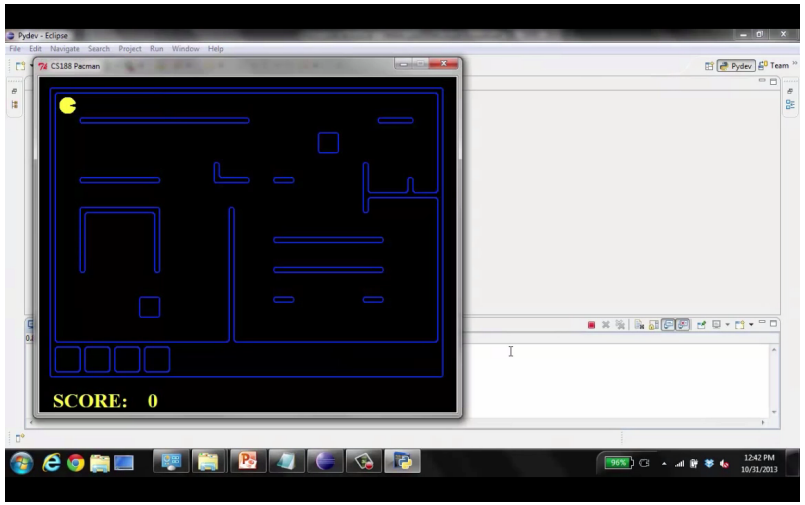
Today.

Continuing Hidden Markov Models!

Hidden Markov Models

Video of Demo Pacman – Sonar (no beliefs)



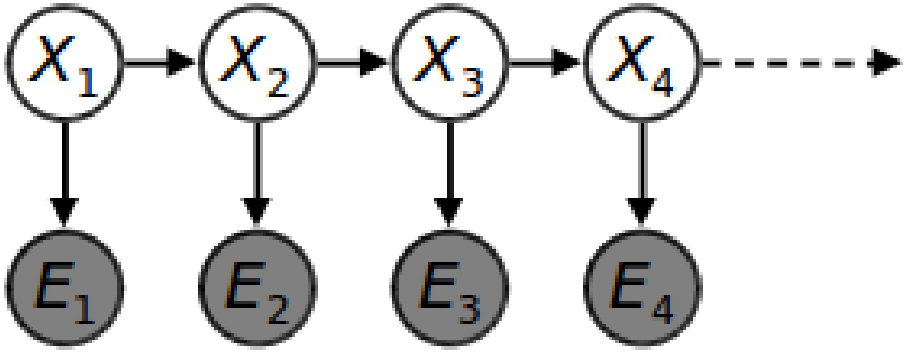
Hidden Markov Models

Markov chains not so useful for most agents

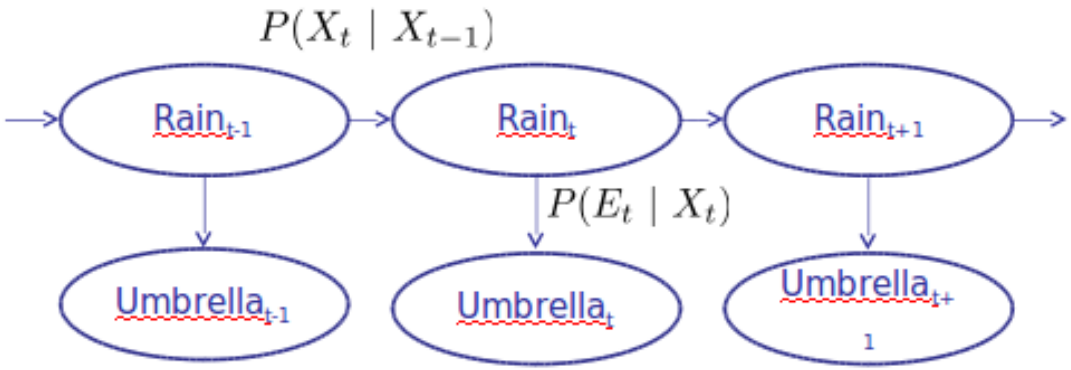
- Need observations to update your beliefs

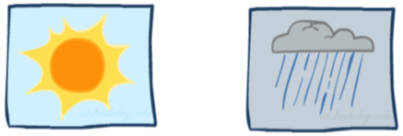
Hidden Markov models (HMMs)

- Underlying Markov chain over states X
- You observe outputs (effects) at each time step



Example: Weather HMM





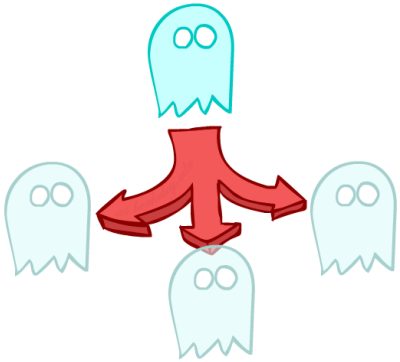
Rt-1	Rt P(Rt|Rt-1)
+r	+r 0.7
+r	-r 0.3
-r	+r 0.3
-r	-r 0.7

Rt	Ut	P(Ut|Rt)
+r	+u	0.9
+r	-u	0.1
-r	+u	0.2
-r	-u	0.8

An HMM is defined by:

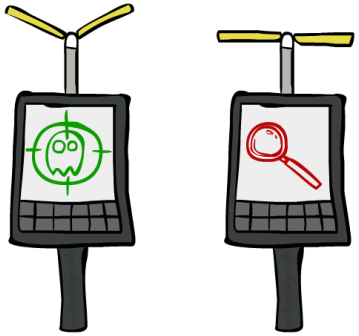
- Initial distribution: $P(X_1)$
- Transitions:$P(X_t|X_{t-1})$
- Emissions: $P(E_t|X_t)$

Example: Ghostbusters HMM



1/9	1/9	1/9
1/9	1/9	1/9
1/9	1/9	1/9

$P(X_1)$



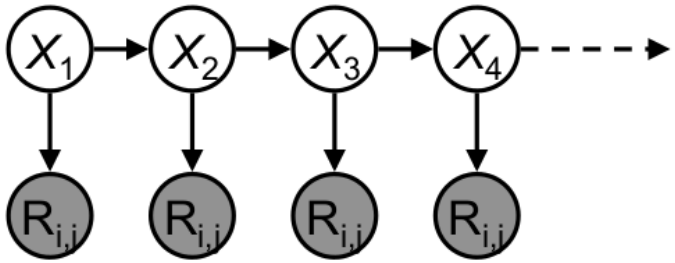
1/6	1/6	0
1/6	1/2	0
0	0	0

$P(X|X' = \langle 1,2 \rangle)$

$P(X_1)$ = uniform

$P(X|X')$ = usually move clockwise, but sometimes move in random direction or stay in place

$P(R_{ij}|X)$ = same sensor model as before: red means close, green/yellow means farther away.



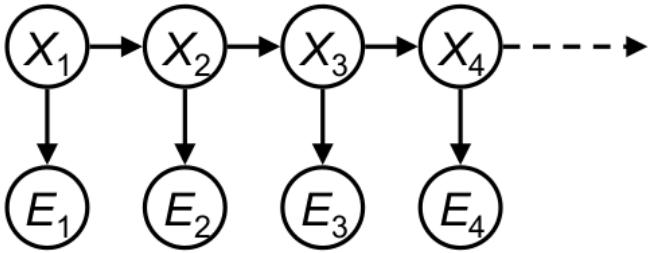
[Demo: Ghostbusters – Circular Dynamics – HMM (L14D2)]

Video of Demo Ghostbusters – Circular Dynamics – HMM

Conditional Independence

HMMs have two important independence properties:

- Markov hidden process: future depends on past via the present
- Current observation independent of all else given current state



Are E_1 and E_3 independent now? No!!
Need to condition on X_1, or X_2 or X_3.

Real HMM Examples

Robot tracking:

- Observations are range readings (continuous)
- States are positions on a map (continuous)

Speech recognition HMMs:

- Observations are acoustic signals (continuous valued)
- States are specific positions in specific words (so, tens of thousands)

Machine translation HMMs:

- Observations are words (tens of thousands)
- States are translation options

Filtering / Monitoring

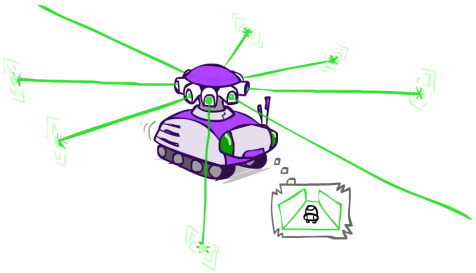
Filtering, or monitoring, is the task of tracking the distribution $B_t(X) = P_t(X_t|e_1, \ldots, e_t)$ (the belief state) over time

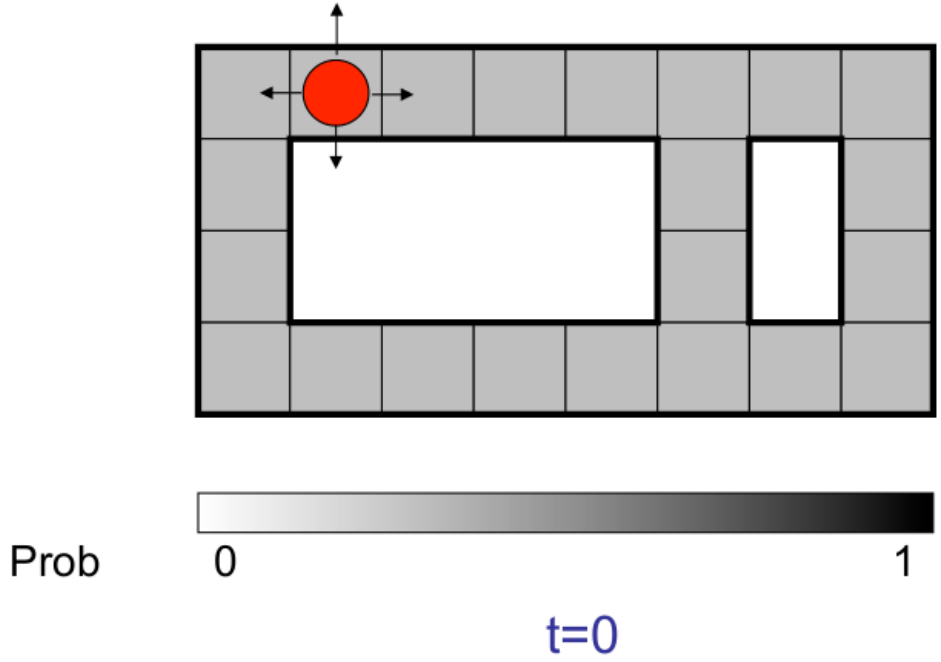
We start with $B_1(X)$ in an initial setting, usually uniform

As time passes, or we get observations, we update $B(X)$

The Kalman filter was invented in the 60's and first implemented as a method of trajectory estimation for the Apollo program

Example: Robot Localization



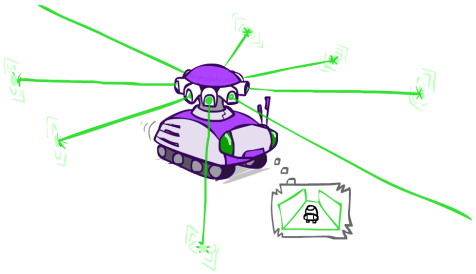


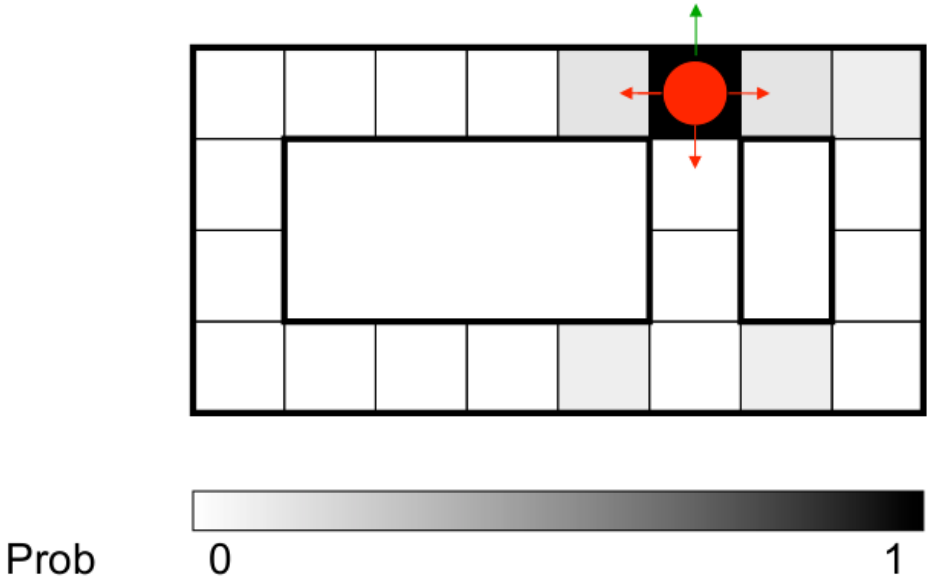
Sensor model: can read in which directions there is a wall, never more than 1 mistake

Motion model: may not execute action with small prob.

Example from Michael Pfeiffer

Example: Robot Localization





t=5

Lighter grey: was possible to get the reading, but less likely b/c required 1 mistake

Inference: Find State Given Evidence

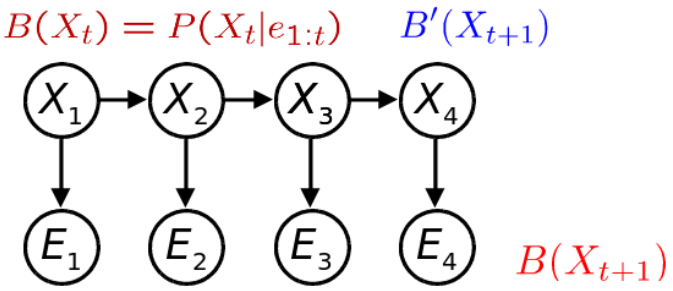
We are given evidence at each time and want to know

$$B_t(X) = P(X_t|e_{1:t})$$

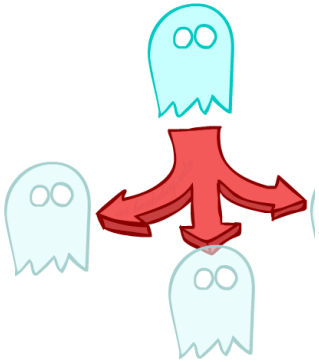
Idea: start with $P(X_1)$ and derive B_t in terms of B_{t-1}

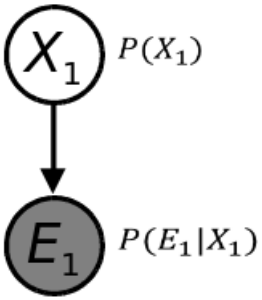
- equivalently, derive B_{t+1} in terms of B_t.

Two Steps: Passage of Time + Observation

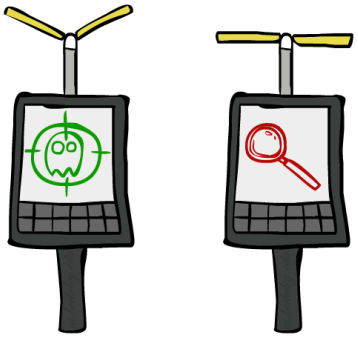


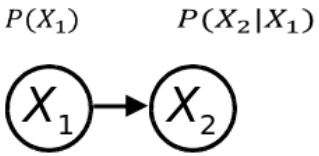
Inference: Base Cases





$$P(X_1|e_1) = P(X_1, e_1)/P(e_1)$$
$$\propto P(X_1)P(e_1|X_1)$$

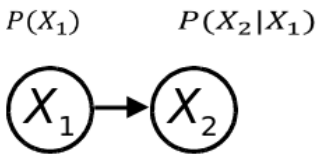




$$P(X_2) = \sum_{X_1} P(x_1, X_2)$$
$$= \sum_{X_1} P(X_2|x_1)P(x_1)$$

Passage of Time

Assume we have current belief $P(X|\text{evidence to date})$



$$B(X_t) = P(X_t|e_{1:t})$$

Then, after one time step passes:

$$\begin{aligned} P(X_{t+1}|e_{1:t}) &= \sum_{x_t} P(X_{t+1}, x_t|e_{1:t}) \\ &= \sum_{x_t} P(X_{t+1}|e_{1:t}, x_t) P(x_t|e_{1:t}) \\ &= \sum_{x_t} P(X_{t+1}|x_t) P(x_t|e_{1:t}) \end{aligned}$$

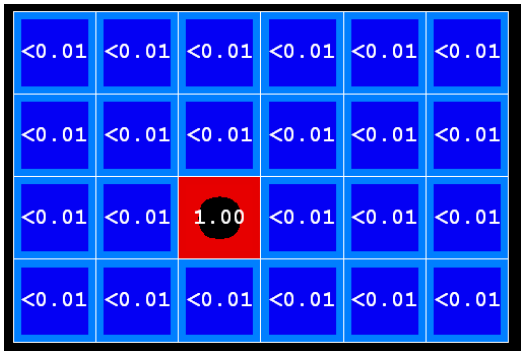
Or, compactly: $B'_{t+1}(X_{t+1}) = \sum_{x_t} P(X_{t+1}|x_t) B_t(x_t)$

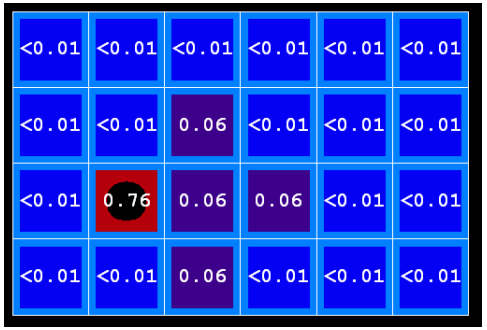
Basic idea: beliefs get "pushed" through the transitions

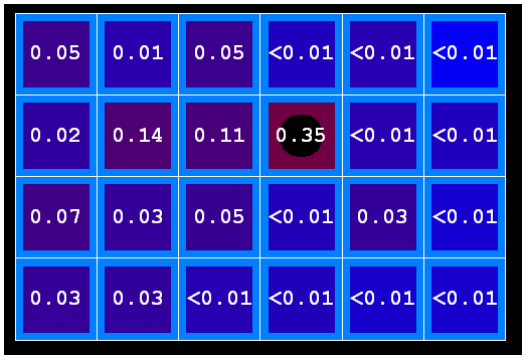
- With the "B" notation: $B'_t(\cdot)$, without e_t, $B_t(\cdot)$, with e_t.

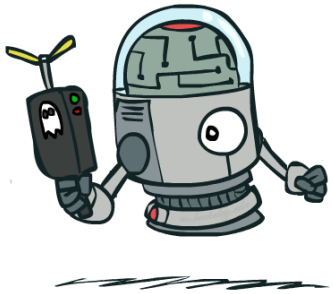
Example: Passage of Time

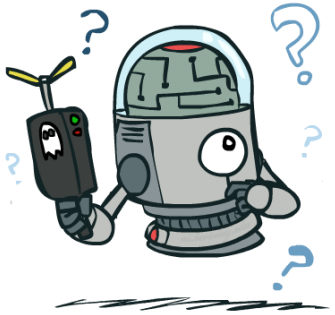
As time passes, uncertainty “accumulates”.



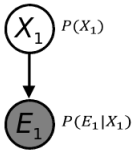








Observation



Assume we have current belief $P(X|\text{previous evidence})$:

$$B'(X_{t+1}) = P(X_{t+1}|e_{1:t})$$

Then, after evidence comes in:

$$\begin{aligned} P(X_{t+1}|e_{1:t+1}) &= P(X_{t+1}, e_{t+1}|e_{1:t})/P(e_{t+1}|e_{1:t}) \\ &\propto P(e_{t+1}|e_{1:t}, X_{t+1})P(X_{t+1}|e_{1:t}) \\ &= P(e_{t+1}|X_{t+1})P(X_{t+1}|e_{1:t}) \end{aligned}$$

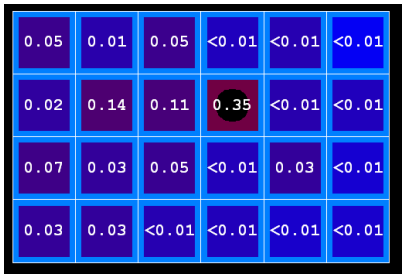
Or, compactly: $B(X_{t+1}) \propto P(e_{t+1}|X_{t+1})B(X_t)$

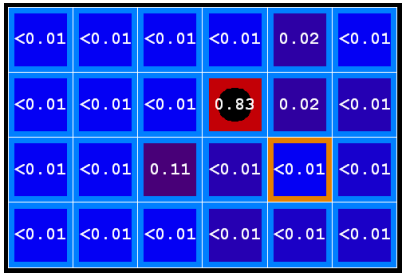
Basic idea: beliefs “reweighted” by likelihood of evidence

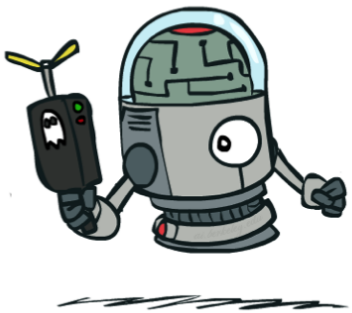
Unlike passage of time, we have to renormalize

Example: Observation

Before obser-vation





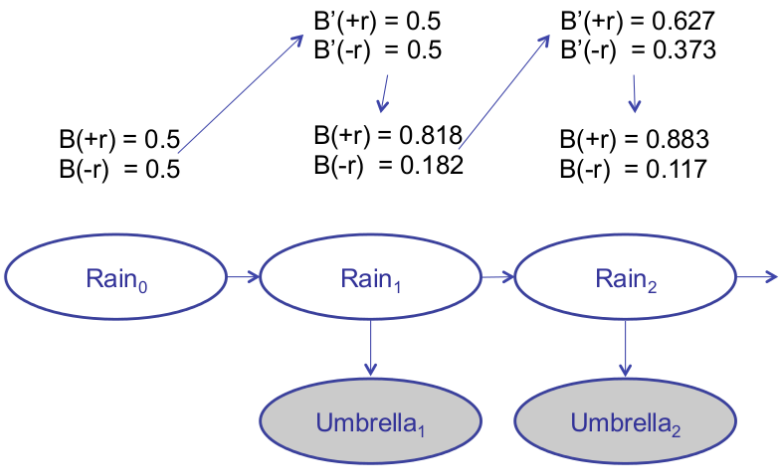


After obser-vation

As we get observations, beliefs get reweighted, uncertainty "decreases"

$$B(X) \propto P(e|X)B'(X).$$

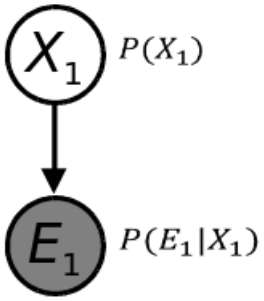
Example: Weather HMM:  



R_t	R_{t+1}	$P(R_{t+1} \mid R_t)$
+r	+r	0.7
+r	-r	0.3
-r	+r	0.3
-r	-r	0.7

R_t	U_t	$P(U_t \mid R_t)$
+r	+u	0.9
+r	-u	0.1
-r	+u	0.2
-r	-u	0.8

Online Belief Updates



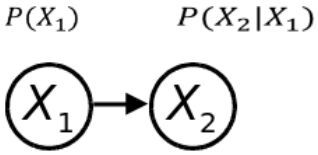
Every time step, we start with current $P(X|\text{evidence})$

We update for time:

$$P(x_t|e_{1:t}) = \sum_{x_{t-1}} P(x_{t-1}|e_{1:t-1}) \cdot P(x_t|x_{t-1})$$

We update for evidence:

$$P(x_t|e_{1:t}) \propto_X P(x_{t-1}|e_{1:t-1}) \cdot P(x_t|x_{t-1})$$



The forward algorithm does both at once (and doesn't normalize)

The Forward Algorithm

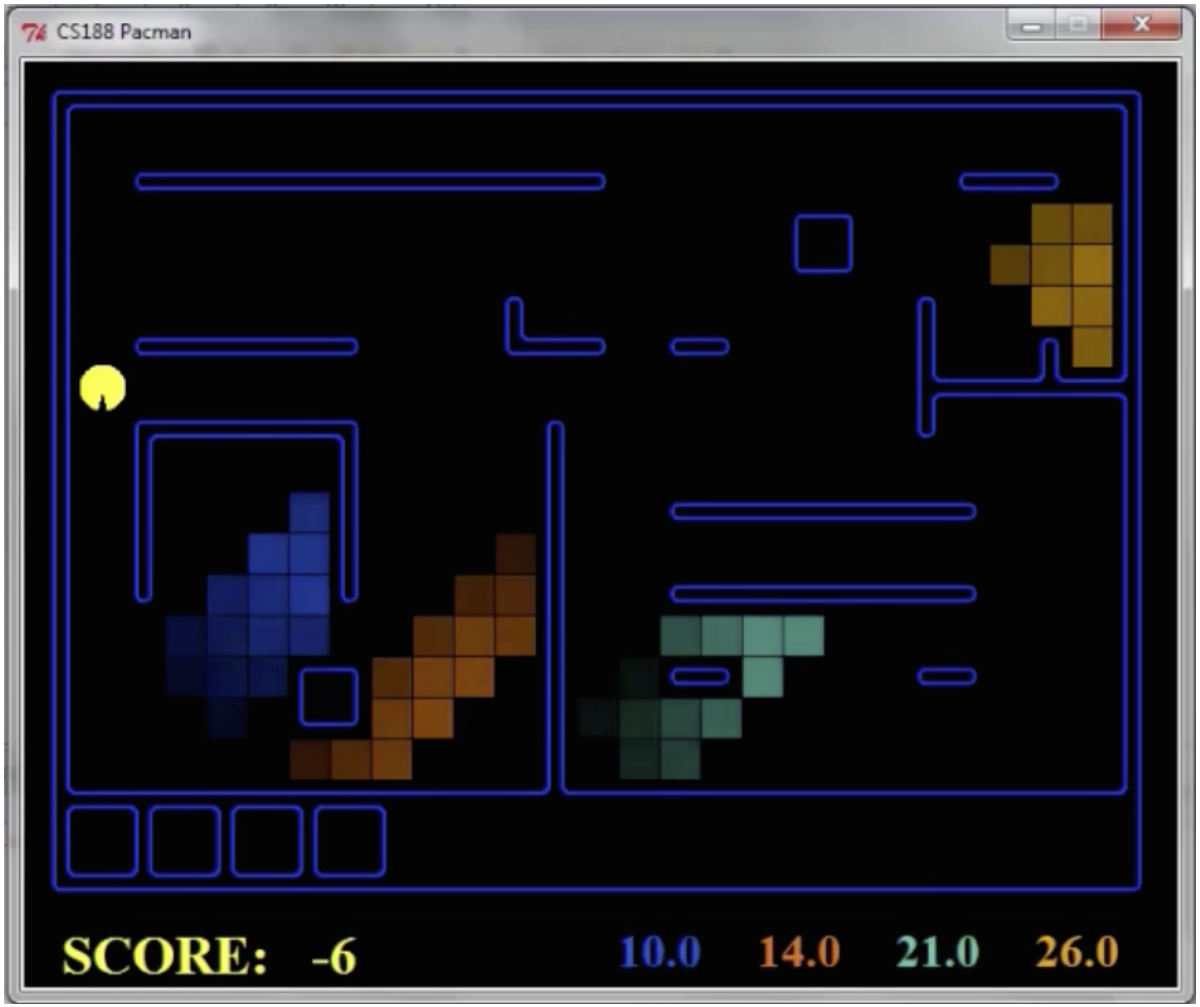
We are given evidence at each time and want to know:

$$B_t(X) = P(X_t|e_{1:t})$$

We can derive the following updates

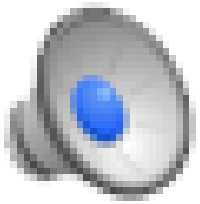
$$\begin{aligned} P(X|e_{1:t}) \propto P(X_t|e_{1:t}) & \text{ Normalize anytime.} \\ = \textstyle\sum_{x_{t-1}} P(x_{t-1}, x_t|e_{1:t}) & \\ = \textstyle\sum_{x_{t-1}} P(x_{t-1}|e_{1:t-1})P(x_t|x_{t-1})P(e_t|x_t) & \\ = P(e_t|x_t)\textstyle\sum_{x_{t-1}} P(x_t|x_{t-1})P(x_{t-1}|e_{1:t-1}) & \end{aligned}$$

Pacman – Sonar (P4)



[Demo: Pacman – Sonar – No Beliefs(L14D1)]

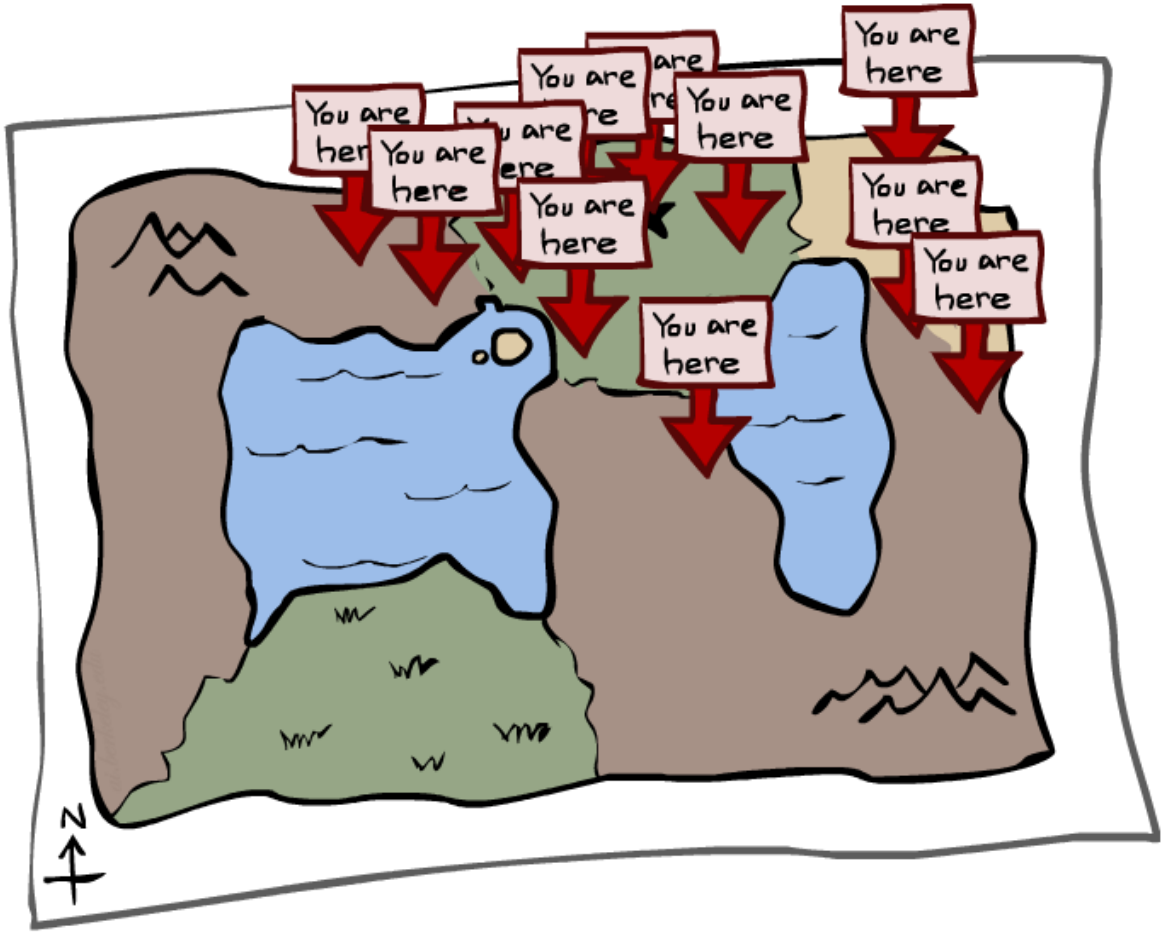
Video of Demo Pacman – Sonar (with beliefs)



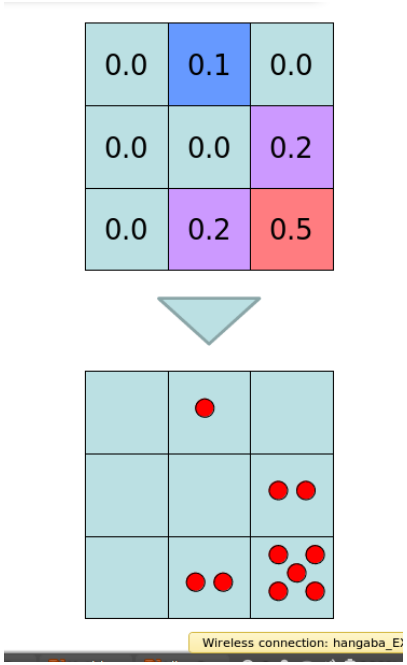
Next Up

Particle Filtering and Applications of HMMs

Particle Filtering



Particle Filtering



Filtering: approximate solution

Sometimes $|X|$ is too big to use exact inference

- $|X|$ may be too big to even store $B(X)$
- E.g. X is continuous

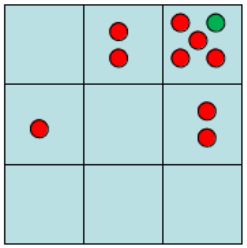
Solution: approximate inference

- Track samples of X, not all values
- Samples are called particles
- Time per step is linear in the number of samples
- But: number needed may be large
- In memory: list of particles, not states

This is how robot localization works in practice

Particle is just new name for sample

Representation: Particles



Particles:
(3,3)
(2,3)
(3,3)
(3,2)
(3,3)
(3,2)
(1,2)
(3,3)
(3,3)
(2,3)

Our representation of P(X) is now a list of N particles (samples)

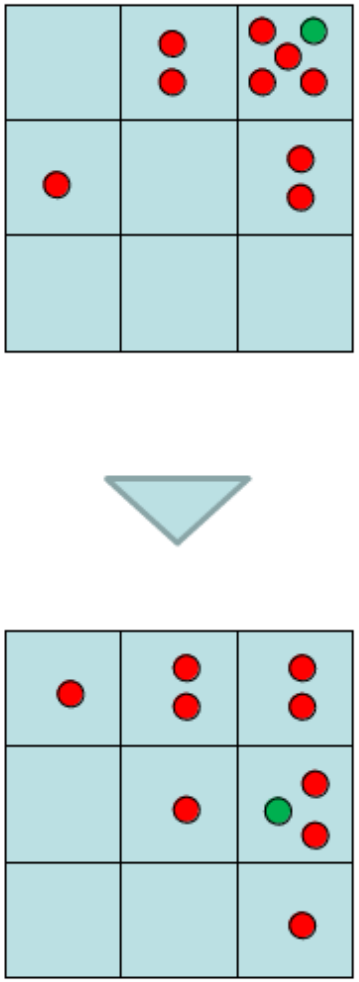
- Generally, $N << |X|$
- Storing map from X to counts would defeat the point

$P(x)$ approximated by number of particles with value x

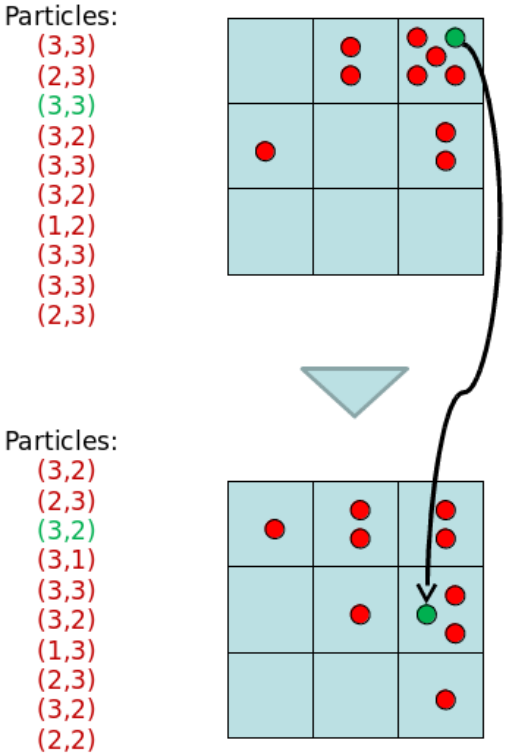
- So, many x may have $P(x) = 0$!
- More particles, more accuracy

For now, all particles have a weight of 1

Particle Filtering: Elapse Time



Particle Filtering: Elapse Time



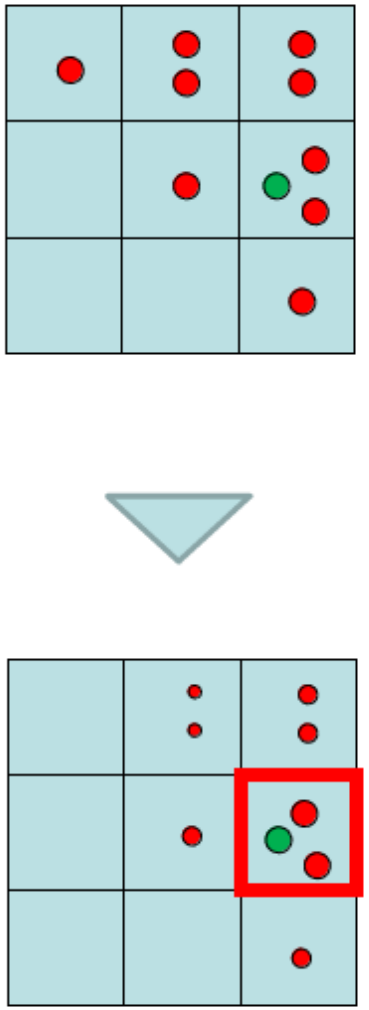
Each particle is moved by sampling its next position from the transition model

- This is like prior sampling – samples' frequencies reflect the transition probabilities
- Here, most samples move clockwise, but some move in another direction or stay in place

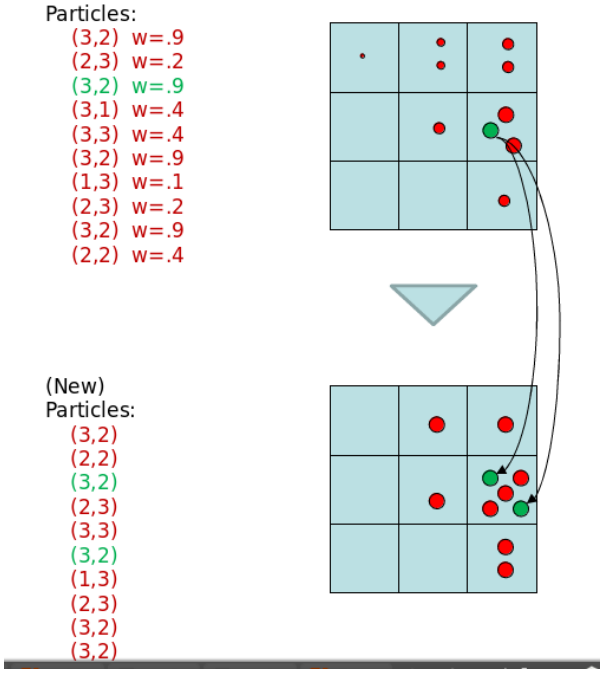
This captures the passage of time

- If enough samples, close to exact values before and after (consistent)

Particle Filtering: Observe



Particle Filtering: Observe



Slightly trickier.

- Don't sample observation, fix it
- Similar to likelihood weighting, downweight samples based on the evidence

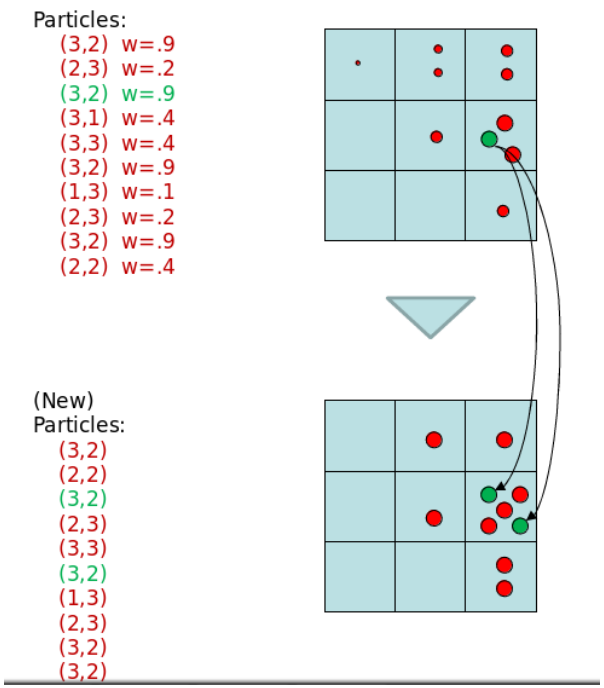
$$w(s) = P(e|x)$$

$$B(X) \propto P(e|X)B'(X)$$

- As before, probabilities don't sum to one, since downweighted

(sum to (N times) an approximation of $P(e)$)

Particle: Resample



Rather than tracking weighted samples, we resample

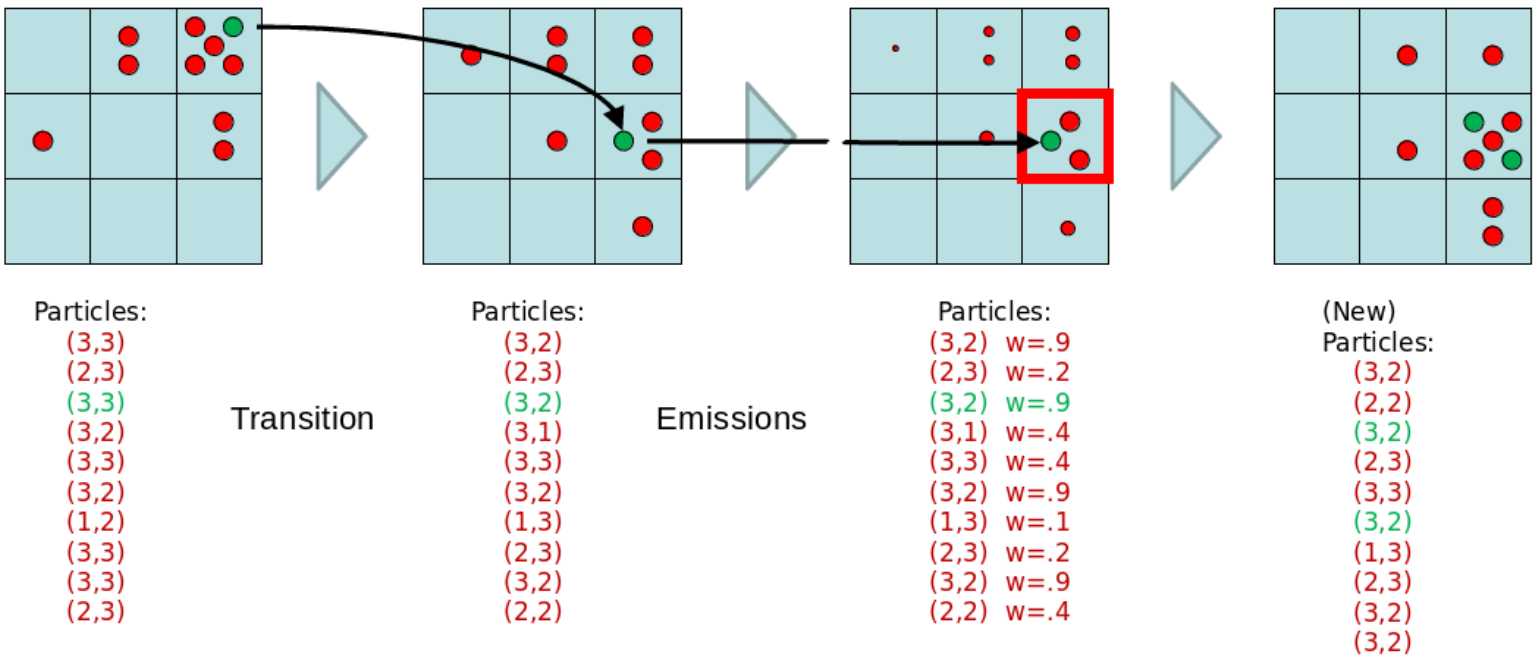
N times, we choose from our weighted sample distribution (i.e. draw with replacement)

This is equivalent to renormalizing the distribution

Now the update is complete for this time step, continue with the next one

Recap: Particle Filtering

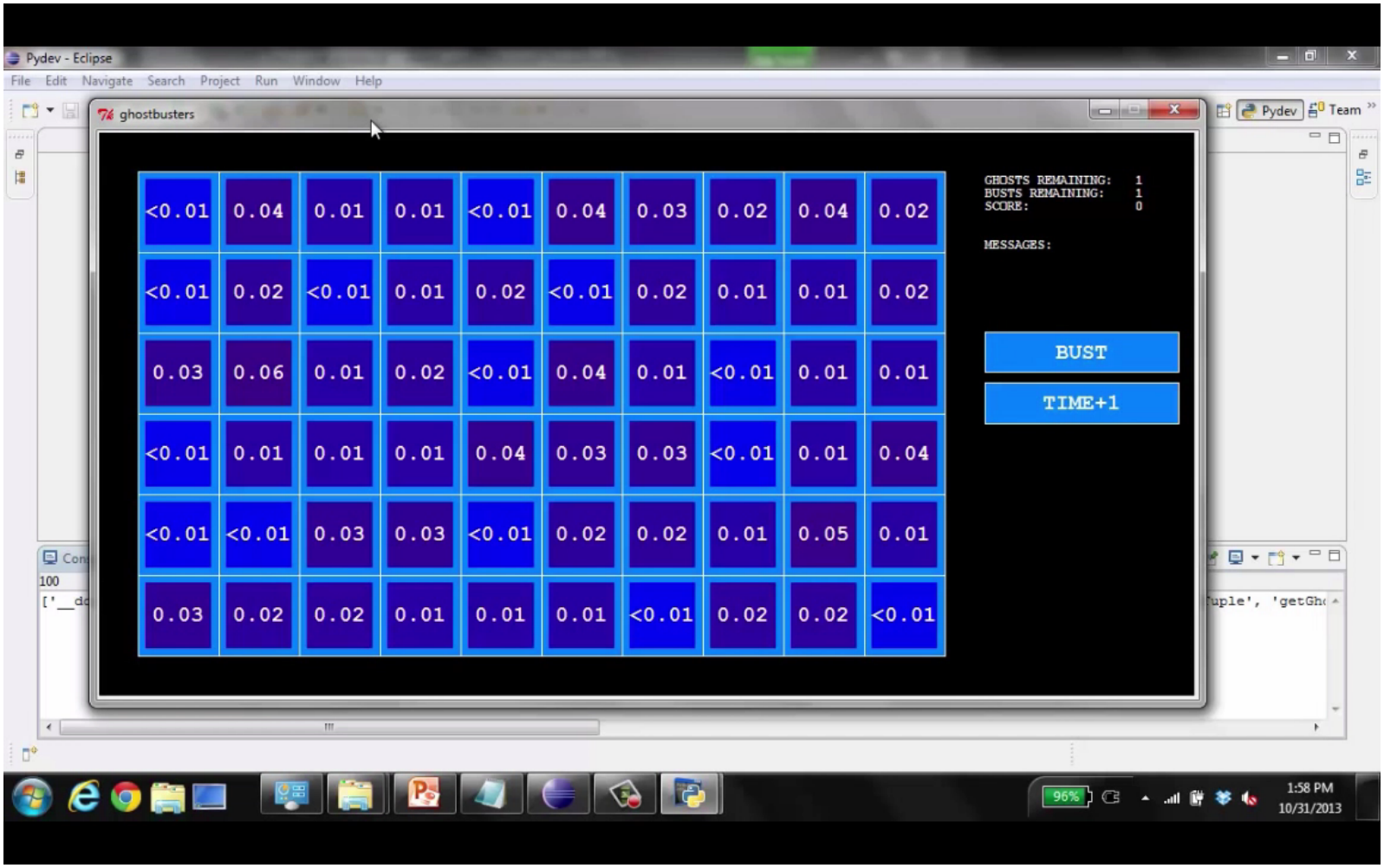
Particles: track samples of states rather than an explicit distribution



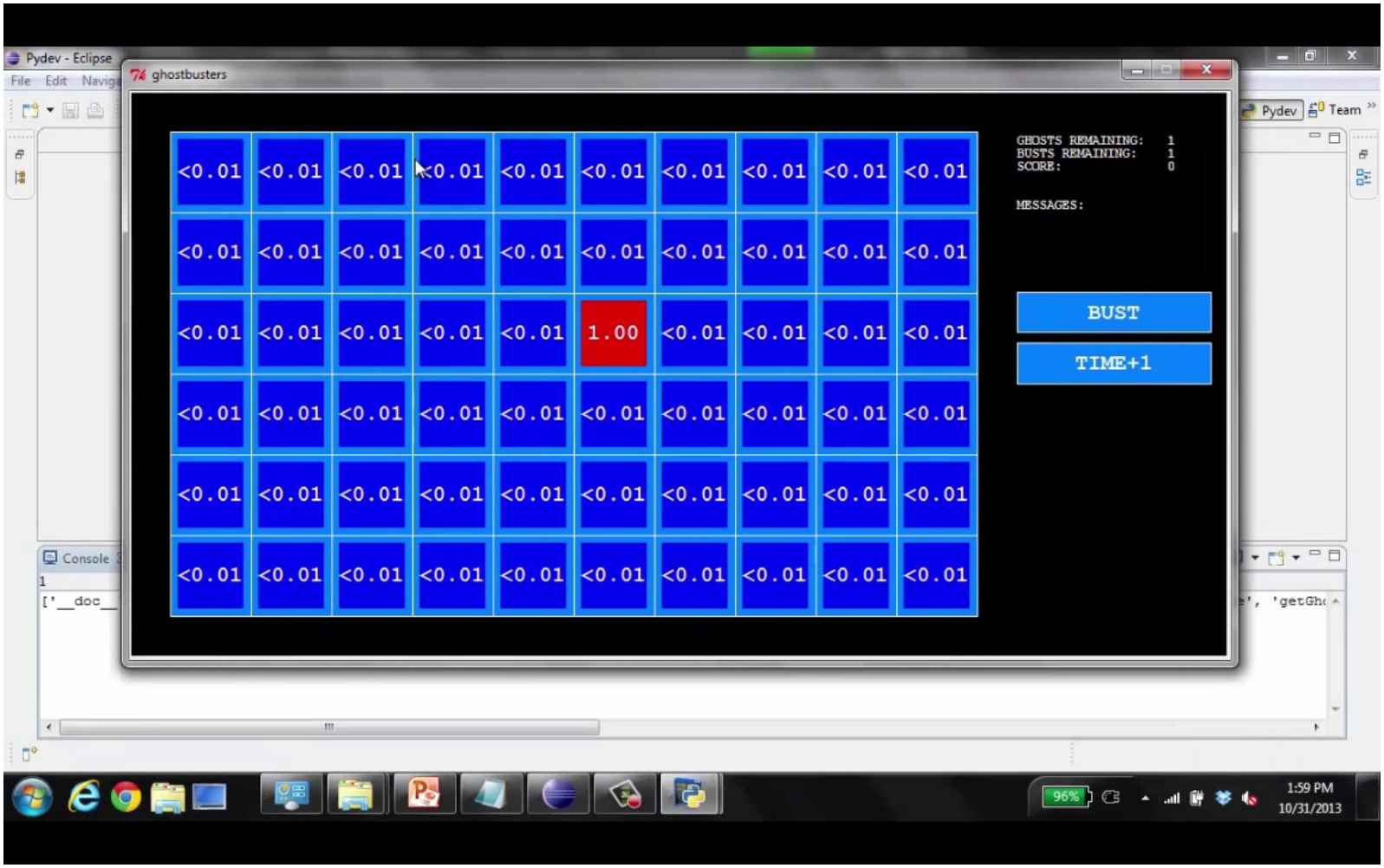
Elapse Weight Resample

[Demos: ghostbusters particle filtering (L15D3,4,5)]

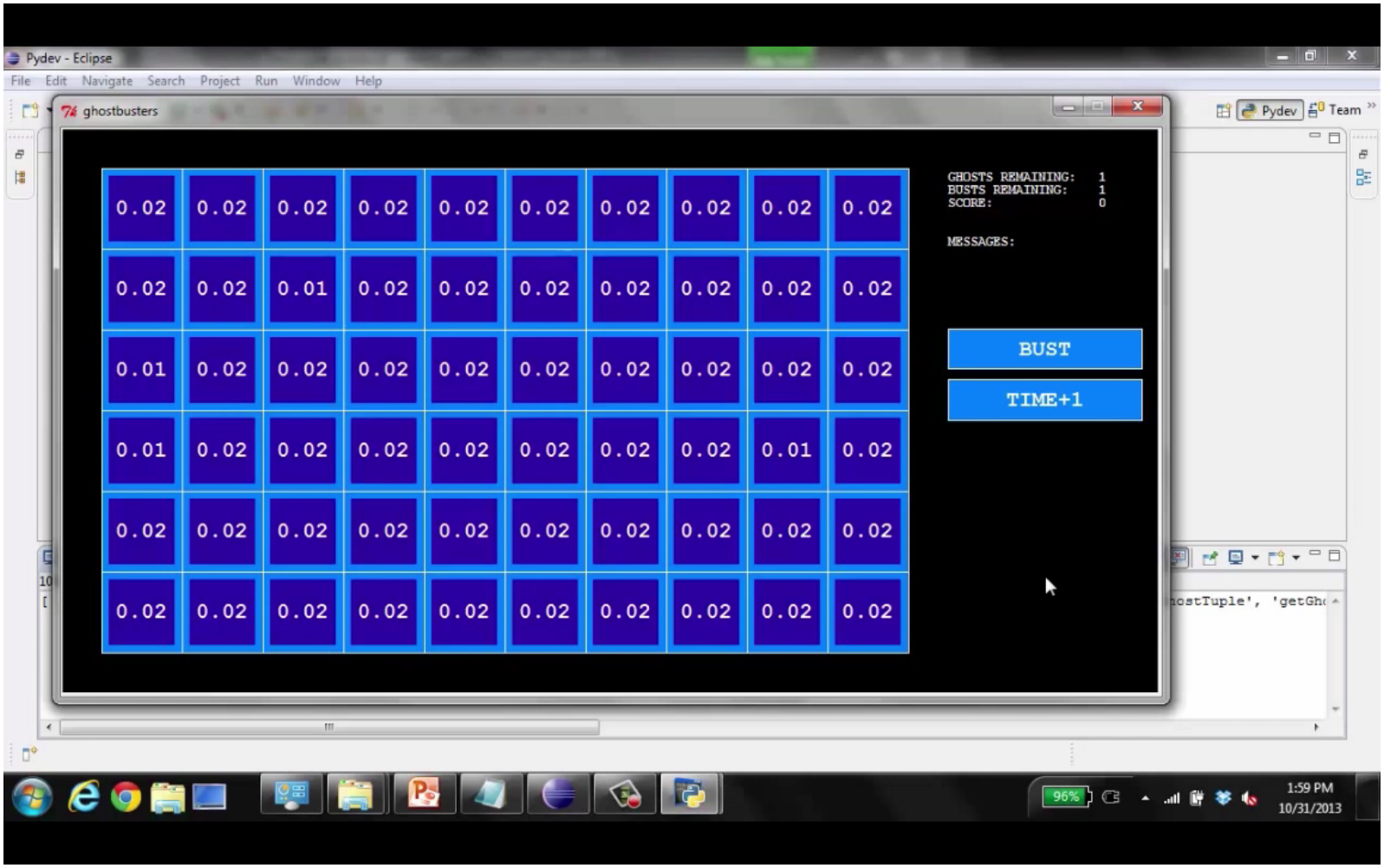
Video of Demo – Moderate Number of Particles



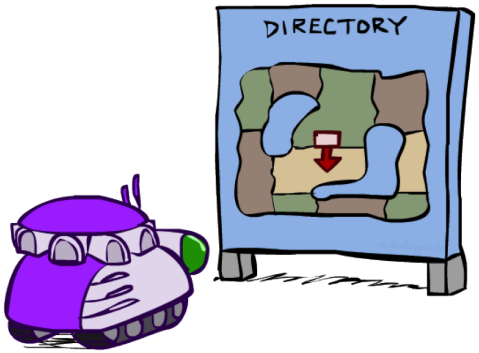
Video of Demo – One Particle

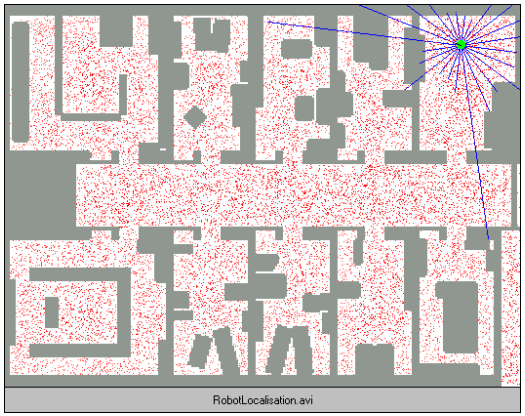


Video of Demo – Huge Number of Particles



Robot Localization





In robot localization:

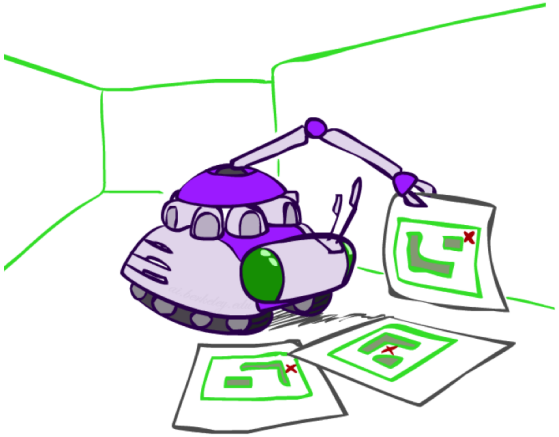
- We know the map, but not the robot's position
- Observations may be vectors of range finder readings
- State space and readings are typically continuous (works basically like a very fine grid) and so we cannot store $B(X)$
- Particle filtering is a main technique

Particle Filter Localization (Sonar)

[Video: global-sonar-uw-annotated.avi]

[Dieter Fox, et al.]

Robot Mapping



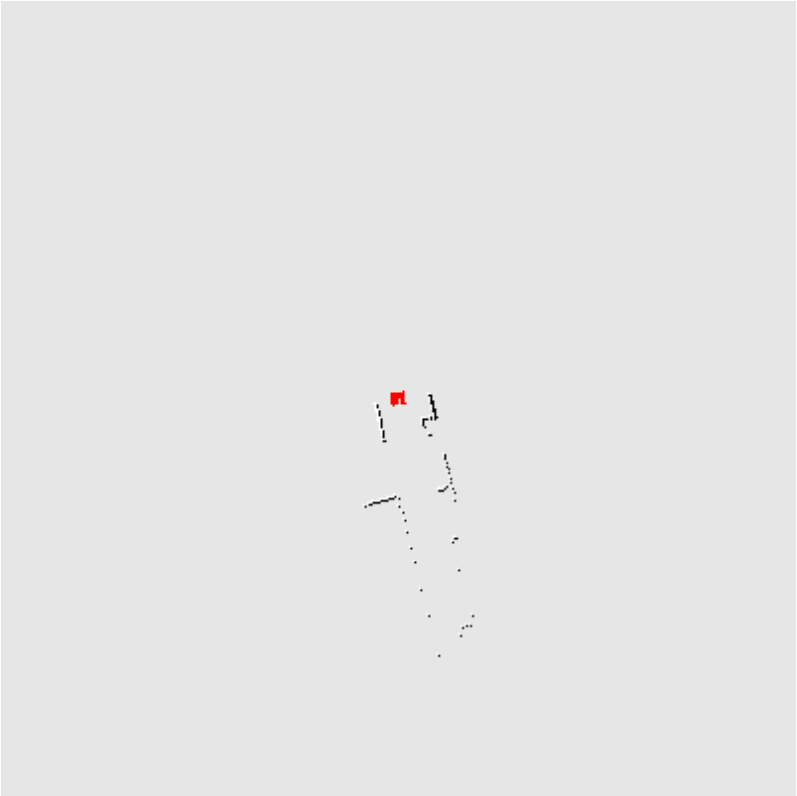
SLAM: Simultaneous Localization And Mapping

- We do not know the map or our location
- State consists of position AND map!
- Main techniques: Kalman filtering (Gaussian HMMs) and particle methods

DP-SLAM, Ron Parr

[Demo: PARTICLES-SLAM-mapping1-new.avi]

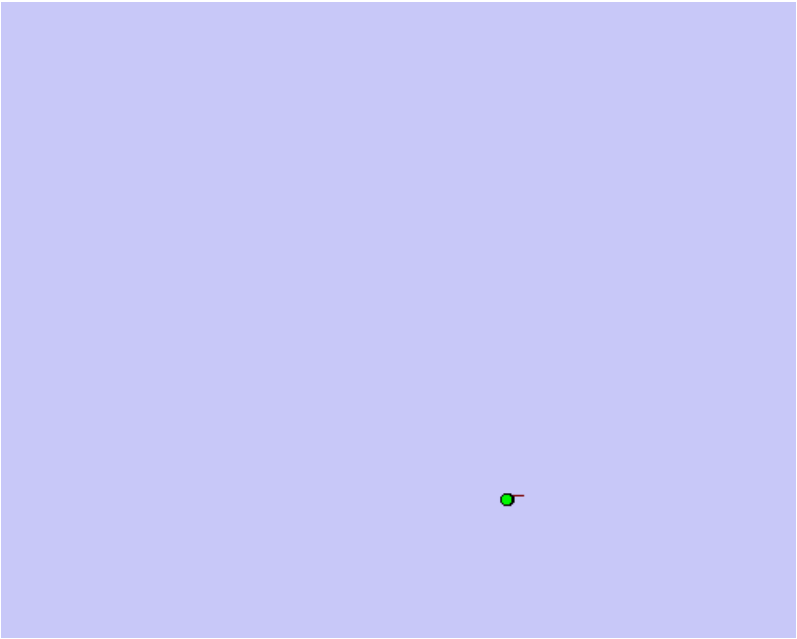
Particle Filter SLAM – Video 1



[Demo: PARTICLES-SLAM-mapping1-new.avi]

[Sebastian Thrun, et al.]

Particle Filter SLAM – Video 2



[Demo: PARTICLES-SLAM-fastslam.avi]

[Dirk Haehnel, et al.]