1. Suppose you have two qubits in some arbitrary entangled state $|\psi\rangle$. You apply the teleportation protocol to each of the two qubits separately. What is the resulting state obtained by the other party? Why?

2. Create a "cloning circuit" that correctly clones the states $|+\rangle$ and $|-\rangle$. i.e. it takes as input two qubits $|\psi\rangle$ and $|0\rangle$, and in the case that $|\psi\rangle$ is either $|+\rangle$ or $|-\rangle$ it outputs $|\psi\rangle$ and $|\psi\rangle$.

3. Write down the observable A that measures the qubit in the standard basis, with measurement outcome 0 if the qubit is in state $|0\rangle$, and 1 if it is in state $|1\rangle$. Also write the observable B that measures in the Fourier basis, with outcome 1 if the qubit is in state $|+\rangle$, and -1 if it is in state $|-\rangle$.

4. a) Find the eigenvectors, eigenvalues, and diagonal representations of the Pauli matrices I, X, Y, and Z, where

$$I \equiv \sigma_0 \equiv \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad X \equiv \sigma_x \equiv \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad Y \equiv \sigma_y \equiv \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad Z \equiv \sigma_z \equiv \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

   Show that $X^2 = Y^2 = Z^2 = I$ (so that calculating X^n, Y^n or Z^n becomes really simple...)

   b) Find the points on the Bloch sphere that correspond to the normalized eigenvectors of the Pauli matrices.

   c) Find the action of the Z operator on a general qubit state $|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle$ and describe this action on the Bloch sphere (i.e. how does the vector representing $|\Psi\rangle$ get rotated on the Bloch sphere?).

   d) Form the matrix representation of the exponentiated operator $e^{-i\gamma Z/2}$ and show how this exponentiated operator acts on the Bloch sphere vector for $|\Psi\rangle$.

   e) Similarly form the matrix representations of the exponentiated operators $e^{-i\gamma X/2}$ and $e^{-i\gamma Y/2}$. Show explicitly how these act on the vector at the North pole of the Bloch sphere, i.e. on the qubit state $|\Psi\rangle = |0\rangle$, specifying the nature of the resulting rotation.