Quantum Teleportation using photons (Zeilinger experiment)

Quantum Teleportation

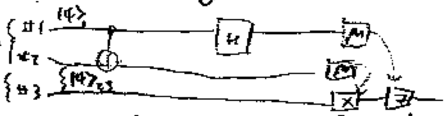
(1)

Crommie Lecture #9

Quantum Teleportation Using Photons

Based on Bouwmeester, et al, Nature 390, 575 (1997) [Zeilinger's gp].

Idea: Alice has qubit state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ which she wants to send to Bob with the help of 2 other entangled qubits and some classical information.



Qubit System: Polarization state of Photons, $\vec{E}$

Basis states: $\vec{E}$-Horizontal $= |0\rangle$ $(= |H\rangle)$, $\vec{E}$-Vertical $= |1\rangle$ $(= |V\rangle)$

$\vec{E}$-field polarization states.

$\Rightarrow$ Photon having $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ has probability $|\alpha|^2$ to be found in Horiz. polarization state and $|\beta|^2$ prob. to be found in vertical polariz. state

$\Rightarrow$ Need 3 photons (3 qubits)

Photon #1: $|\psi\rangle_1 = \alpha|0\rangle_1 + \beta|1\rangle_1$ belongs to Alice. This is the state she wants to teleport.

Photons #2 & #3: $|\psi\rangle_{23} = |\beta_{11}\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle_2|1\rangle_3 - |1\rangle_2|0\rangle_3\right)$

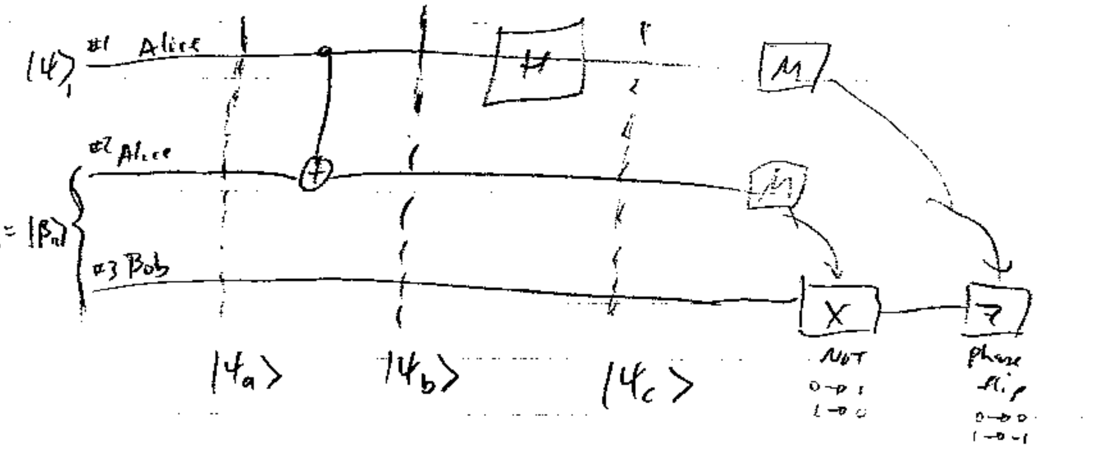
This is an entangled pair. Alice has one of the photons (Photon #2) and Bob has the other (Photon #3).

$\Rightarrow$ Alice wants to teleport the state of photon #1 to photon #3 in Bob's Possession.

⇒ How it works in theory

Consider Teleportation Circuit we saw before in class

Inputs → $\begin{cases} |\psi\rangle_1 = \alpha|0\rangle + \beta|1\rangle \\ |\psi\rangle_{23} = |\beta_{11}\rangle = \frac{1}{\sqrt{2}}\left(|01\rangle - |10\rangle\right) \end{cases}$



$$\boxed{|\psi_a\rangle = |\psi\rangle_1 \otimes |\beta_{11}\rangle} = \left(\alpha|0\rangle + \beta|1\rangle\right)\cdot\frac{1}{\sqrt{2}}\left(|01\rangle - |10\rangle\right)$$

$$\Rightarrow |\psi_a\rangle = \boxed{\frac{1}{\sqrt{2}}\alpha|0\rangle\left(|01\rangle - |10\rangle\right) + \frac{1}{\sqrt{2}}\beta|1\rangle\left(|01\rangle - |10\rangle\right)}$$

$$\Rightarrow \boxed{|\psi_b\rangle = \frac{1}{\sqrt{2}}\alpha|0\rangle\left(|01\rangle - |10\rangle\right) + \frac{1}{\sqrt{2}}\beta|1\rangle\left(|11\rangle - |00\rangle\right)}$$

$$\Rightarrow \boxed{|\psi_c\rangle = \frac{1}{2}\alpha\left(|0\rangle + |1\rangle\right)\left(|01\rangle - |10\rangle\right) + \frac{1}{2}\beta\left(|0\rangle - |1\rangle\right)\left(|11\rangle - |00\rangle\right)}$$

$$= \frac{1}{2}\left[\alpha|001\rangle - \alpha|010\rangle + \alpha|101\rangle - \alpha|110\rangle + \beta|011\rangle - \beta|000\rangle - \beta|111\rangle + \beta|100\rangle\right]$$

$$\boxed{= \frac{1}{2}\Big(|00\rangle\left[\alpha|1\rangle - \beta|0\rangle\right] - |01\rangle\left[\alpha|0\rangle - \beta|1\rangle\right] + |10\rangle\left[\alpha|1\rangle + \beta|0\rangle\right] - |11\rangle\left[\alpha|0\rangle + \beta|1\rangle\right]\Big)}$$

⇒ Usual thing:
Now Do measurement of qubits #1 & #2 (in standard basis) ⇒ Act on Qubit #3 depending on results of this measurement

⇒ Suppose measurement of qubits #1 & #2 registers "1" and "1" (i.e., $|11\rangle$)

⇒ $\boxed{|\psi\rangle_3 = \alpha|0\rangle + \beta|1\rangle}$ [since $|\psi\rangle \propto |11\rangle\langle 11|\psi_c\rangle$]

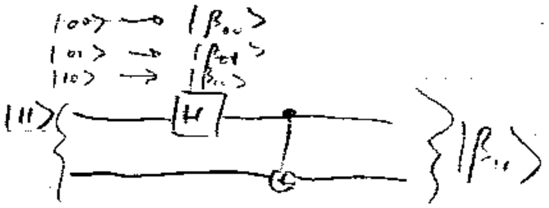
In this case the qubit is teleported and Bob doesn't have to do anything to qubit #3... (no "X" or "Z")

This is the situation that has been performed experimentally with photons [other cases haven't been done yet to best of my knowledge]

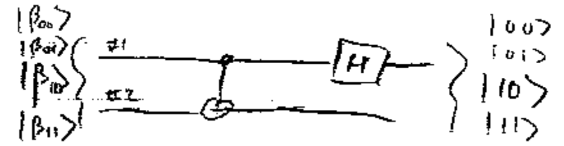
But it's Done w/ a small conceptual trick

Notice :

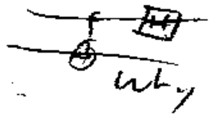
This gate transforms the standard basis into the Bell state basis



And so the reversed gate transforms Bell states into standard basis



Note: This is exactly what we have in the teleportation circuit. This means that if qubits #1, #2 are projected onto $|\beta_{11}\rangle$ before hitting this gate and the gate is removed, then $|\beta_{11}\rangle$ will play exactly same role that $|11\rangle$ would have played if gate stayed. We still get teleportation to qubit #3, only now it's tensored w/ $|\beta_{11}\rangle$ instead of $|11\rangle$. Basically we just replace role of $|11\rangle$ w/ $|\beta_{11}\rangle$.

So, instead of running qubits #1 & #2 through  and projecting them onto $|11\rangle$ at the output, why not just directly project qubits #1 & #2 onto $|\beta_{11}\rangle$ directly at the beginning?

[easy to show it's same mathematically, see supplementary stuff]

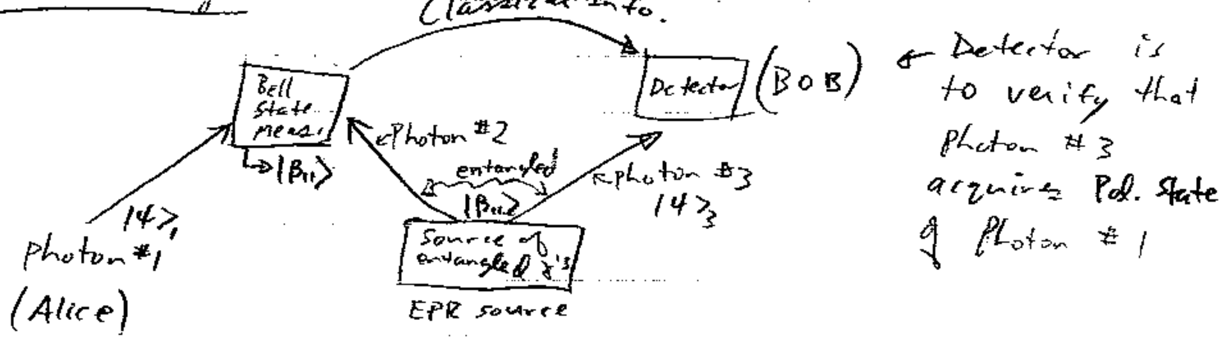
This is what is done experimentally with photons to achieve Quantum teleportation. A "Bell state measurement" is performed which projects qubits #1 & #2 onto $|\beta_{11}\rangle$. Alice's state is then "Teleported" onto qubit #3 without any further work.

So, the new Q.Teleportation scheme is as follows: Alice performs a Bell state measurement on photons #1 & #2 and projects them onto the Bell state $|\beta_{11}\rangle = |\psi\rangle_{12} = \frac{1}{\sqrt{2}}\left(|0\rangle_1|1\rangle_2 - |1\rangle_1|0\rangle_2\right)$

This forces photon #3 to be in the former state of photon #1 ($|\psi\rangle_1$)

$\Rightarrow \; |\psi\rangle_3 \rightarrow \alpha|0\rangle + \beta|1\rangle$

Diagramatically:



Classical info. is needed to verify that photons #1 and #2 have been projected onto the correct Bell state ($|\beta_{11}\rangle$). The teleportation will only work for projection on to this state [i.e. this is the one choice out of 4 where Bob doesn't have to change the state of photon #3 to obtain $|\psi\rangle_1$]

So, how do you do this in real life?

There are 2 important experimental tricks:

1) Must Create 2 entangled photons in $|\beta_{11}\rangle$ state:

$$|\beta_{11}\rangle = \frac{1}{\sqrt{2}}\Big(\underset{\#1\ \text{photon}}{|H\rangle}\underset{\#2\ \text{photon}}{|V\rangle} - \underset{\#1\ \text{photon}}{|V\rangle}\underset{\#2\ \text{photon}}{|H\rangle}\Big)$$

$$|H\rangle \to |0\rangle, \quad |V\rangle \to |1\rangle$$

$$\Rightarrow \quad |\beta_{11}\rangle = \frac{1}{\sqrt{2}}\Big(|01\rangle - |10\rangle\Big)$$

2) Must Perform Bell state measurement on 2 photons. Project 2 photons onto $|\beta_{11}\rangle$ state. How?

1^{st} let's talk about how to

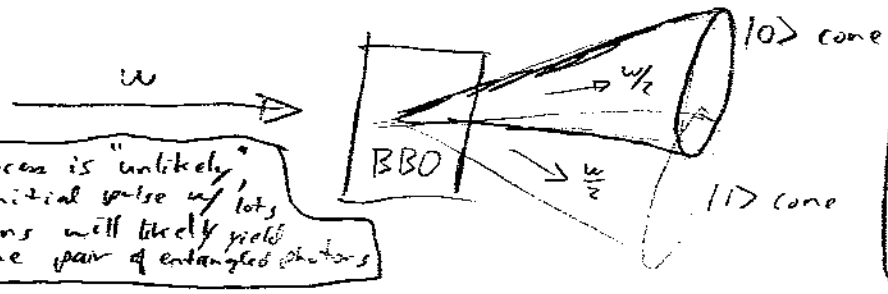
create entangled photons;

1 | How do you get an EPR pair of photons? i.e., an entangled photon state?

Answer: Use Nonlinear crystal capable of "parametric down conversion"

Special Nonlinear Xtals (such as barium borate) have the property that you can send in 1 photon at freq. ω and get out 2 photons at freq. $\omega/2$.

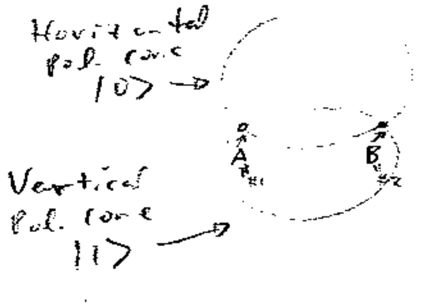
The 2 photons come out in 2 cones. One cone has a Horiz. polarized photon ($|0\rangle$) and the other has a vert. polarized photon ($|1\rangle$)



This process is "unlikely", so an initial pulse w/ lots of photons will likely yield only one pair of entangled photons

The 2 photons have opposite polarization and total transv. momentum = 0

Entangled photons come from capturing photons at the intersection of the cones:



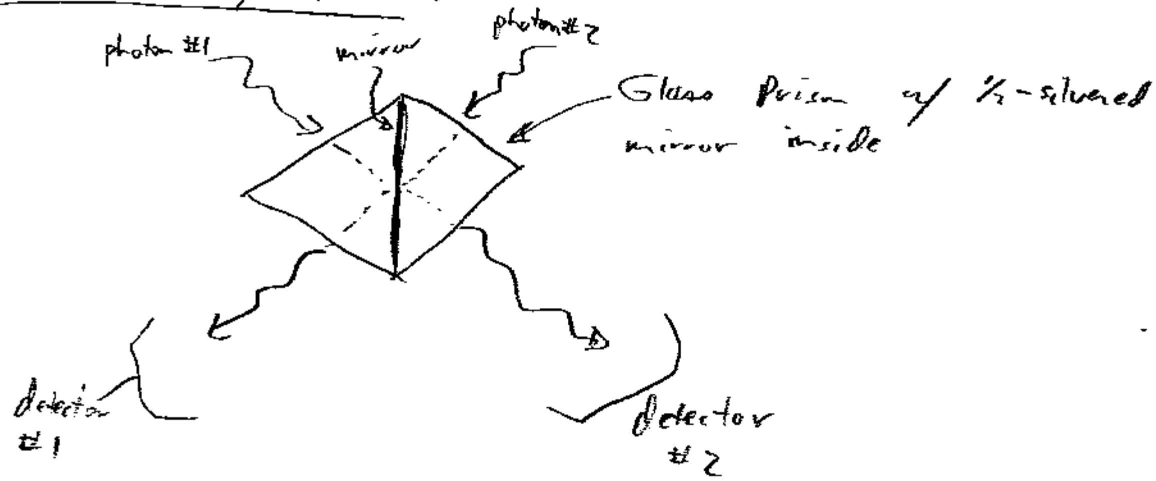
The 2 photons coming from points A and B have indeterminate polarization but they must have opposite polarization

$\Rightarrow$ they are entangled:

$$|\psi\rangle_{AB} = \frac{1}{\sqrt{2}}\left(|0\rangle_{\underset{\#1}{A}}|1\rangle_{\underset{\#2}{B}} - |1\rangle_{\underset{\#1}{A}}|0\rangle_{\underset{\#2}{B}}\right)$$

7) How do you perform Bell State measurement?

Use Beam splitter!



Assuming photons #1 & #2 are incident on the beamsplitter at exactly the same time ⇒ there are 4 possible polarization states in the BELL STATE BASIS:

$$|\beta_{00}\rangle = \tfrac{1}{\sqrt{2}}\left(|H\rangle|H\rangle + |V\rangle|V\rangle\right) = \tfrac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

[initial 2 photon state is a superposition of these] ⇒

$$|\beta_{01}\rangle = \tfrac{1}{\sqrt{2}}\left(|H\rangle|V\rangle + |V\rangle|H\rangle\right) = \tfrac{1}{\sqrt{2}}\left(|01\rangle + |10\rangle\right)$$

$$|\beta_{10}\rangle = \tfrac{1}{\sqrt{2}}\left(|H\rangle|H\rangle - |V\rangle|V\rangle\right) = \tfrac{1}{\sqrt{2}}\left(|00\rangle - |11\rangle\right)$$

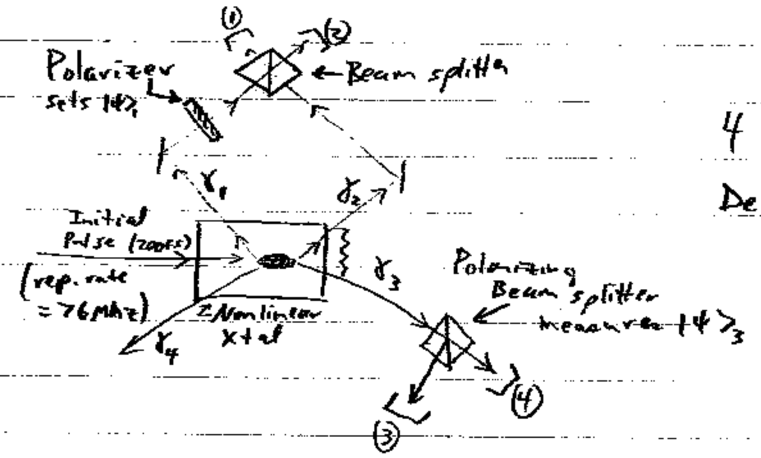
$$|\beta_{11}\rangle = \tfrac{1}{\sqrt{2}}\left(|H\rangle|V\rangle - |V\rangle|H\rangle\right) = \tfrac{1}{\sqrt{2}}\left(|01\rangle - |10\rangle\right)$$

It turns out that the properties of a beam splitter are such that if detectors #1 & #2 click simultaneously ⇒ photons are in $|\beta_{11}\rangle$ state. Only one detector or the other would click for the other Bell states.

Showing why this is true takes some tricky Quantum optics (ref. ?) Not discussed here.

So, how do we actually do the exp. in the lab??

Use laser, parametric down conversion, beamsplitters, single photon detection:

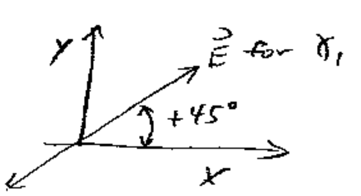


4 single Photon Detectors (1), (2), (3), (4)

Experimental Results:

1) Initial pulse hits Nonlinear Xtal and creates 2 entangled pairs: $|\gamma_1 \gamma_4\rangle$ and $|\gamma_2 \gamma_3\rangle$. γ_4 is thrown away, and γ_1 is used as qubit #1. $\gamma_1, \gamma_2, \gamma_3, \gamma_4$ created same time

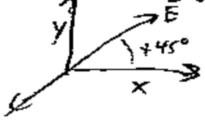
2) γ_1 is passed through a polarizer that sets $|\psi\rangle_1$ at a polarization of $+45°$. This is the state we want to teleport to γ_3.



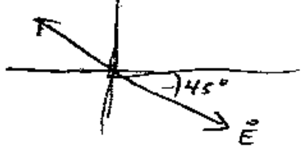
3) γ_2 and γ_3 are entangled into $|\beta_{11}\rangle$ state $= |\psi\rangle_{23}$

4) γ_1 and γ_2 hit beam splitter at same time. Bell state measurement is performed by looking for simultaneous firing of detectors #1 and #2. If they fire at the same time then we know that Bell state measurement has been performed and $|\psi\rangle_{12} = |\beta_{11}\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle_1|1\rangle_2 - |1\rangle_1|0\rangle_2\right)$ has been detected (these detectors provide the 2 classical bits of information).

5) The state of γ_3 is measured with polarizing beam splitter. If γ_3 is polarized at $+45°$ then detector (4) fires.



If γ_3 is polarized at $-45°$ then detector (3) fires.



This is a complete basis, so need to measure a predominance of detector #4 clicks to know that state is selected.

A successful Teleportation event occurs when detectors 1, 2 and 4 go off at the same time. This shows that qubit state for γ_1 and γ_2 have been projected onto the Bell state $|\Psi^-\rangle_{12}$ and the qubit state for γ_1 has been transported to γ_3.

It is also important that detector 3 NOT go off for repeated measurements, so that we know that the state of γ_3 is really an eigenstate of $+45°$, and not just being projected onto it.

Stop here!

What is time uncertainty in order ??

[just have ~~OO~~ γ_1, γ_2 measured before γ_3]

Supplementary:

Teleportation Algebra for Zeilinger Experiment:

$|\psi\rangle_1 = \alpha|0\rangle_1 + \beta|1\rangle_1$ (qubit state we want to transport to Bob via $|\psi\rangle_3$)

$|\psi\rangle_{23} = \frac{1}{\sqrt{2}}\left(|0\rangle_2|1\rangle_3 - |1\rangle_2|0\rangle_3\right)$ (entangled qubits split betw Alice and Bob)

$\Rightarrow$ Initial State of 3 photons:

$$|\psi\rangle_{123} = \left(\alpha|0\rangle_1 + \beta|1\rangle_1\right)\cdot\frac{1}{\sqrt{2}}\left(|0\rangle_2|1\rangle_3 - |1\rangle_2|0\rangle_3\right)$$

$$= \frac{1}{\sqrt{2}}\left[\alpha|0\rangle_1|0\rangle_2|1\rangle_3 - \alpha|0\rangle_1|1\rangle_2|0\rangle_3 + \beta|1\rangle_1|0\rangle_2|1\rangle_3 - \beta|1\rangle_1|1\rangle_2|0\rangle_3\right)$$

Now, do Bell state Measurement of photons #1 & #2 by projecting them onto $|\psi\rangle_{12} = \frac{1}{\sqrt{2}}\left(|0\rangle_1|1\rangle_2 - |1\rangle_1|0\rangle_2\right)$

$\Rightarrow$ Define $|\phi\rangle = |\psi\rangle_{12}\,{}_{12}\langle\psi|\psi\rangle_{123}$ [Proj. of $|\psi\rangle_{123}$ onto $|\psi\rangle_{12}$]

$\Rightarrow$ After measurement $\Rightarrow$ New $|\psi\rangle_{123} = \dfrac{|\phi\rangle}{\sqrt{\langle\phi|\phi\rangle}}$

So, what is $|\phi\rangle = ?$

$$|\phi\rangle = |\psi\rangle_{12}\left[\frac{1}{\sqrt{2}}\left({}_1\langle 0|\,{}_2\langle 1| - {}_1\langle 1|\,{}_2\langle 0|\right)\cdot\frac{1}{\sqrt{2}}\left(\alpha|0\rangle_1|0\rangle_2|1\rangle_3 - \alpha|0\rangle_1|1\rangle_2|0\rangle_3 + \beta|1\rangle_1|0\rangle_2|1\rangle_3 - \beta|1\rangle_1|1\rangle_2|0\rangle_3\right]\right.$$

$$= |\psi\rangle_{12}\cdot\frac{1}{2}\cdot\left(\alpha\cdot 0 - \alpha|0\rangle_3 + \beta\cdot 0 - \beta\cdot 0 - \alpha\cdot 0 + \alpha\cdot 0 - \beta|1\rangle_3 + \beta\cdot 0\right)$$

$$|\phi\rangle = -\tfrac{1}{2}|\psi\rangle_{12}\left(\alpha|0\rangle_3 + \beta|1\rangle_3\right)$$

$\Rightarrow \langle \phi | \phi \rangle = \frac{1}{4} \Rightarrow \sqrt{\langle \phi | \phi \rangle} = \frac{1}{2}$

So, New $|\psi'\rangle_{123} = \frac{|\phi\rangle}{\sqrt{\langle \phi | \phi \rangle}} = -|\psi\rangle_{12} \left(\alpha |0\rangle_3 + \beta |1\rangle_3 \right)$

$\Rightarrow$ By inspection we see $|\psi'\rangle_{123} = |\psi'\rangle_{12} |\psi'\rangle_3$

AND $\boxed{|\psi'\rangle_3 = \alpha |0\rangle_3 + \beta |1\rangle_3}$ Exactly what we wanted!

Alice's original qubit #1 state has teleported to qubit #3