Quantum Computation Using Optical Lattices

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Physics 191
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Abstract

- **A detailed explanation of the physics behind optical lattices is presented. Cooling and trapping techniques are discussed. Finally, applications to Quantum Computation, specifically involving groups at Yale and UNM, are summarized.**

An optical lattice is much like a typical solid state crystalline lattice in that both lattices are repeatable arrays of particles stabilized by a minimum energy configuration. However, in the crystalline case the particles are held together via coulombic fields, while in the optical case, the particles are trapped in harmonic wells created by light fields.

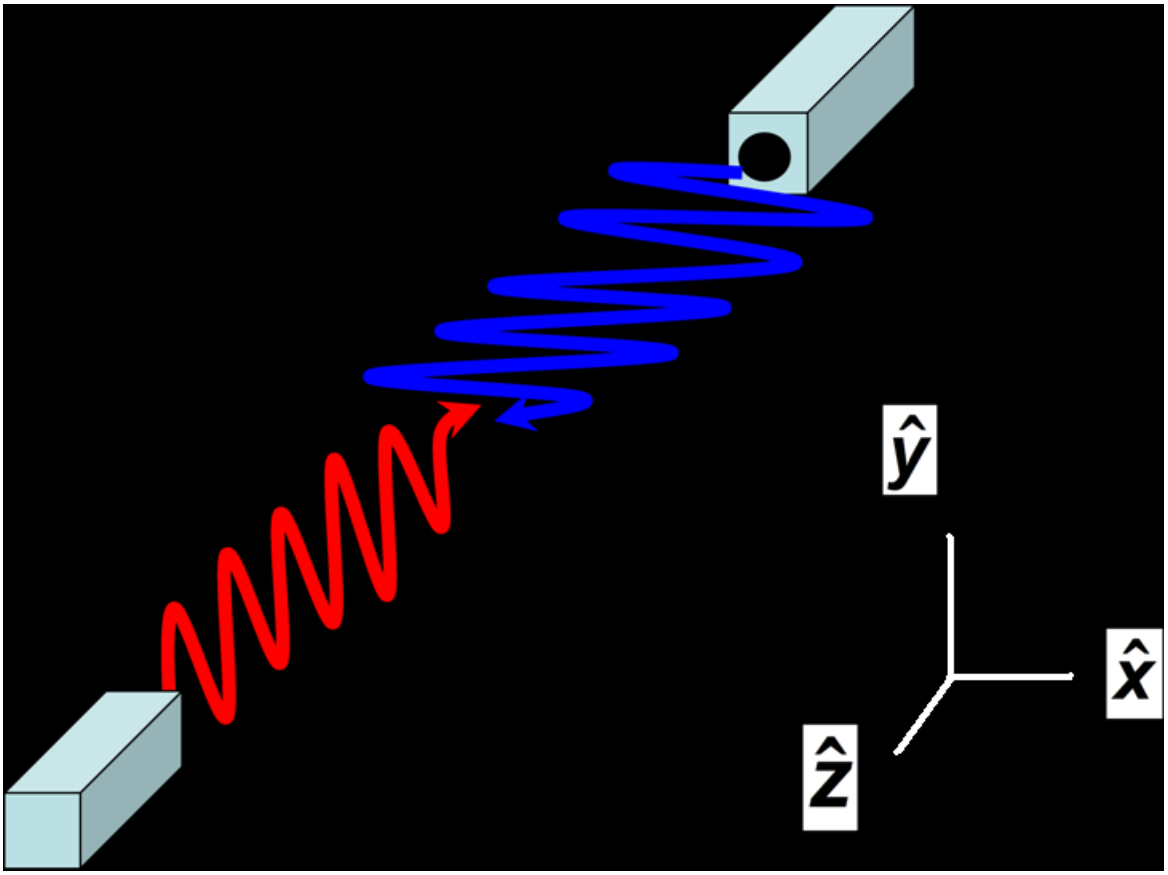

Figure 1: Lin-Perp-Lin Laser Setup for 1-D Optical Lattice. Blue laser has $\hat{x}$ linear polarization and red laser is counterpropagating in $\hat{z}$ direction with $\hat{y}$ polarization.

To see how these lattice sites are created, consider the simple case of a 1-D lin-perp-lin laser setup. In this setup (Figure 1), counterpropagating light waves are linearly polarized but in directions perpendicular to one another. The resulting field can be described by:

$$\boxed{1}\ \vec{E}[z,\,t] = \text{Sin}[k\,z + \omega_m\,t]\,\hat{x} + \text{Sin}[\,k\,z - \omega_m\,t]\,\hat{y}$$
$$= \text{Cos}[k\,z]\,\text{Sin}[\omega_m\,t]\,(\hat{x} - \hat{y}) + \text{Sin}[k\,z]\,\text{Cos}[\omega_m\,t]\,(\hat{x} + \hat{y})$$
$$\equiv \text{Cos}[k\,z]\,\hat{\epsilon_R} + \text{Sin}[k\,z]\,\hat{\epsilon_L}$$
$$\equiv\ |\,\sigma_-\rangle + |\,\sigma_+\rangle$$

Here k is the wave number, ω_m is the laser frequency, and $\hat{x}$ and $\hat{y}$ are perpendicular linear axes. Inspection of $\boxed{1}$ reveals that this is just an equation for two circularly polarized standing light waves with polarization vectors, $\hat{\epsilon_R} = \text{Sin}[\omega_m\,t]\,(\hat{x} - \hat{y})$, and $\hat{\epsilon_L} = \text{Cos}[\omega_m\,t]\,(\hat{x} - \hat{y})$. We've used $|\,\sigma_-\rangle$ and $|\,\sigma_+\rangle$ to denote the circularly polarized standing waves by convention. Figure 2 shows how these waves are separated by a phase shift corresponding to $\frac{\lambda}{4}$ where λ is the light wavelength.

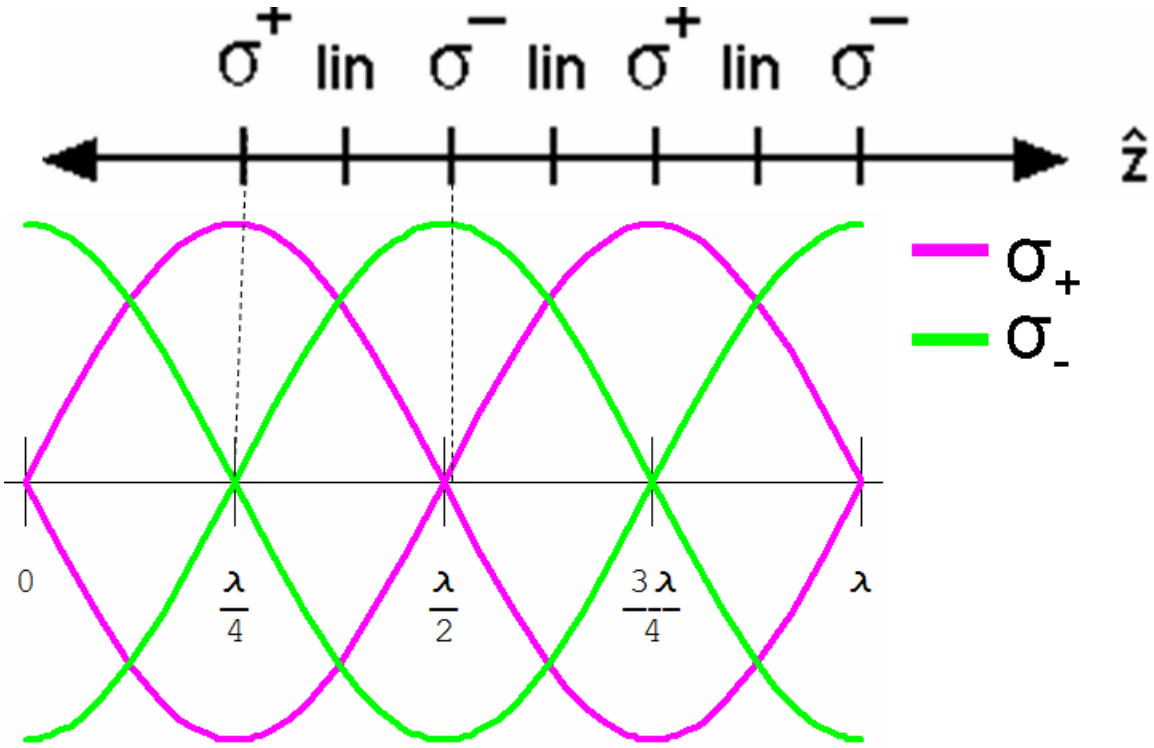

Figure 2: Two Circularly Polarized Standing Waves (denoted by $|\sigma_-\rangle$ and $|\sigma_+\rangle$). The figure clearly illustrates how at $z = \frac{n}{2}\lambda$, $n \in \mathbb{Z}$, we have exclusively $|\sigma_-\rangle$ light. In other words, the $|\sigma_-\rangle$ component of the standing wave configuration is at an antinode while the $|\sigma_\mp\rangle$ component is at a node). Similarly, at $z = \frac{2n+1}{4}\lambda$, we have exclusively $|\sigma_+\rangle$ light. At $z = \frac{2n+1}{8}$, we have $\text{Sin}[k\,z] = \text{Cos}[k\,z] = \frac{1}{\sqrt{2}}$. Thus the light is in an equally weighted superposition of circularly polarized states resulting in a true linear polarization. This is readily verified by inserting $z = \frac{2n+1}{8}$ into [1]. The result is light polarized in $\hat{x} \pm \hat{y}$ direction.

Thus far, we have explored the formation of a periodic spatially varying electric field. In order to see how this field leads to an array of harmonic potential wells, we need to understand the AC Stark Shift, also called simply "Light Shift". The Light Shift is an energy shift in the levels of a particle (anything with an electric dipole moment) caused by the introduction of a time dependent electric field due to, for instance, a laser beam. For the purposes of a more concise derivation, we assume a 2-level atom and take care of more complicated systems later.

We naturally start with the Time Dependent Schroedinger Equation:

$$\boxed{2}\quad H\,\psi = i\,\dot{\psi}$$

Here we've let $\hbar = 1$ in accordance with typical atomic units. The variables H and ψ represent the Hamiltonina and spatial wavefunction, respectively. Now the time-dependent Hamiltonian for an atom in an electric field is:

$$\boxed{3}\quad H_{\text{AC}}[t] = \vec{r}\cdot\overrightarrow{E_o}\,\text{Cos}[\omega_m\,t]$$

In this dipole-field coupling, ω_m is the laser frequency, $\vec{r}$ is the electron position with respect to the nuclear center of mass, and $\overrightarrow{E_o}$ is the amplitude of the laser electric field. Now we invoke the 2-level atom assumtpion to express the total wavefunction as:

$$\boxed{4}\quad \psi = c_1\,|\,1\rangle\,e^{-i\,\omega_i\,t} + c_2\,|\,2\rangle\,e^{-i\,\omega_2\,t} = \begin{pmatrix} c_1\,e^{-i\,\omega_1\,t} \\ c_2\,e^{-i\,\omega_2\,t} \end{pmatrix}$$

In the last step we used a matrix notation in the $\begin{pmatrix} |\,1\rangle \\ |\,2\rangle \end{pmatrix}$ basis. Furthermore, ω_n denotes the energy (modulo $\hbar$) of state $|\,n\rangle$ while c_n denotes the complex time-dependent coefficient of that state. The Hamiltonian acting on this space becomes:

$$\boxed{5}\ H_{\text{eff}} = \begin{pmatrix} 0 & \chi_{12} \mid \overrightarrow{E_o} \mid \text{Cos}[\omega_m t] \\ \chi_{21} \mid \overrightarrow{E_o} \mid \text{Cos}[\omega_m t] & 0 \end{pmatrix}$$

Here $\chi_{\text{ij}} \equiv \left\langle i \,\middle|\, \vec{r} \cdot \overrightarrow{E_o} \,\middle|\, j \right\rangle$. The diagonals are effectively 0 since $\chi_{\text{ii}} = 0$. This is because when the $\vec{r} \cdot \overrightarrow{E_o}$ term is expanded into sphereical harmonics, the raising action of these harmonics causes diagonal elements to vanish. Inserting [4]-[5] in [2] and simplifying gives the coupled differential equations:

$$\boxed{6}\ \begin{aligned} i\,\dot{c_1} &= \Omega_R \,\text{Cos}[\omega_m t]\, e^{-i\,\omega_o\, t}\, c_2 \\ i\,\dot{c_2} &= {\Omega_R}^* \,\text{Cos}[\omega_m t]\, e^{i\,\omega_o\, t}\, c_1 \end{aligned} \Longrightarrow i\,\frac{d}{dt}\,\vec{c} = \begin{pmatrix} 0 & \Omega_R \,\text{Cos}[\omega_m t]\, e^{-i\,\omega_o\, t} \\ {\Omega_R}^* \,\text{Cos}[\omega_m t]\, e^{i\,\omega_o\, t} & 0 \end{pmatrix} \cdot \vec{c}$$

The variable $\Omega_R \equiv \chi_{12} \mid E_o \mid$is the famous Rabi frequency for a two-level system, $\omega_o \equiv \omega_2 - \omega_1$ is the energy spacing between ground ($|1\rangle$) and excited ($|2\rangle$) states. The vectors denote the standard two-level basis. Next we use a substitution of variables to simplify matters:

$$\boxed{7}\ \vec{\tilde{c}} = e^{-i\,\delta\, t/2}\,\vec{c}$$

The variable $\delta \equiv \omega_m - \omega_o$. Next, we turn our attention back to [6]. The Rabi frequency, Ω_R,is real, thus ${\Omega_R}^* = \Omega_R$. Now we can expand the remaining critical term:

$$\boxed{8}\ \Omega_R \,\text{Cos}[\omega_m t]\, e^{\pm i\,\omega_o\, t} = \tfrac{1}{2}\,\Omega_R (e^{\pm i\,\delta\, t} + e^{\pm i\,(\omega_m + \omega_o)\, t}) \simeq \tfrac{1}{2}\,\Omega_R\, e^{\pm i\,\delta\, t/2}$$

The second term vanishes as it is a rapidly oscillating term that must necessarily average to zero over any reasonable interaction time scales. Now we may insert [7]-[8] in [6] to obtain (after some tedious algebraic manipulation):

$$\boxed{9}\ i\,\frac{d}{dt}\,\vec{\tilde{c}} = \tfrac{1}{2} \begin{pmatrix} \delta & \Omega_R \\ \Omega_R & -\delta \end{pmatrix} \cdot \vec{\tilde{c}}$$

Solution of [9] gives the general form $\vec{\tilde{c}} \propto e^{i\,\lambda/t}$ where λ are the eigenvalues of $\frac{1}{2}\begin{pmatrix} \delta & \Omega_R \\ \Omega_R & -\delta \end{pmatrix}$. It then becomes clear that these eigenvalues are precisely the energy shifts (from [4] and [7] its clear that $\vec{\tilde{c}} \propto \psi$ and thus the time evolution must be of the form $e^{i\,E\,t}$ leads us to se require that $\lambda = \Delta\text{E}$). The specific eigenvalues are then:

$$\boxed{10}\ \Delta\text{E}_{\text{Stark}_\pm} = \pm\,\tfrac{1}{2}\,\sqrt{\delta^2 + {\Omega_R}^2} \simeq \pm\left(\tfrac{\delta}{2} + \tfrac{{\Omega_R}^2}{4\,\delta}\right).$$

Here we've assumed that the $\frac{\delta}{\Omega_R} >> 1$. This large detuning is important for most experiments utilizing light shift, as absorption is negligible, enhancing the AC Starf shifts. The first term in the simplification of [10], $\pm\,\frac{\delta}{2}$ can be thought of as the creation of new states incorporating both atom and photons, but unperturbed by the light. The second term is the light perturbation. In general for more complicated atomic systems (more than 2 levels), Ω_R must be replaced by $C\,\Omega_R$ where C is the Clebsch-Gordon corresponding to the dipole matrix elements between the two states effected by the perturbation (recall that Ω_R resulted in simplifying the dipole matrix, something that in general does not occur for more complicated systems). Figure 3 does a better job of describing this picture.

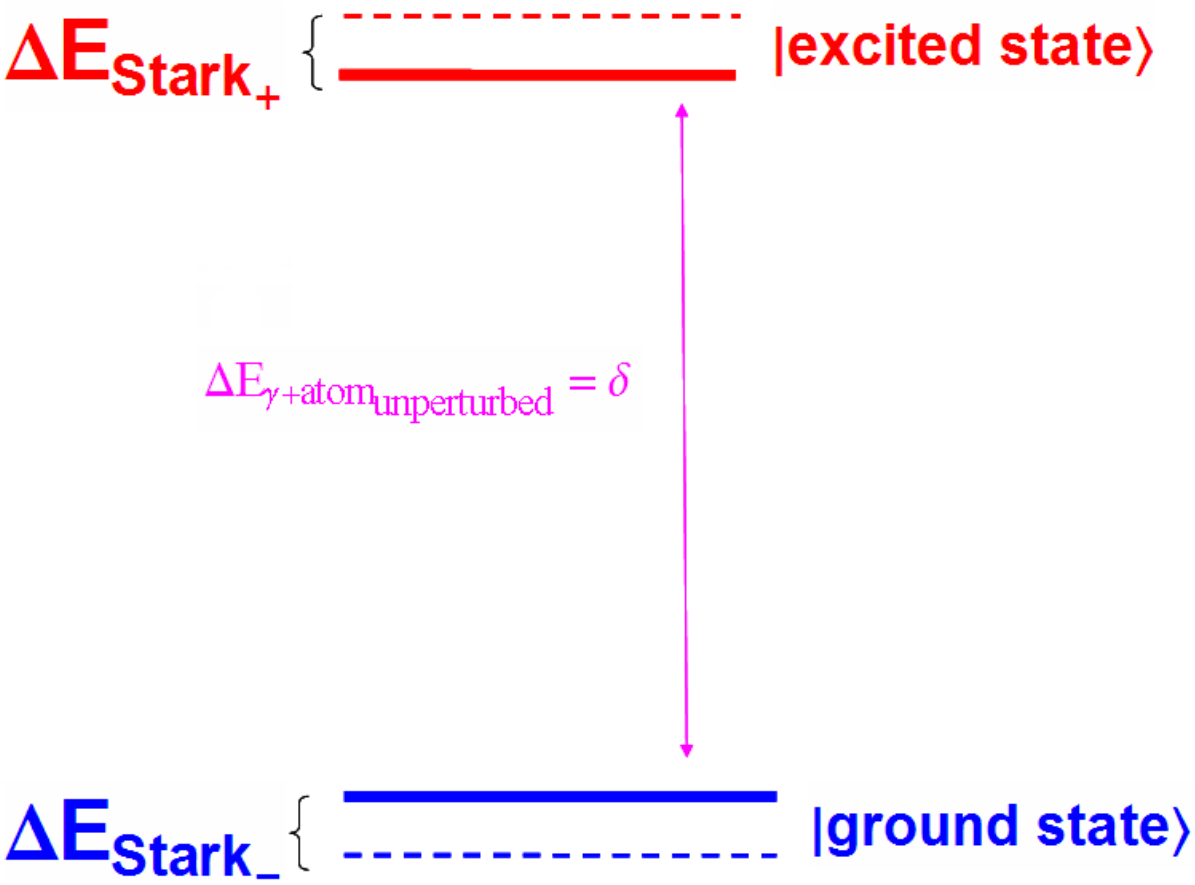

Figure 3: Energy Level Diagram for Light Shifts. The original two level system can be thought of has perturbing from a splitting of ω_o to ω_m. Then the light perturbation is added onto these new combination states shown here as $\Delta E_{Stark_\pm}$. In this case the detuning is red (δ>0). If the detuning had been blue, the energy shifts would go in the opposite directions (i.e. the ground state would increase energy while the excited state would decrease).

The dependence of the AC Stark shifts on the Clebsch-Gordan coefficients is the ultimate cause for the spatially varying harmonic well lattice sites. Figure 4 illustrates a level diagram for $J = \frac{1}{2} \rightarrow J = \frac{3}{2}$ transition with the square of the Clebsch-Gordon coefficients listed.

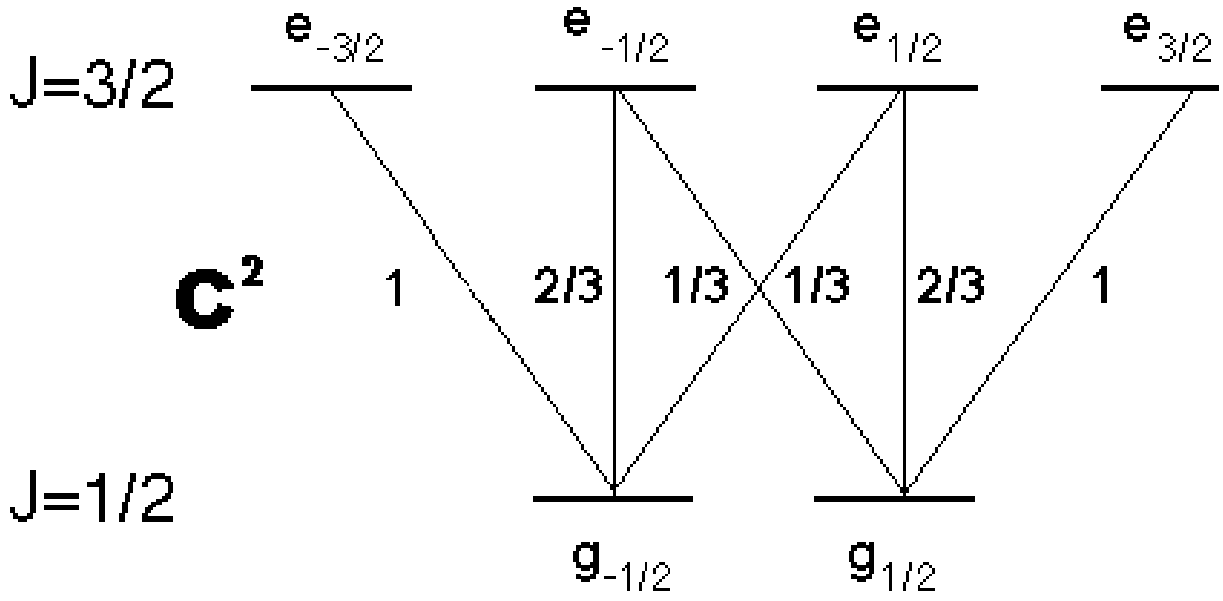

Figure 4: The Level Diagram for $J=\frac{1}{2} \rightarrow J=\frac{3}{2}$ Transition. The squre of the Clebsch-Gordon coeffiecients (C^2) are listed showing the relative coupling between states. Since Δm=±1 corresponds to $|\sigma_{\pm}\rangle$ light and Δm=0 corresponds to linear light, one can see that the presence of differences in coupling strengths causes the Light Shift to vary based on the type of light the atoms are exposed to. Since this type of light was shown to vary spatially in Figure 2, we now have a spatially varying potential.

In Figure 4, one can easily see that $|\sigma_{\pm}\rangle$ and linear light all are capable of providing a different AC Stark shift on the atoms in the beam path. Figure 5 returns to Figure 2 and ammends it to show the energy's dependence on propagation distance.

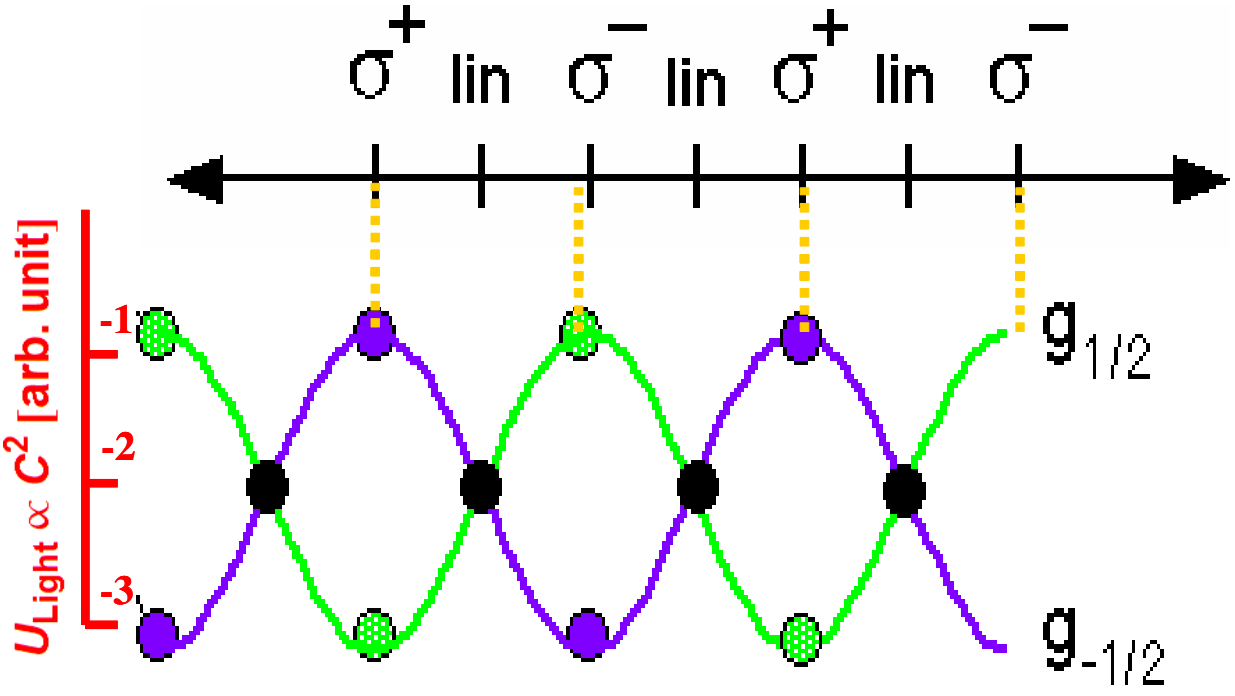

Figure 5: The Spatial Depenedence of the Potential due to Lin-Perp-Lin Setup. The vertical axis shows the change in energy for red detuned light from the unperturbed ground state. The degeneracy in ground states is now evident due to the varying strengths of coupling between these two states. Furthermore, the states with the strongest coupling for $g_{\pm 1/2}$ are Δm=±1 corresponding to $|\sigma_{\pm}\rangle$. Since this laser is red detuned, this corresponds to $|\sigma_{\pm}\rangle$ light having a greater Light Shift and consequently a lower ground state potential. Note that the degeneracy remains at the nodes (linear regime) as the coupling strengths are equal for both Δm=0 transitions ($C^2 = 2/3$, see Figure 4).

Figure 5 nicely illustrates the periodic harmonic wells created first by the spatial variation in types of light and then the dependence of the atomic potential on type of light due to AC Stark Shifts.

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To be able to keep atoms or molecules in an optical lattice, we must have a way to reduce their kinetic energy to less than the potential energy required to leave each individual trap. This is actually a multi-step process involving pre-cooling using techniques such as optical molasses in a magneto-optical trap followed by an extreme cooling by optical pumping technique known as Sisyphus cooling. As we will see in a moment, the setup of our optical lattice is actually already well set to perform both of these cooling techniques.

The main idea behind optical molasses is to use the momentum of incoming photons on a particle to slow that particle. This requires a source of light that will be the correct frequency to interact with a certain particle as well as a way to make sure that the momentum of the photon absorbed by a particle is anti-parallel to the motion of the particle. If we did not have some way to ensure opposite momentums of the photon and the particle, then the photon would be just as likely to add momentum to the particle as take it away, and the average energy of all the particles would not change. How we deal with this problem is to select a laser frequency near a resonance of our particle, so we know the particle will interact with the light. We then slightly detune the laser to a slightly lower frequency than resonance. Due to the Doppler effect, particles moving towards the laser to see a slightly higher frequency of light, and a little closer to resonance, and particles moving away will see a slightly lower frequency, even farther from resonance. The closer the frequency of light that the particle sees is to resonance, the more likely it is to interact with that photon. We can see that particles moving towards the laser see a frequency closer to resonance than those moving away, and are therefore more likely to interact with the laser light and absorb some of its momentum. The particles will absorb the momentum of the light and begin to slow, and as they slow, the Doppler effect will become smaller and smaller, and the particles will interact less and less with the light, until they are moving at speeds that are very small fractions of their original speed.

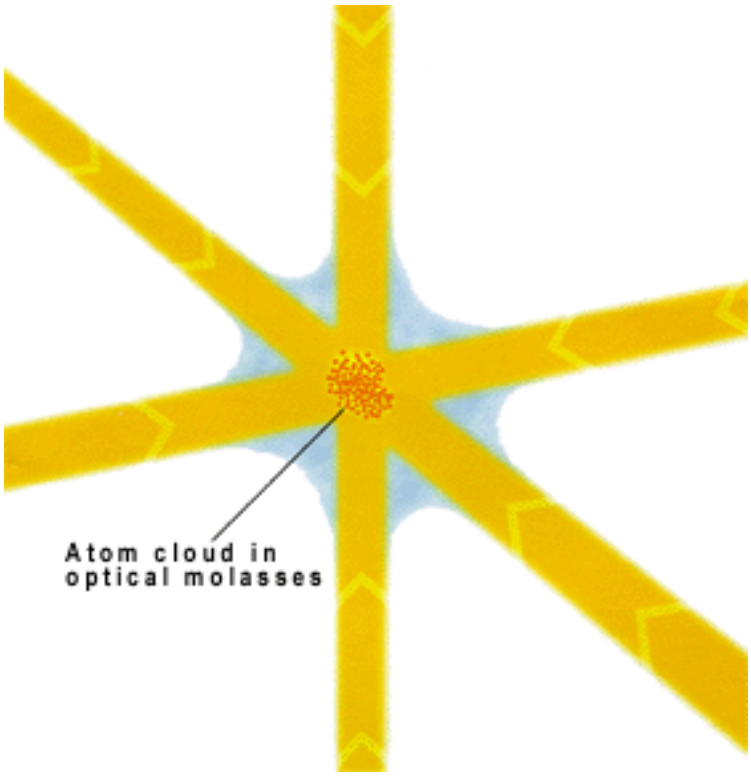

Figure B1: A diagram of a magneto-optical trap. The lasers all point towards the center and the shading around the center represents the magnetic field gradient.

If we use three sets of anti-parallel lasers all perpendicular to each other, we can set up a region at the intersection of these lasers in which a particle will see an anti-parallel photon approaching it no matter what direction it is moving (Figure B1). We have also seen before that we can use anti-parallel lasers to form our optical lattice, so with some careful planning, it is not to difficult to tune our lasers to the proper settings for an optical lattice once the particles have been trapped. In order to slow these particles enough for us to perform Sisyphus cooling on them, a magnetic field gradient is introduced around the intersection of the lasers that further prevents particles from leaving that area.

After pre-cooling the particles, we can adjust the lasers to create the polarization and spatially dependent potentials of the optical lattice. The particles will fall into the potential wells, but have enough energy to “climb” the walls of that well. Once a particle moves up the wall to a higher potential and a different position, it will absorb a photon and be excited (Figure B2).

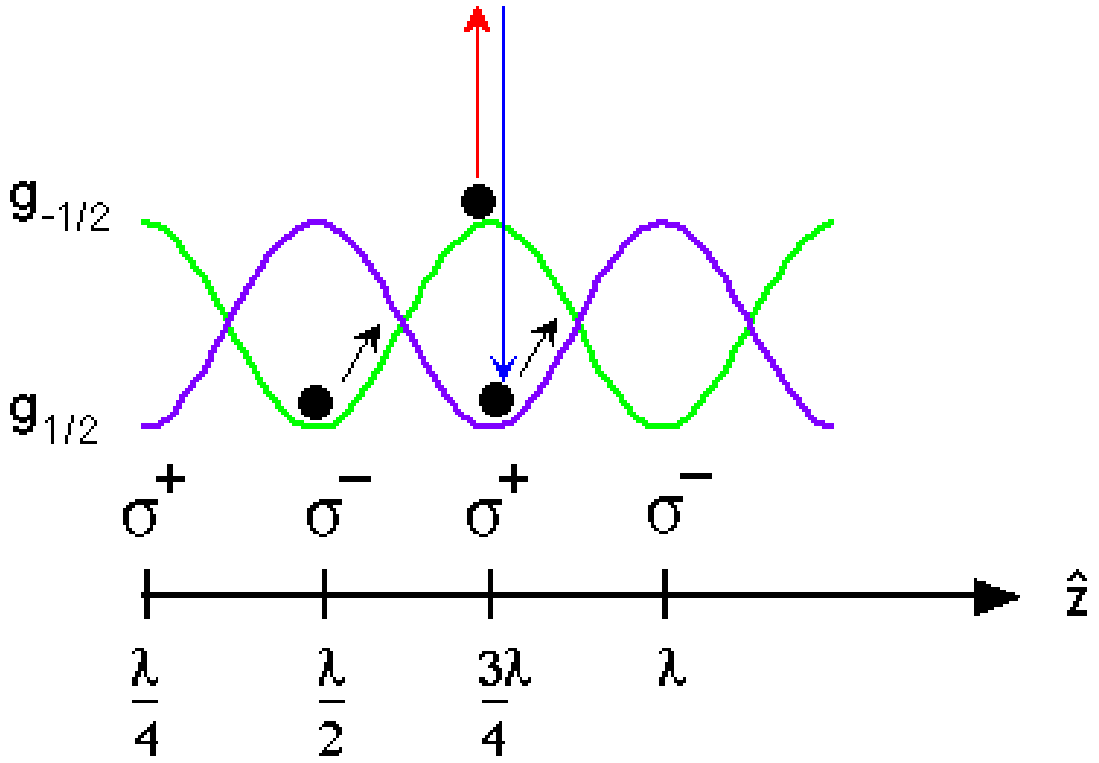

Figure B2: As a particle moves out of the well, it is excited (red line) and then drops back into a lower ground state with less energy (blue line)

When this particle drops back into the ground state due to spontaneous emission, the ground state potential is lower than the potential that the particle was excited from. This means that the particle loses more energy than it gains, and this loss is lost in the kinetic energy of the particle. After this process repeats many times, the particle will lose enough energy that it will become trapped in the potential well, and can then be manipulated to our desire.

Once we have trapped our particles we can utilize the properties of these particles as qubits in a register. There is currently research going on that hopes to utilize the properties of neutral atoms, polar molecules and ions as qubits in an optical lattice. I will go into the particular research of a couple of physicists as specific examples of how they hope to implement qubits and gates. Professor Demille works at Yale University and hopes to use polar molecules as qubits and Professor Deutsch does research at the University of New Mexico on using neutral atoms as qubits, and these will be the examples I will describe.

Both of these professors hope to utilize the dipole-dipole interaction between particles as a means of entangling qubits. The dipole-dipole interaction is a force that is inversely proportional to the cube of the distance(r) between those dipoles (d_1,d_2).

$V_{dipole}\ d_1 d_2/r^3$

Due to this fact, either the dipole moments need to be very close together to be entangled or their dipole moments must be very large. The disadvantage of the dipole-dipole interaction is that large dipoles interact strongly with the environment, and this coupling to the environment will introduce large errors into the process of quantum information transfer. Therefore

it is the job of the researchers to develop a method to maximize the coupling to other dipoles and minimize the coupling to the environment.

Professor DeMille hopes to utilize the strong dipole moments of polar molecules as qubits in an optical lattice. These molecules will be arranged in a 1-D optical lattice with a differential applied electric field that will allow for spectroscopic addressing of individual sites (Figure B3). They would like to use short microwave pulses as single qubit not gates, an applied electric field will cause a rotation of the dipole. They would also like to use longer microwave pulses to act as a two qubit CNOT gate. As it turns out, we can utilize the coupling between dipoles to help enable the CNOT gate: by leaving the proper spacing between pulses, we can use the coupling between dipoles as to create the proper rotation of the dipoles.

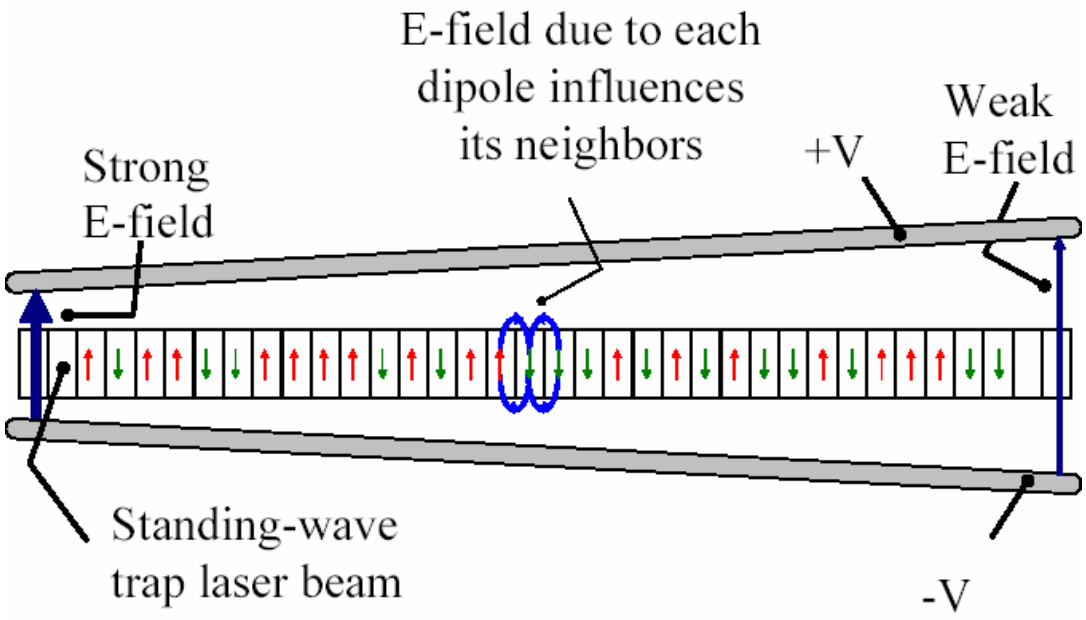

Figure B3: Arrangement of polar atoms in an electric field gradient allows for spectroscopic addressing of individual sites

The strong dipole moments of the polar molecules generates strong coupling between qubits but also creates an unwanted evolution of nearby qubits during gate actions. We could address all of these unwanted rotations individually with short laser pulses, but this would be extremely inefficient. Luckily, by performing what we referred to in class as “refocusing”, we can address the all of the unwanted rotations as a system. By properly applying the correct pulses, we can “act” the Hadamard rotation before and after our desired action, which will negate the unwanted changes to other qubits in the system. Professor DeMille also believes that this is a good setup to try and build a quantum computer because the technology required has all been tested and requires no dramatic breakthroughs.

At the University of New Mexico, Professor Deutsch is doing research on using neutral atoms as qubits. This group also hopes to utilize the dipole -dipole interaction between atoms to make a two-qubit gate and uses short laser pulses as single-qubit gates. Due to the small magnitude of the dipole moments of neutral atoms, the coupling between dipoles is only strong for very nearby qubits. This means that to entangle qubits, we need to bring them together and when we don’t want them to interact, we can move them apart. We can move the particles by slowly changing the polarization of one laser

which acts to bring two potential wells closer to each other (Figure B4).

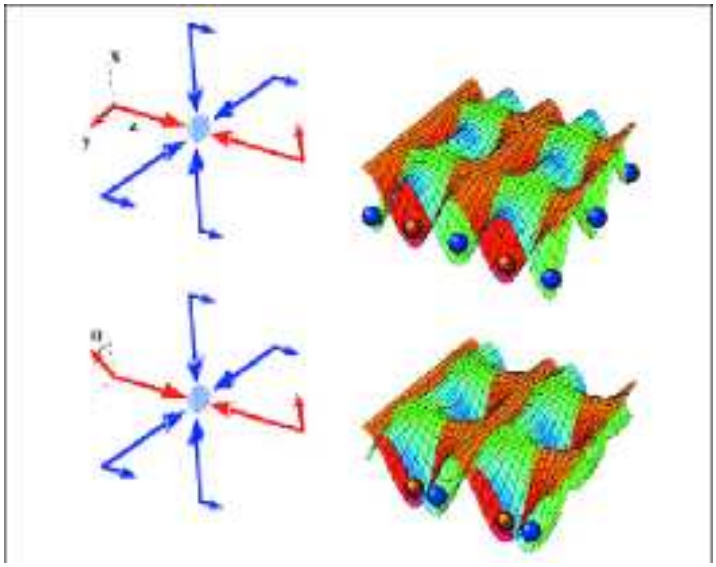

Figure B4: By changing the polarization of one laser, we can move two wells closer to each other and then entangle the qubits

Once we have brought two qubits together, we can use laser pulses to excite dipoles of the atoms, and sufficiently timed laser pulses can act as a controlled phase gate (CPHASE) on the qubits. The CPHASE gate is another two-qubit gate that can be used to entangle states for quantum computation. Though there has been much research on how to implement single-qubit gates, there is a great deal of work to be done on implementation of two-qubit gates as well as quantum entanglement. There are many good ideas out there that still need to be tested and the Deutsch group spends much of its time studying the interaction between nearby qubits and their coupling, and hope to soon see major progress towards enabling a two-qubit gate.

In conclusion, we can see that optical lattices hold good promise for a quantum register in the future. Our ability to manipulate the structure of the lattice as well as our ability to target individual spots on the lattice conforms to the constraints necessary to perform gate actions on our qubits. It also allows for a variety of different types of qubits, including neutral atoms, ions and polar molecules. Due to the flexibility of an optical lattice, I believe we will be able utilize its properties to greatly advance our understanding of entanglement as well as our understanding of how to implement quantum gates, if not enable to us actually build a quantum computer.

I would like to thank the following websites for the use of their information and diagrams:

(B1) http://nobelprize.org/physics/laureates/1997/illpres/
(B2) http://quaser.physics.lsa.umich.edu/projects/lattice/
(B3) http://pantheon.yale.edu/%7Edpd5/quantumcomp.htm
(B4) http://www.ieee.org/organizations/pubs/newsletters/leos/apr02/academic.html