1. Show that if U and V are unitary, then so is $U \otimes V$.

2. Write the 4×4 matrix of the unitary operation on two qubits resulting from performing a Hadamard transform on the first qubit and a phase flip on the second qubit.

3. Consider the unitary operation U resulting from applying the Hadamard gate to each of n qubits. Describe U by giving a formula for its $(x,y)^{th}$ entry.

4. Consider two qubits interacting with the Hamiltonian

$$H_I = g\sigma_z^{(1)} \otimes \sigma_z^{(2)}.$$

This is referred to as an Ising interaction and is a typical interaction between physical spins, e.g., between nuclear spins in liquids.

Show that $X^{(2)}U(t)X^{(2)} = U^{-1}(t)$, where $U(t) = e^{-iH_I t}$.

This result implies that $X^{(2)}U(t)X^{(2)}U(t) = 1$: the single qubit operations have effectively removed the interaction between the two qubits. This is referred to as 'refocusing' and can be used to remove undesired time evolution of interacting qubits when the interaction cannot be switched off.