The goal of today's handout is to help prepare you with Hidden Markov Models in your project. To do this, we will formally derive the equations for *filtering, prediction, smoothing, and viterbi.*

Remember,

$$P(q_t|y) = \frac{P(y_1, \dots, y_t, q_t)P(y_{t+1}, \dots, y_T)}{P(y)} = \frac{\alpha(q_t)\beta(\mathrm{q_t})}{\mathrm{P(y)}}$$

$$\alpha(q_t) = P(y_1, \dots y_t, q_t)$$

$$\beta(q_t) = P(y_{t+1}, \dots, y_T|q_t)$$

Filtering

Given observation sequence $E_0, E_1, \dots, E_{T-1}$, compute $P(X_{T-1}|E_0, \dots, E_{T-1})$.

Prediction

Given observation sequence $E_0, E_1, \dots, E_{T-1}$, compute $P(X_T|E_0, \dots, E_{T-1})$.

Smoothing

Given observation sequence $E_0, E_1, \dots, E_{T-1}$, compute $P(X_k|E_0, \dots, E_{T-1})$ for 0 <= k <= T-1..

Viterbi

Given observation sequence $E_0, E_1, \dots, E_{T-1}$, compute the most likely sequence of states: $X_0, X_1, \dots, X_{T-1}$