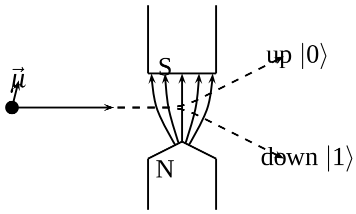
1 Stern-Gerlach Apparatus

A Stern-Gerlach device is simply a magnet set up to generate a particular inhomogeneous $\vec{B}$ field.



When a particle with spin state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ is shot through the apparatus from the left, its spin-up portion is deflected upward, and its spin-down portion downward. The particle's spin becomes entangled with its position! Placing detectors to intercept the outgoing paths therefore measures the particle's spin.

Why does this work? We'll give a semiclassical explanation – mixing the classical $\vec{F} = m\vec{a}$ and the quantum $H\psi = E\psi$ – which is quite wrong, but gives the correct intuition. [See Griffith's § 4.4.2, pp. 162-164 for a more complete argument.] Now the potential energy due to the spin interacting with the field is

$$E = -\vec{\mu} \cdot \vec{B} \ ,$$

so the associated force is

$$\vec{F}_{\text{spin}} = -\vec{\nabla} E = \vec{\nabla}(\vec{\mu} \cdot \vec{B}) \ .$$

At the center $\vec{B} = B(z)\hat{z}$, with $\frac{\partial B}{\partial z} < 0$, so $\vec{F} = \vec{\nabla}(\mu_z B(z)) = \mu_z \frac{\partial B}{\partial z}\hat{z}$. The magnetic moment $\vec{\mu}$ is related to spin $\vec{S}$ by $\vec{\mu} = \frac{gq}{2m}\vec{S} = -\frac{e}{m}\vec{S}$ for an electron. Hence

$$\vec{F} = \frac{e}{m}|\frac{\partial B}{\partial z}|S_z\hat{z} \ ;$$

if the electron is spin up, the force is upward, and if the electron is spin down, the force is downward.

2 Initialize a Qubit

- How can we create a beam of qubits in the state $|\psi\rangle = |0\rangle$? Pass a beam of spin-$\frac{1}{2}$ particles with randomly oriented spins through a Stern-Gerlach apparatus oriented along the z axis. Intercept the downward-pointing beam, leaving the other beam of $|0\rangle$ qubits.

  Note that we *measure* the spin when we intercept an outgoing beam – after this measurement, the experiment is probabilistic and not unitary.

- How can we create a beam of qubits in the state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$? First find the point on the Bloch sphere corresponding to $|\psi\rangle$. That is, write

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}|1\rangle$$

(up to a phase), where

$$\tan\frac{\theta}{2} = \left|\frac{\beta}{\alpha}\right| \qquad e^{i\varphi} = \frac{\beta/|\beta|}{\alpha/|\alpha|} \quad .$$

The polar coordinates θ, φ determine a unit vector $\hat{n} = \cos\varphi \sin\theta \hat{x} + \sin\varphi \sin\theta \hat{y} + \cos\theta \hat{z}$. Now just point the Stern-Gerlach device in the corresponding direction on the Bloch sphere, and intercept one of the two outgoing beams. That is, a Stern-Gerlach device pointed in direction $\hat{n}$ measures $S_{\hat{n}} = \hat{n} \cdot \hat{\vec{S}}$.

- How can we implement a unitary (deterministic) transformation? We need to evolve the wave function according to a Hamiltonian $\hat{H}$. Then

  $$|\psi(t)\rangle = e^{-\frac{i}{\hbar}\hat{H}t}|\psi(0)\rangle$$

  solves the Schrödinger equation (if $\hat{H}$ is time-independent). In the next lecture we will show how to accomplish an arbitrary single-qubit unitary gate (a rotation on the Bloch sphere) by applying a precise magnetic field for some precise amount of time: Larmor precession.