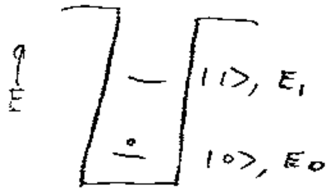
Atomic qubits
~~→~~ controlling
& measuring
$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

Crommie Lecture #7

(1)

A new Qubit: Electronic states of an atom!

Consider an atom where only 2 atomic states are important:



$\Rightarrow H_0|1\rangle = E_1|1\rangle$

$H_0|0\rangle = E_0|0\rangle$

$|0\rangle, |1\rangle$ refer to atomic orbitals:

$|0\rangle, |1\rangle \longrightarrow \psi_{n\ell m}(r,\theta,\phi)\,|s, m_s\rangle = R_{n\ell}(r)\,Y_{\ell m}(\theta,\phi)\,|s, m_s\rangle$

Atomic wavefcn can be quite complex, but let's not worry about this detail (look it up in book!); We just assume it exists

$\Rightarrow$ What does $\hat{H}_0$ look like in Basis of $|0\rangle, |1\rangle$?

We showed before that it looks like $\hat{H}_0 = \begin{pmatrix} E_0 & 0 \\ 0 & E_1 \end{pmatrix}$ $\Rightarrow$ Difference $E_1 - E_0$ plays same role as B_0 did for spin!

$\Rightarrow$ Now Consider arbitrary electronic state: $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

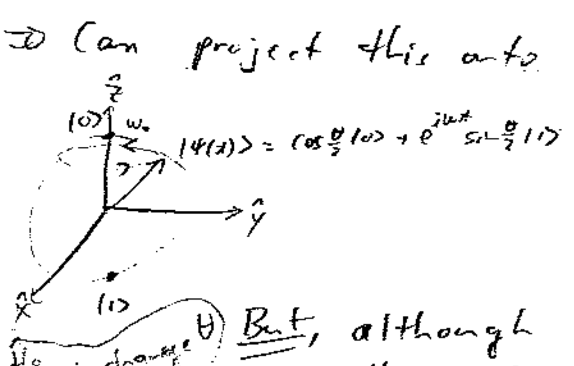
$\Rightarrow$ Can Project onto B. Sphere [illegible]

How does it change in time? $\Rightarrow |\psi(t)\rangle = e^{-iH_0 t/\hbar}\,|\psi\rangle$

$\Rightarrow |\psi(t)\rangle = \alpha e^{-iE_0 t/\hbar}|0\rangle + \beta e^{-iE_1 t/\hbar}|1\rangle$

(just like spin!)

$\Rightarrow$ Can project this onto Bloch Sphere: $|\psi(t)\rangle \to \alpha|0\rangle + \beta e^{-i(E_1 - E_0)t/\hbar}|1\rangle$



$\Rightarrow$ State vector spins around $\hat{z}$-axis w/ $\omega_0 = \frac{E_1 - E_0}{\hbar}$ (very fast!)

H_0 Doesn't change θ

But, although $\hat{H}_0$ causes $|\psi(t)\rangle$ to spin around $\hat{z}$-axis, it will never cause it to change "latitude" on Bloch Sphere. i.e., No "spin-flips" or transitions. How do we do this? ???

$\hat{H}_0$ causes us to spin around Bloch sphere at constant lattitude (θ = const.), just like $\vec{B} = B_0 \hat{z}$

Question: How do we change our atomic qubit state in a way that θ changes where $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle$??

(i.e., change latitude on Bl. sphere)

Answer: Need to perturb our system with a "Force Field" so that Hamiltonian gets off-diagonal matrix elements!! (like $\vec{B}_\perp$ for spin)

$$H = \begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix}$$

In order to change ratio between occupancy of $|0\rangle$ & $|1\rangle$, we must have $H_{12} \neq 0$

BUT, $H_{12} = 0$ for $\hat{H}_0$, so we need a NEW term in the Hamiltonian.

How do we get it? Turn on external $\vec{E}$-field!! (Like a capacitor)

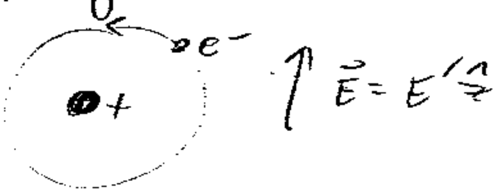
This applies a force that causes $H_{12} \neq 0$ and induces "vertical" rotations on Bloch sphere in exactly the same manner as $\vec{B} = \vec{B}_\perp$ did for spin.

$\Rightarrow$ To Understand this, we must Understand what happens when we subject an atom to an $\vec{E}$-field. How do we do this?? $\rightarrow$ SOLVE SCHR. EQN !!!!

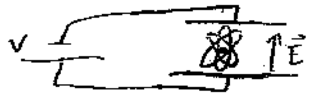
Solve Schr. Eq'n !! $\boxed{1^{st} \text{ Must Find } \hat{H}}$

⇒ What is classical energy of an atom in an $\vec{E}$-field? Must find the energy of this "perturbation".

⇒ Consider static E-field :



Experimentally :



⇒ Only worry about electron, since it is much lighter than nucleus.

$$\Rightarrow \vec{F} = -e\vec{E} = -eE'\hat{z} \quad \Rightarrow \text{Energy} = U = -\int \vec{F}\cdot d\vec{r}$$

$$\Rightarrow U = -\int -eE'\hat{z}\cdot d\vec{r} = \int eE'\,dz = eE'z$$

$\Rightarrow U = eE'z$ Potential energy depends on z-location of electron

⇒ This is the energy term that causes transitions between $|0\rangle$ and $|1\rangle$!!

Now, $\hat{H} = \hat{H}_0 + \hat{H}'$, $\hat{H}' = eE'z$

$\hat{H}'$ is called the "dipole" Hamiltonian since if we define an electric dipole $\vec{p} = -e\vec{r}$:

$$\Rightarrow H' = -\vec{p}\cdot\vec{E} = -(-e\vec{r})\cdot E'\hat{z} = eE'z$$

⇒ How does $\hat{H}'$ change the 2×2 representation of $\hat{H}$, and how can this be used to control $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$??

To see how Qubit changes under influence of Perturbation $\Rightarrow$ Must <u>Find $\hat{H}$</u> and <u>solve TDSE</u>:

$\Rightarrow \hat{H} = \hat{H}_0 + eE'z \rightarrow$ what does $\hat{H}$ look like in qubit basis $|0\rangle, |1\rangle$??

$$\hat{H} = \begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix} \Rightarrow$$

<u>Calculate these matrix elements:</u>

$\underset{\#1}{|0\rangle}, \underset{\#2}{|1\rangle}$ $\quad |0\rangle \rightarrow R_{n\ell}(r)\, Y_{\ell m}(\theta,\phi)$

$|1\rangle \rightarrow R_{n'\ell'}(r)\, Y_{\ell' m'}(\theta,\phi)$

$$H_{11} = \langle 0| (\hat{H}_0 + eE'z) |0\rangle = \langle 0|H_0|0\rangle + \langle 0|eE'z|0\rangle$$

$$= E_0 + \langle 0|eE'z|0\rangle$$

$\Rightarrow$ Find $\langle 0|eE'z|0\rangle$ $\leftarrow$ <u>Calculate Matrix element in Sph. Coords.</u>

$\langle 0| \rightarrow R^*_{n\ell}(r)Y^*_{\ell m}(\theta,\phi)$, $z \rightarrow r\cos\theta$, $|0\rangle \rightarrow R_{n\ell}(r)\, Y_{\ell m}(\theta,\phi)$

$$\Rightarrow \langle 0|eE'z|0\rangle = \iiint_{r,\theta,\phi} R^*_{n\ell}(r)\, Y^*_{\ell m}(\theta,\phi)\, eE' r\cos\theta\, R_{n\ell}(r)\, Y_{\ell m}(\theta,\phi)\, r^2 \sin\theta\, dr\, d\theta\, d\phi$$

$= 0$! From Electric Dipole Selection Rule: Need $\Delta\ell = \pm 1$ to be nonzero integral

<u>AND here $\Delta\ell = 0$</u>

So, $H_{11} = E_0$

Similarly, $H_{22} = E_1$

<u>BUT</u>, what about the crucial H_{12} $(= H_{21}^*)$ term?

Calculate OFF-Diagonal Matrix Element:

$$H_{12} = \langle 0 | \hat{H}_0 + eE'z | 1 \rangle = \langle 0 | \hat{H}_0 | 1 \rangle + \langle 0 | eE'z | 1 \rangle$$

$\Rightarrow \langle 0 | H_0 | 1 \rangle = E_1 \langle 0 | 1 \rangle = 0$

$\Rightarrow$ Must Find $\langle 0 | eE'z | 1 \rangle$ $\Rightarrow$ Calculate integral:

$$\langle 0 | eE'z | 1 \rangle = \iiint_{r,\theta,\phi} R^*_{n,\ell}(r) Y_{\ell m}(\theta,\phi)\, eE' r\cos\theta\, R_{n'\ell'}(r) Y_{\ell' m'}(\theta,\phi)\, r^2 \sin\theta\, dr\, d\theta\, d\phi$$

Here, however, states $|0\rangle$ & $|1\rangle$ can be chosen so that $\ell \neq \ell'$ and $\Delta \ell = \ell' - \ell = \pm 1$ so that selection rule does not make integral $= 0$.

$\Rightarrow$ Can solve integral (use Mathematica, or look up in book) and we find that $\langle 0 | eE'z | 1 \rangle \neq 0$

Assume integral is known $\Rightarrow$ Define: $\langle 0 | eE'z | 1 \rangle = V_1$

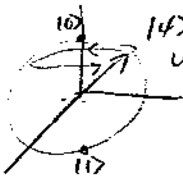
$\Rightarrow$ $\boxed{H_{12} = V_1}$ $\Rightarrow$ Since $\hat{H}$ = hermitian $\Rightarrow$ we know

$\boxed{H_{21} = V_1^*}$

So, $\hat{H} = \begin{pmatrix} E_0 & V_1 \\ V_1^* & E_1 \end{pmatrix}$ $\Rightarrow$ Off-diag. matrix elements are $\neq 0$!!

$\Rightarrow$ This induces transitions between $|0\rangle$ & $|1\rangle$!!

$\Rightarrow$ What does this look like geometrically on Bloch Sphere ??



$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\theta/2\,|1\rangle$

$\omega_0 = \frac{E_1 - E_0}{\hbar}$

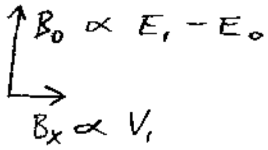
Use Spin analogy

Remember, $\hat{H}_0$ causes $|\psi\rangle$ to rotate about $\hat{z}$-axis at $\omega_0 = \frac{E_1 - E_0}{\hbar}$

$\Rightarrow E_1 - E_0$ plays role of $\vec{B} = B_0\hat{z}$:

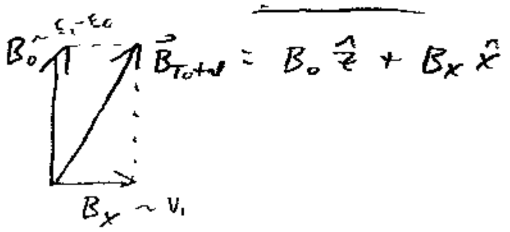
$\Rightarrow$ $\vec{E}$-field plays role of $\vec{B}_\perp$!

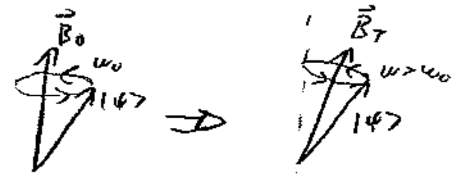
Off-diag. matrix element, V_1, plays role of $\vec{B}_\perp = B_x\hat{x}$



$\Rightarrow$ What Happens?

B_x "= V_1" causes total $\vec{B}$-field to "tilt", and $|\psi\rangle$ rotates around New total "$\vec{B}$-field":



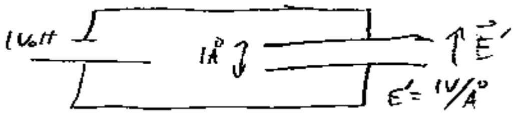


So, "latitude" on Bloch Sphere DOES change, i.e., "θ" changes for $|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle$

BUT, θ will NOT change by much unless $\vec{B}_x \approx \vec{B}_0$. In other words, For a significant change in "θ" $\Rightarrow$ need $V_1 \approx E_1 - E_0$

But Physically this is VERY Difficult

Since it would require electric fields on the order of Volts/Angstrom, since the size of an atom is ~ 1 Å and $E_1 - E_0 \sim 1\,eV$

∘ Need Cap. plates to be very close 

UNREASONABLE ! →

So, what else can we do to control the state of our electronic qubit?? $|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle$

How can we change "θ", or "tilt" the $|\psi\rangle$ vector more on Bloch sphere?

Answer: Use Resonance Technique!!!
Do what we did for Spin Resonance!

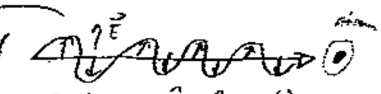
Oscillate the $\vec{E}$-field at the frequency $\omega_0 = \frac{E_1 - E_2}{\hbar}$

⇒ Can control $\theta(t)$ very precisely.

$\Rightarrow \underset{\text{Change}}{} \; \vec{E} = E'\hat{z} \xrightarrow{\text{to}} \vec{E}(t) = E'\cos\omega_0 t\,\hat{z}$ ← { Make $\vec{E}$-field Oscillate at ω_0

BUT, ω_0 here is very large! $E_1 - E_0 \approx 1\,eV \Rightarrow \omega_0 \sim 10^{15}\,Hz$

That's the Frequency of light!!

So, to create $\vec{E} = E'\cos\omega_0 t\,\hat{z}$, we don't use a capacitor plate, we shine light on the atom!! Light is an oscillating $\vec{E}$-field! 

Light polarized in $\hat{z}$-direction

So, what does Atomic qubit do when we shine light on it?

1. Must solve Schr. Eq'n : $i\hbar \frac{\partial}{\partial t} |\psi(t)\rangle = \hat{H} |\psi(t)\rangle$

$$\Rightarrow \hat{H} = \begin{pmatrix} E_0 & V_1 \cos\omega_0 t \\ V_1^* \cos\omega_0 t & E_1 \end{pmatrix}$$

$V_1 \to V_1 \cos\omega_0 t$ by shining light on atom
$V_1 \propto$ light intensity

$\Rightarrow$ This is essentially the <u>same</u> <u>problem</u> that we solved before for an oscillating $\vec{B}_1$ applied to spin:

<u>Spin Hamiltonian</u> : $\hat{H} = \frac{e\hbar}{2m} \begin{pmatrix} B_0 & B_1 \cos\omega_0 t \\ B_1 \cos\omega_0 t & -B_0 \end{pmatrix}$

$\Rightarrow$ Before we found that

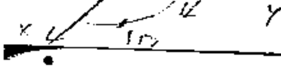
$$|\psi(t)\rangle = \cos\frac{\omega_1 t}{2} |0\rangle + e^{i(\omega_0 t + \pi)} \sin\frac{\omega_1 t}{2} |1\rangle$$

where $\omega_0 \propto B_0$, $\omega_1 \propto B_1$, $\boxed{\omega_1 = \frac{\partial\theta}{\partial t}}$

Now we can map our atomic system onto the spin problem, and we see that

$$\omega_0 = \frac{E_1 - E_0}{\hbar} \sim \text{qubit energy splitting}$$

& $\omega_1 = \frac{V_1}{\hbar} \sim$ intensity of light at ω_0 shined on qubit.



$|\psi\rangle$ now changes lattitude (θ) on Bloch sphere at rate $\omega_1 = \frac{V_1}{2\hbar}$. Can now control $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ very precisely!

OK, so now we see that we can control

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle = \cos\tfrac{\theta}{2}|0\rangle + e^{i\phi}\sin\tfrac{\theta}{2}|1\rangle$$

Very precisely by shining well-timed pulses of light at the atomic qubit.

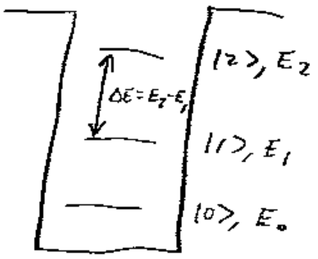
⇒ Suppose we put atom into arbitrary state

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

⇒ How do we <u>measure</u> α and β ?

<u>Answer</u>: Use Fluorescence!

Pick an atom that has a 3^{rd} state, $|2\rangle$, that couples to $|1\rangle$ but NOT to $|0\rangle$!!



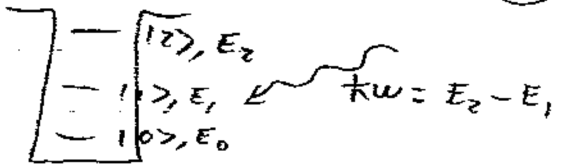
⇒ Shine light on atom at Frequency $\omega = \frac{\Delta E}{\hbar} = \frac{E_2 - E_1}{\hbar}$

⇒ <u>If</u> atom is in state $|0\rangle$
⇒ NOTHING Happens (<u>Dark state</u>)

<u>BUT</u>, if atom is in state $|1\rangle$ ⇒ electron will absorb photon and get pushed up to state $|2\rangle$. Electron will then tend to fall back down and re-radiate the photon. <u>This can be detected!</u>

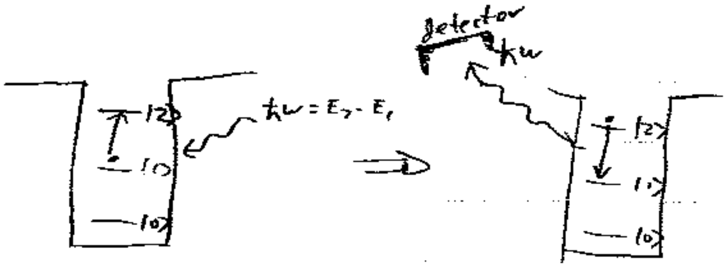
$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

Measurement Scheme:



If electron in state $|0\rangle$: No absorption

If electron in state $|1\rangle$: Absorption + Re-radiation & detection



$\Rightarrow$ This is JUST Like Stern-Gerlach, in the sense that we get a DIFFERENT signal if electron is in state $|0\rangle$ or $|1\rangle$.

Recipe for measuring $|\alpha|, |\beta|$: Prepare atom in state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ $\Rightarrow$ Shine light on it $\Rightarrow$ ask: did it absorb photon?

$\Rightarrow$ Prepare state again $\Rightarrow$ shine light $\Rightarrow$ did it absorb photon?

$\Rightarrow$ " " "

Suppose you do this 1000 times and it absorbs the photon 800 times. $\Rightarrow$ what is β ??

$$\Rightarrow |\beta|^2 = \text{probability} = \frac{800}{1000} = \frac{8}{10} \Rightarrow |\beta| = \sqrt{\frac{8}{10}}$$

$$\Rightarrow |\alpha| = \sqrt{\frac{2}{10}}$$

But what about relative phase between α and β? $\Rightarrow$ More difficult: Have to rotate $|\psi\rangle$ by $90°$ around $\hat{y}$ and measure $\langle S_z\rangle$ to get $\langle S_x\rangle$ & Rotate by $90°$ around $\hat{x}$ and measure $\langle S_z\rangle$ to get $\langle S_y\rangle$ $\Rightarrow$ $\langle S_x\rangle$ & $\langle S_y\rangle$ define ϕ!! Tricky!