SUPERCONDUCTING QUBITS

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MICROWAVE OPTICS & SUPERCONDUCTING ARTIFICIAL ATOMS

ATOM/CAVITY

LIGHT SOURCE

DETECTOR

HOMODYNE

COUNTING

M. Hatridge et al., *Phys. Rev. B* 83 (2011)


Tunable $f$, $\phi$

$|1\rangle$

4 – 20 GHz

$|0\rangle$
4-8 GHz LINEAR CAVITIES

3D Waveguide
Q ~ 10^6-10^9...

2D Planar
Q ~ 10^5-10^6...
THE NON-DISSIPATIVE JOSEPHSON JUNCTION OSCILLATOR

\[ I(\delta) = I_0 \sin(\delta) \]
\[ V(t) = \frac{\hbar}{2e} \frac{d}{dt}(\delta) \]
\[ U(\delta) = -\frac{\hbar}{2e} I_0 \cos(\delta) \]

\[ I_0 \sim nA \]

\[ I_0 > \mu A \]

Quantized Levels

Non-linear Oscillator

\[ U(\pm\hbar I_0/2e) \]
\[ \frac{\delta}{2\pi} \]

\[ \text{harmonic osc.} \]
\[ \omega_p \]
Coherent control of macroscopic quantum states in a single-Cooper-pair box

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Observation of High Coherence in Josephson Junction Qubits Measured in a Three-Dimensional Circuit QED Architecture

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SUPERCONDUCTING TRANSMON QUBIT

\[ L_J \sim 13 \text{ nH} \quad C \sim 70 \text{ fF} \]

\[ \omega_{01} \approx \frac{1}{\sqrt{L_J C}} \]

\[ \omega_{01} \neq \omega_{12} \]

- Tunable qubit frequency
- \( \omega_{01} \sim 5-8 \text{ GHz} \)
- \( T_1, T_2 \sim 100s \mu s \)

Josephson tunnel junctions

500 nm
EXPERIMENTAL SETUP

T = 4K

(~25 added photons)

INPUT

readout/qubit pulses in

OUTPUT

readout w/ qubit information

T = 0.02 K
SYSTEM NOISE TEMPERATURE

Signal power at paramp (dBm)

-145 -140 -135 -130 -125 -120 -115

System noise temperature (K)

0.0 0.5 1.0 1.5 2.0 2.5 3.0

Measurement cavity photon occupation

-145 -140 -135 -130 -125 -120 -115

System noise w/o paramp = 7K

○ Measured noise temperature

- Standard quantum limit
PARAMETRIC AMPLIFICATION

$L_J \sim 0.1 \text{ nH}$  
$C \sim 10000 \text{ fF}$

Al Lumped LC Resonator
4-8 GHz
Coupled to 50 Ω
Q = 26
4th GENERATION CRYOPACKAGE

- Copper cover (prevents any coupling to box mode)
- Aluminum superconducting shield
- Removable CPW launch
- Magnet for tuning frequency
- Thermalizing strap
OUTLINE

• WEAK MEASUREMENT

• SINGLE QUBIT EXPERIMENTS
  – Individual Quantum Trajectories
  – Distribution of Quantum Trajectories

• TWO QUBIT MEASUREMENTS
  – Remote Entanglement
WEAK MEASUREMENT
QUBIT STATE ENCODED IN PHASE SHIFT

\[ A \sin(\omega t + \phi) = A \sin(\omega t) \cos(\phi) + A \cos(\omega t) \sin(\phi) = [A \cos(\phi)] \sin(\omega t) + [A \sin(\phi)] \cos(\omega t) \]

PHASE SENSITIVE HOMODYNE MEASUREMENT
MEASUREMENT: COUPLE TO E-M FIELD OF CAVITY (Jaynes-Cummings)

Cavity Transmission

Frequency

|1⟩  | 0⟩ α n

VARY MEASUREMENT STRENGTH USING DISPERSIVE SHIFT & PHOTON NUMBER

NEED TO DETECT ~ SINGLE MICROWAVE PHOTONS in T₁ ~ μs
STRONG vs. WEAK MEASUREMENT

**Strong**

- READOUT/TOMOGRAPHY

**Weak**

- READOUT/TOMOGRAPHY

![Diagram showing the comparison between strong and weak measurement techniques in quantum mechanics, focusing on control parameters and homodyne voltage distributions.](image-url)
MEASUREMENT STRENGTH

\[ S = \frac{\Delta V^2}{\sigma^2} \]

\[ S = \frac{64\tau\chi^2\bar{n}\eta}{\kappa} \]

\( \tau \): measurement time
\( \chi \): dispersive shift
\( \bar{n} \): photon number
\( \kappa \): cavity decay rate
\( \eta \): detector efficiency

\[ \eta = 0.49 \]
WHAT DO YOU DO WITH THIS WEAK MEASUREMENT SIGNAL?

• Control Signal for Feedback (eg. Stabilized Rab Osc.)

• Construct Trajectories

• Feed it to Another Qubit to Generate Entanglement
CAN WE TRACK A PURE STATE ON THE SURFACE OF THE BLOCH SPHERE?

OBSERVING SINGLE QUANTUM TRAJECTORIES OF A SUPERCONDUCTING QUBIT

BACKACTION OF SINGLE QUADRATURE MEASUREMENT

Cavity Output $\theta=0$

After Amplification $\theta=0$

Cavity Output $\theta=\pi/2$

After Amplification $\theta=\pi/2$

• Obtain qubit state information
• Tip Bloch vector

• Obtain cavity photon number information
• Rotate Bloch vector
• Prepare state along $+x$
• Continuous weak measurement
• Integrated readout is trajectory

$$V_m(t) = \frac{1}{\tau} \int_0^\tau V(t) dt$$
BAYESIAN UPDATE

**Event:**

A = Avalanche  S = Snow > 10 feet

**Likelihood:** Prob. snow accompanied avalanche

**Prior:** Historic chance of an avalanche

**Posterior:** Chance of an avalanche given snow

\[
P(A|S) = \frac{P(S|A)P(A)}{P(S)}
\]

\[
P(|0\rangle | V) = \frac{P(V | |0\rangle) P(|0\rangle)}{P(V)}
\]
WEAK MEASUREMENT OF THE QUBIT STATE

- Initial state along $+X$
- Measure $Z$ (phase quadrature)

Want to evaluate:

\[
\begin{align*}
\langle \sigma_Z \rangle |V_m & \overset{\text{def}}{=} Z^Z \\
\langle \sigma_X \rangle |V_m & \overset{\text{def}}{=} X^Z \\
\langle \sigma_Y \rangle |V_m & \overset{\text{def}}{=} Y^Z
\end{align*}
\]

Bayes Rule:

\[
\begin{align*}
Z^Z &= \tanh\left(\frac{V_m S}{2\Delta V}\right) \\
X^Z &= \sqrt{1 - \langle \sigma_Z \rangle^2} e^{-\gamma \tau} \\
\gamma &= 8\chi^2 \bar{n}(1 - \eta)/\kappa + 1/T_2^* 
\end{align*}
\]
WEAK MEASUREMENT OF THE PHOTON NUMBER

\[ \sigma_Z | V_m \equiv Z^\phi \]
\[ \sigma_X | V_m \equiv X^\phi \]
\[ \sigma_Y | V_m \equiv Y^\phi \]

WANT TO EVALUATE:

BAYES RULE:

\[ X^\phi = \cos \left( \frac{SV_m}{2\Delta V} \right) e^{-\gamma \tau} \]
\[ Y^\phi = \sin \left( \frac{SV_m}{2\Delta V} \right) e^{-\gamma \tau} \]
REALTIME TRACKING

- Prepare qubit along X axis
- Evolve under measurement
- Use Bayes rule to update our guess of the qubit state (dots)
- Perform tomography for each time step (solid)
DISTRIBUTION OF QUANTUM TRAJECTORIES
Initial State along $+x$

Integration time $\tau$ needed to resolve state: 1.25 $\mu$s
MEASUREMENT w. POSTSELECTION

|0⟩

Initial State along +x
Final State at z = -0.85

Can Identify Most Likely Path

Predict with Theory?
EXTREMIZING THE QUANTUM ACTION

Classical Example: Kramer’s Escape
- Consider paths to saddle point $\Lambda$
- Establish canonical phase space $(p,q)$
- Define action $S$
- Calculate most favorable path, etc…

Quantum Case for Pre/Post-Selected Trajectories:
- Consider paths connecting quantum state $q_I \rightarrow q_F$
- Double quantum state space ($\rightarrow$ canonical)
- Express joint probability of measurement & trajectories as path integral
- Minimize action
  $\rightarrow$ ODE for equation of motion

\[
\mathcal{P} = \delta^d(q_0 - q_I)\delta^d(q_n - q_F) \prod_{k=0}^{n-1} P(q_{k+1}, r_k | q_k).
\]

\[
\mathcal{P} = \int \mathcal{D}p e^{S} = \mathcal{D}p \exp\left[\int_0^T dt (-p \cdot \dot{q} + \mathcal{H}[q, p, r])\right]
\]

- Calculate statistical distributions
- Treat case of measurement backaction with control pulses $\Omega$ (Schrödinger dynamics)
Initial State along $+x$
Final State at $z = -0.85$

**EOM for Optimized Path**

\[
\begin{align*}
\dot{x} &= -\gamma x + \Omega z - xzr/\tau, \\
\dot{z} &= -\Omega y + (1 - z^2)r/\tau, \\
\dot{p}_x &= \gamma p_x + \Omega p_z + p_xzr/\tau, \\
\dot{p}_z &= -\Omega p_z + (p_xx + p_yy + 2pz - 1)r/\tau,
\end{align*}
\]

**No Driving ($\Omega = 0$)**

\[
\begin{align*}
\ddot{x}(t) &= e^{-\gamma t}\text{sech}(\ddot{t}/\tau) \\
\ddot{z}(t) &= \tanh(\ddot{t}/\tau)
\end{align*}
\]

$r$: detector output
max likelihood
QUANTUM TRAJECTORIES WITH RABI DRIVING
TRAJECTORIES w. RABI DRIVE: TOMOGRAPHY

- Trajectories w. Rabi drive: two step update (master eqn. + Bayes)
- Individual trajectories show “high purity”
DISTRIBUTION OF RABI TRAJECTORIES

Excellent agreement with ODE solutions
CAN WE ENTANGLE TWO REMOTE SUPERCONDUCTING QUBITS \textit{via} MEASUREMENT?

“TRACKING ENTANGLEMENT GENERATION BETWEEN TWO SPATIALLY SEPARATED SUPERCONDUCTING QUBITS”

N. Roch et al., \textit{PRL} \textbf{112}, 170501, 2014
TWO DISTANT QUBITS

Theoretical proposal:
Kerckoff, Bouten, Silberfarb & Mabuchi,
JOINT DISPERSIVE MEASUREMENT

Theory Proposal:
Kerckhoff et al., Phys Rev A 2009
JOINT DISPERSIVE MEASUREMENT

JOINT DISPERSIVE MEASUREMENT
JOINT DISPERSIVE MEASUREMENT
No classical OR quantum observer can discriminate eigenstates; system is **perturbed, but not projected**, by measurement.

Hatridge et al., *Science* 2013
MEASUREMENT HISTOGRAMS

Read out

Qubits pulses

$R_1^{0,\pi}$

$R_2^{0,\pi}$

$\omega_{RO}$

$\omega_{qb1}$

$\omega_{qb2}$

$|\psi_i\rangle = |00\rangle$

$|\psi_i\rangle = |01\rangle$

$|\psi_i\rangle = |10\rangle$

$|\psi_i\rangle = |11\rangle$

Counts

$V_m (t_m = 0.65 \mu s) \text{ (Volts)}$
MEASUREMENT INDUCED ENTANGLEMENT

Quantifying the entanglement:

\[ C = \max(0, |\rho_{01,10}| - \sqrt{\rho_{00,00}\rho_{11,11}}) \]
TRAJECTORIES

Ensemble measurements:

Single experimental realization:

Quantum trajectory reconstruction allows us to directly observe quantum state evolution under measurement.

Single Qubit Trajectories: Murch et al., Nature 2013
Weber et al., Nature 2014
PULSE SEQUENCE & ANALYSIS

Joint Readout

Digitize and Average

Qubit 1

\( R_y^{n/2} \)

Qubit 2

\( R_y^{n/2} \)

\[ |\psi_i\rangle = \frac{|00\rangle + |01\rangle + |10\rangle + |11\rangle}{2} \]

\[ V_m(t_m) = \frac{1}{t_m} \int_0^{t_m} V_{\text{inst}}(t) \, dt \]
QUANTUM BAYESIAN UPDATE

\[ p(x|y) = \frac{p(x)p(y|x)}{p(y)} \]

\[ p(|ij\rangle|V_m) = \frac{p(|ij\rangle)p(V_m|ij\rangle)}{p(V_m)} \]

\[ f_{i,j}(V_m, t) = \frac{1}{\sqrt{2\pi}\sigma(t)} e^{-\frac{(V_m - S_{ij})^2}{2\sigma^2(t)}} \]

\[ \sigma(t) = \frac{1}{2\sqrt{\eta_{meas}t}} \]
BAYESIAN TRAJECTORY RECONSTRUCTION
BAYESIAN TRAJECTORY RECONSTRUCTION
CONDITIONAL TOMOGRAPHY MAPPING
FUTURE DIRECTIONS

• IMPROVE DETECTION EFFICIENCY (ON-CHIP PARAMPS)

• TRAVELING WAVE AMPLIFIERS (BW ~ 2 GHz)

• FEEDBACK STABILIZATION OF ENTANGLEMENT

• WEAK MEASUREMENT IN QUBIT CHAINS
  → Adaptive State Estimation (cf. tomography)
  → Weak Value Amplification of Errors/Couplings
  → Information Flow / Equilibration / Perturbations
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