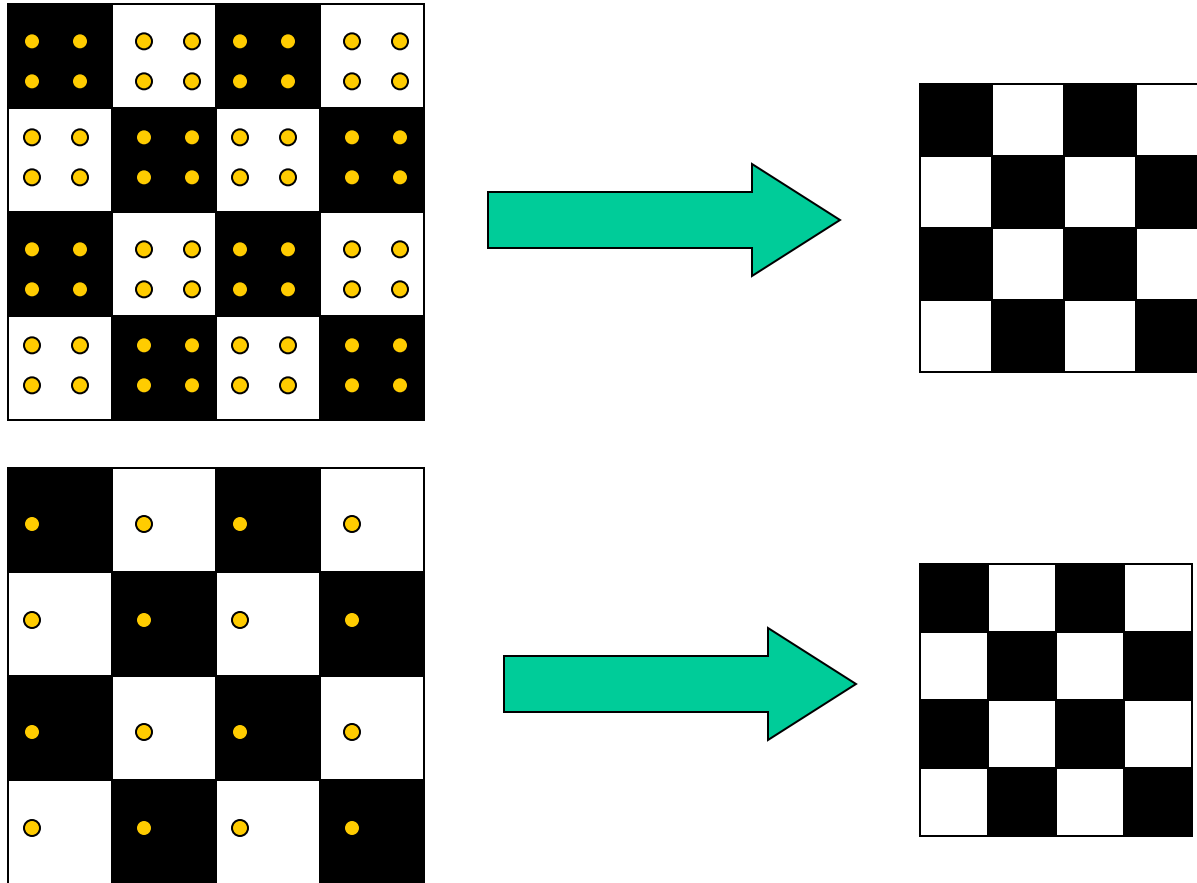


1/8 (4x zoom)

Aliasing! What do we do?

# Sampling an image

---

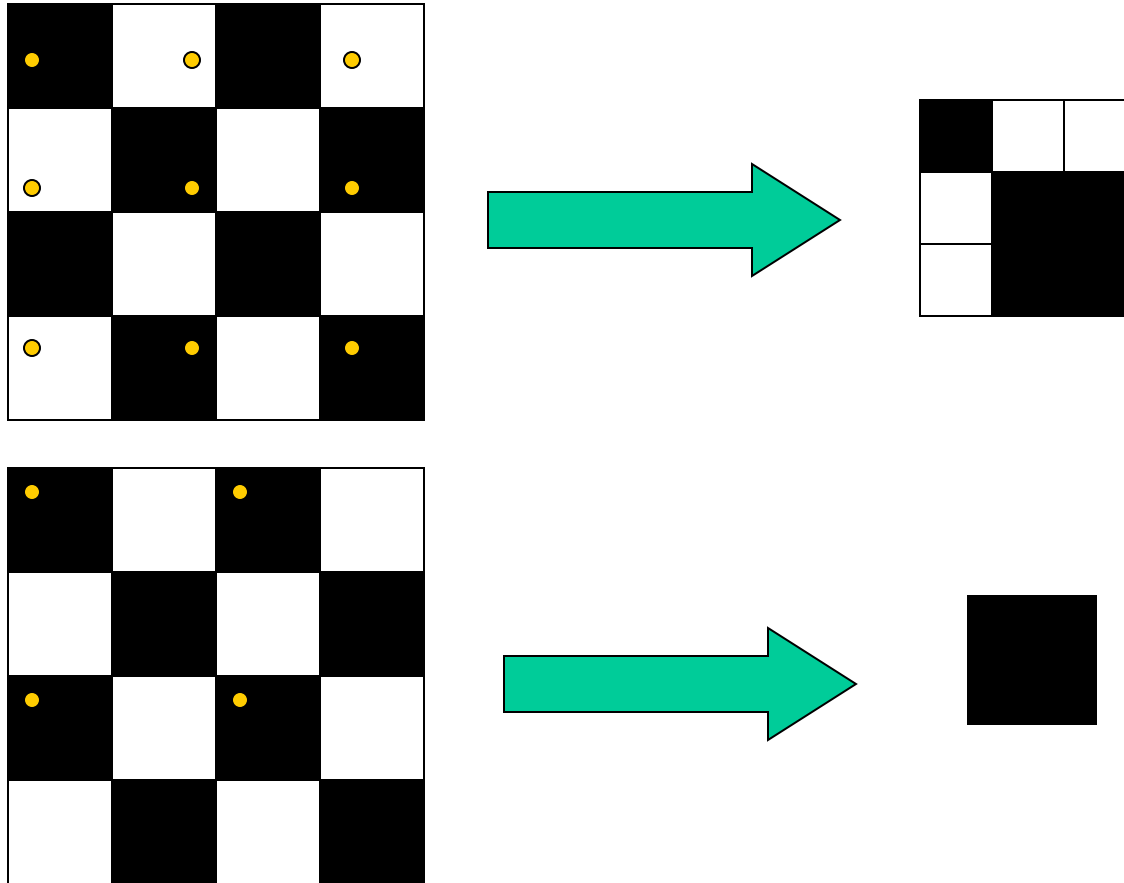


Examples of GOOD sampling



# Undersampling

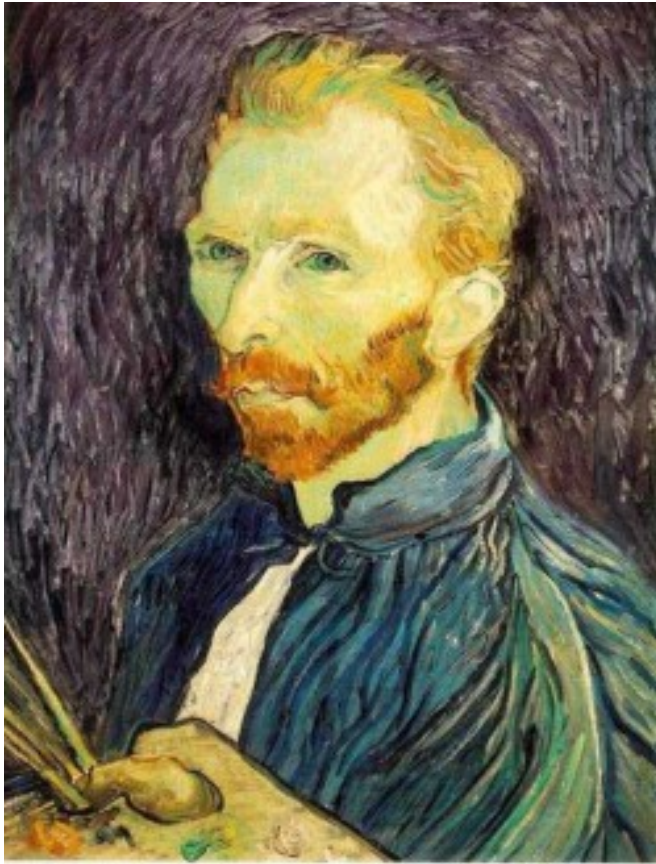
---



Examples of BAD sampling -> Aliasing

# Gaussian (lowpass) pre-filtering

---



Gaussian 1/2



G 1/4



G 1/8

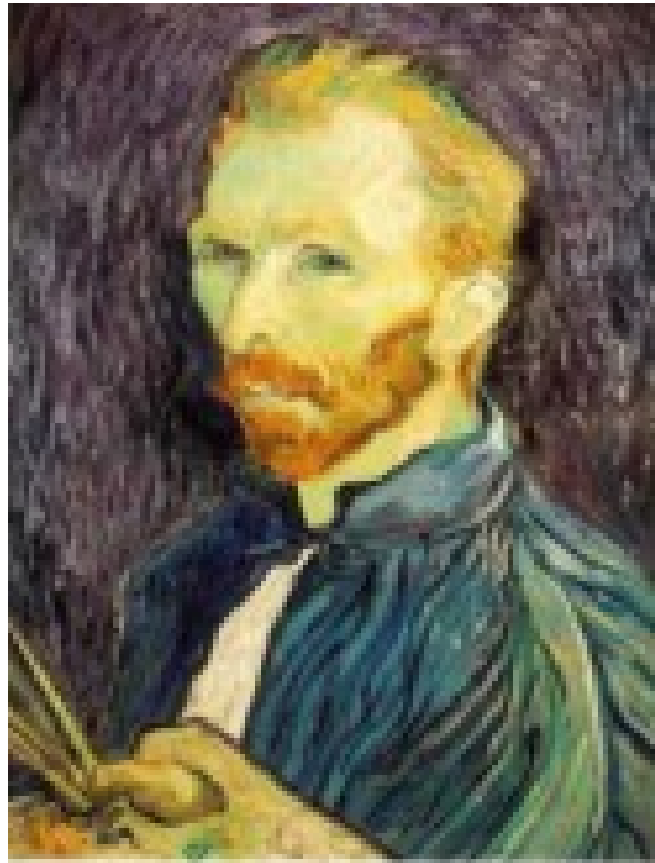
Solution: filter the image, *then* subsample

- Filter size should double for each  $\frac{1}{2}$  size reduction. Why?

# Subsampling with Gaussian pre-filtering



Gaussian  $1/2$



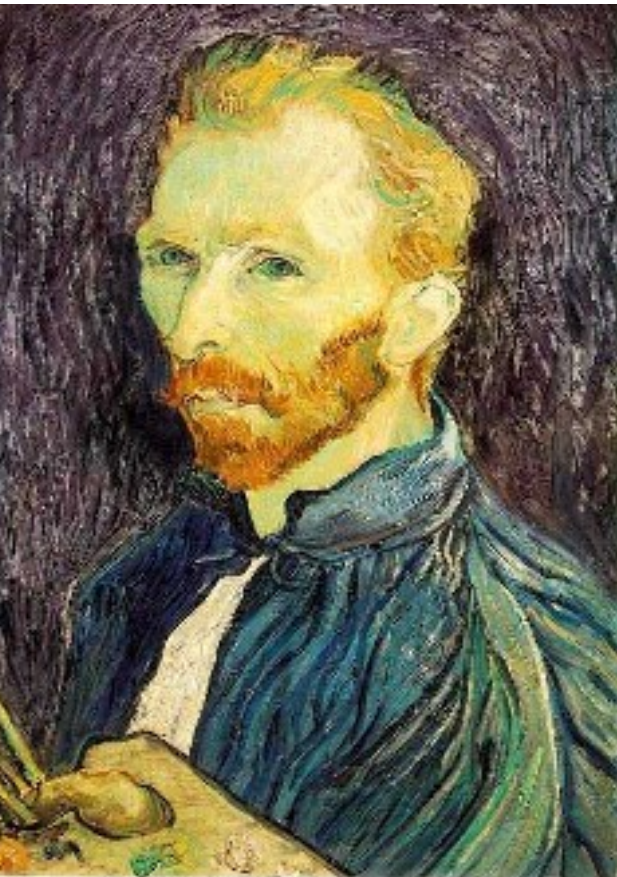
G  $1/4$



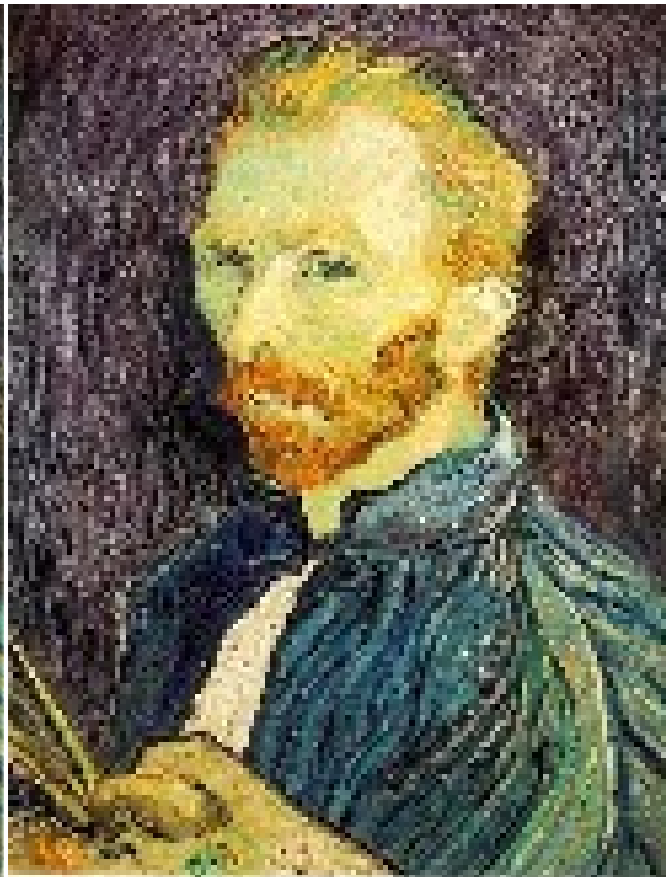
G  $1/8$

# Compare with...

---



1/2



1/4 (2x zoom)

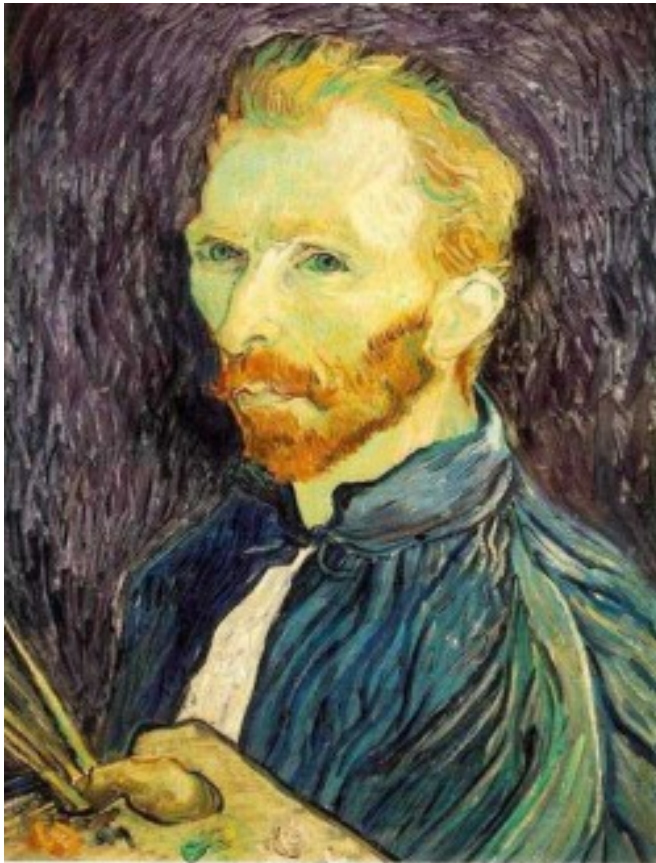


1/8 (4x zoom)



# Gaussian (lowpass) pre-filtering

---



Gaussian 1/2



G 1/4



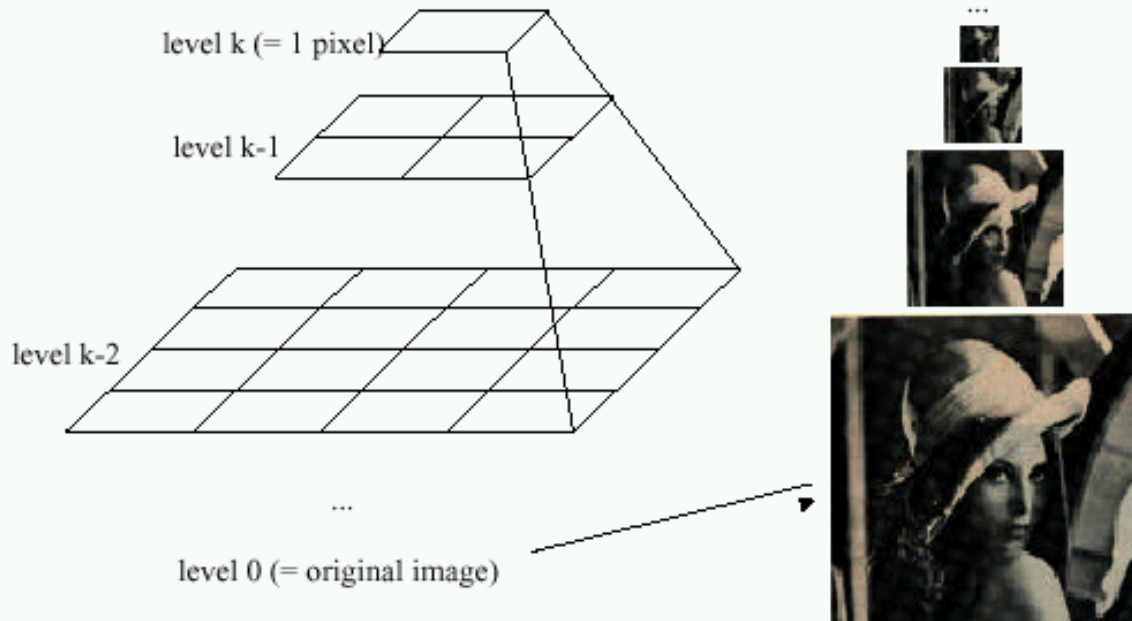
G 1/8

Solution: filter the image, *then* subsample

- Filter size should double for each  $\frac{1}{2}$  size reduction. Why?
- How can we speed this up?

# Image Pyramids

Idea: Represent  $N \times N$  image as a “pyramid” of  $1 \times 1, 2 \times 2, 4 \times 4, \dots, 2^k \times 2^k$  images (assuming  $N=2^k$ )



Known as a **Gaussian Pyramid** [Burt and Adelson, 1983]

- In computer graphics, a *mip map* [Williams, 1983]
- A precursor to *wavelet transform*



512

256

128

64

32

16

8

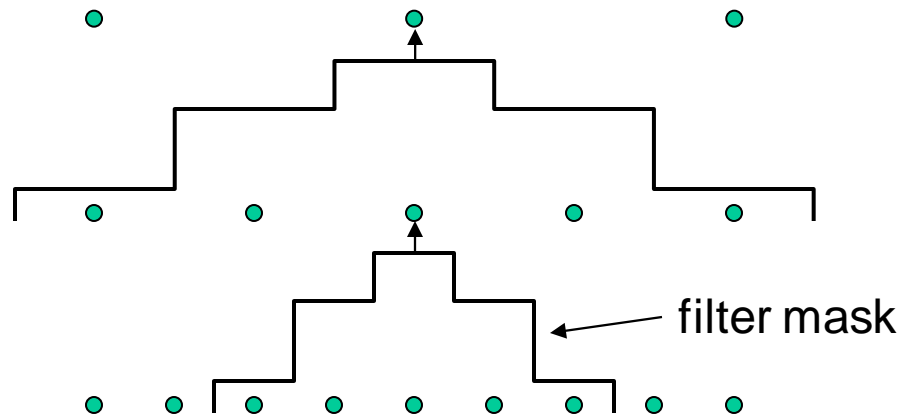
A bar in the big images is a hair on the zebra's nose; in smaller images, a stripe; in the smallest, the animal's nose



Figure from David Forsyth

# Gaussian pyramid construction

---



Repeat

- Filter
- Subsample

Until minimum resolution reached

- can specify desired number of levels (e.g., 3-level pyramid)

The whole pyramid is only  $\frac{4}{3}$  the size of the original image!



# What are they good for?

---

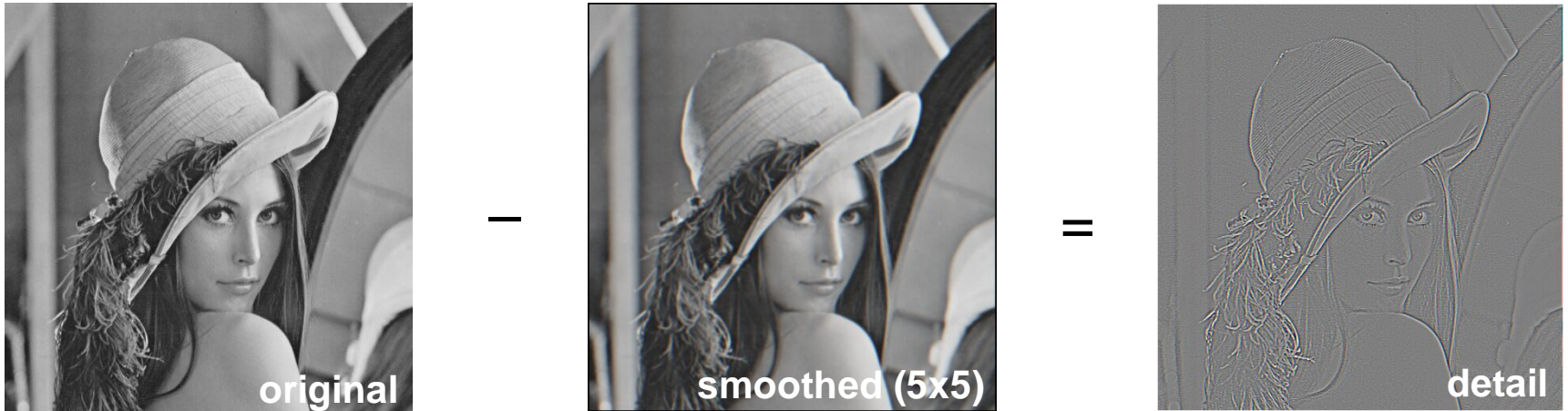
## Improve Search

- Search over translations
  - Classic coarse-to-fine strategy
- Search over scale
  - Template matching
  - E.g. find a face at different scales

# Sharpening

---

What does blurring take away?



Let's add it back:



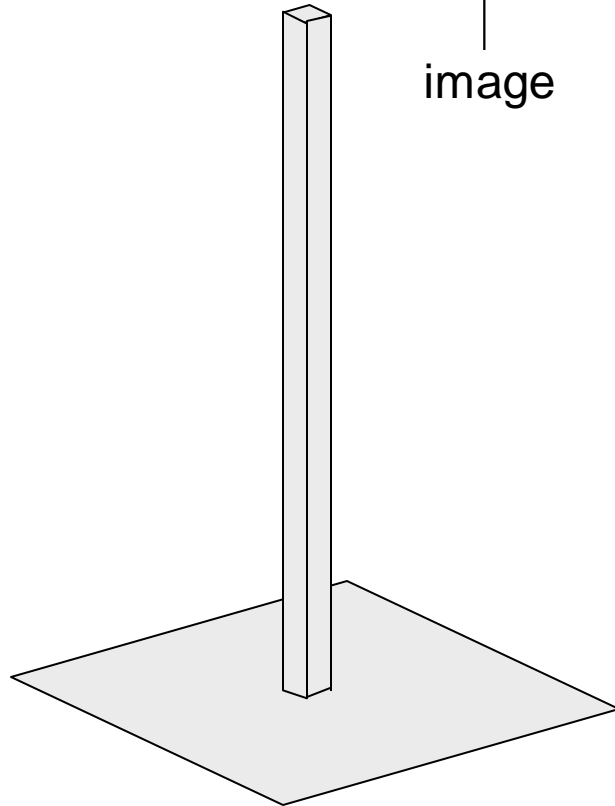
# Unsharp mask filter

$$f + \alpha(f - f * g) = (1 + \alpha)f - \alpha f * g = f * ((1 + \alpha)e - \alpha g)$$

image

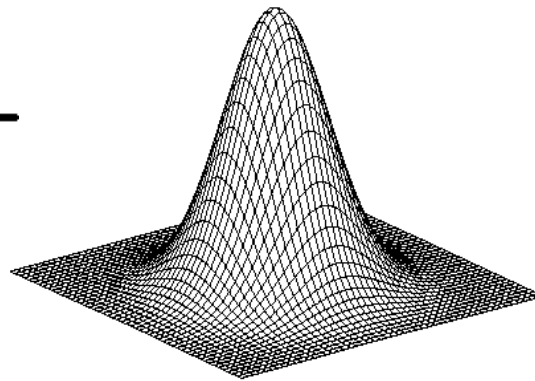
blurred  
image

unit impulse  
(identity)



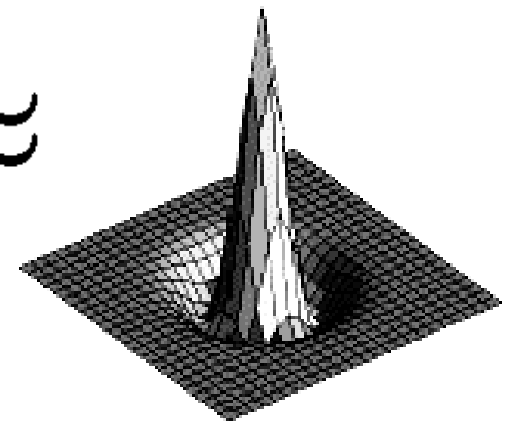
unit impulse

—



Gaussian

$\approx$



Laplacian of Gaussian