1. (a) Consider a CNOT gate whose control qubit (first input) is \( \frac{1}{\sqrt{2}} |0\rangle + \frac{1}{\sqrt{2}} |1\rangle \) and target qubit (second input) is \( \frac{1}{\sqrt{2}} |0\rangle - \frac{1}{\sqrt{2}} |1\rangle \). What are the states of the two output qubits? Repeat when the control qubit is \( |\rangle = \frac{1}{\sqrt{2}} |0\rangle - \frac{1}{\sqrt{2}} |1\rangle \). Can you describe the action of the CNOT gate on the first qubit when the target qubit is \( |\rangle = \frac{1}{\sqrt{2}} |0\rangle - \frac{1}{\sqrt{2}} |1\rangle \). What about when the target qubit is \( |\rangle \).

(b) This brings us to an important concept. We can consider what the action of a quantum gate looks like when we write the qubits in a different basis. Show that if the CNOT gate is applied in the Hadamard basis - i.e. apply the Hadamard gate to the inputs and outputs of the CNOT gate - then the result is a CNOT gate with the control and target qubit swapped.

2. Suppose \( |\rangle = \alpha_{00} |\phi^+\rangle + \alpha_{01} |\phi^-\rangle + \alpha_{10} |\psi^+\rangle + \alpha_{11} |\psi^-\rangle \), where \{ \( |\phi^+\rangle, |\phi^-\rangle, |\psi^+\rangle, |\psi^-\rangle \) \} is the Bell basis. Give a quantum circuit that maps \( |\rangle \) to \( \alpha_{00} |00\rangle + \alpha_{01} |01\rangle + \alpha_{10} |10\rangle + \alpha_{11} |11\rangle \).

3. (a) Create a ”copying circuit” that correctly copies the states \( |+\rangle \) and \( |-\rangle \). i.e. it takes as input two qubits \( |\rangle \) and \( |0\rangle \), and in the case that \( |\rangle \) is either \( |+\rangle \) or \( |-\rangle \) it outputs \( |\rangle \) and \( |0\rangle \).

(b) Now suppose that the two states in question are \( |+\rangle \) and \( |0\rangle \). Can you create a ”copying circuit” that correctly copies these two states?

Hint: What is the inner product between \( |+\rangle |0\rangle \) and \( |0\rangle |0\rangle \)? And what is the inner product between \( |+\rangle |+\rangle \) and \( |0\rangle |0\rangle \)?

4. Suppose you have two qubits in some arbitrary entangled state \( |\rangle \). You apply the teleportation protocol to each of the two qubits separately. What is the resulting state obtained by the other party? Why?