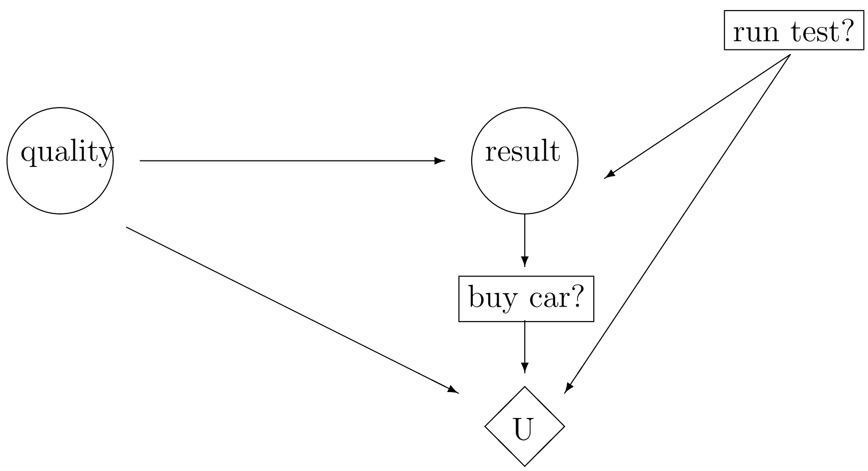
CS 188 Homework 4 - Solutions

Problem 16.11

(a)



Here the result node denotes the result of the test, which could be "pass", "fail", or "unknown" (in the case that the test wasn't run).

(b)

$$\begin{aligned} E(gain) &= -\$1500 + P(q+) * (\$2000) + P(q-) * (\$2000 - \$700) \\ &= -\$1500 + 0.70 * (\$2000) + 0.30 * (\$1300) \\ &= \$290 \end{aligned}$$

(c)

$$\begin{aligned} P(pass(c_1, t_1)) &= P(pass(c_1, t_1)|q^+(c_1))P(q^+(c_1)) + P(pass(c_1, t_1)|q^-(c_1))P(q^-(c_1)) \\ &= 0.8 * 0.70 + 0.35 * 0.30 \\ &= 0.665 \end{aligned}$$

$$P(fail(c_1,t_1)) = 1 - P(pass(c_1,t_1)) = 0.335$$

$$\begin{aligned} P(q^+(c_1)|pass(c_1,t_1)) &= \frac{P(pass(c_1,t_1)|q^+(c_1))P(q^+(c_1))}{P(pass(c_1,t_1))} \\ &= 0.8 * 0.70/0.665 \\ &= 0.8421 \end{aligned}$$

$$P(q^-(c_1)|pass(c_1,t_1)) = 1 - P(q^+(c_1)|pass(c_1,t_1)) = 0.1579$$

$$\begin{aligned} P(q^+(c_1)|fail(c_1,t_1)) &= \frac{P(fail(c_1,t_1)|q^+(c_1))P(q^+(c_1))}{P(fail(c_1,t_1))} \\ &= (1 - 0.8) * 0.70/0.335 \\ &= 0.4179 \end{aligned}$$

$$P(q^-(c_1)|fail(c_1,t_1)) = 1 - P(q^-(c_1)|fail(c_1,t_1)) = 0.5821$$

(d) Assuming we run the test then since the test costs \$50,

$$E(gain|fail, not\ buy) = E(gain|pass, not\ buy) = -\$50$$

$$\begin{aligned} E(gain|pass, buy) &= -\$1500 + P(q^+|pass) * (\$2000) + P(q^-|pass) * (\$2000 - \$700) - \$50 \\ &= -\$1500 + 0.8421 * (\$2000) + 0.1579 * (\$1300) - \$50 \\ &= \$339.47 \end{aligned}$$

$$\begin{aligned} E(gain|fail, buy) &= -\$1500 + P(q^+|fail) * (\$2000) + P(q^-|fail) * (\$2000 - \$700) - \$50 \\ &= -\$1500 + 0.4179 * (\$2000) + 0.5821 * (\$1300) - \$50 \\ &= \$42.53 \end{aligned}$$

The optimal decision in both cases is to buy the car.

Problem 17.1

This problem is related to the task of projecting a Hidden Markov Model as follows. We want to maintain a probability distribution over the 12 states of the 4 by 3 grid world. Let vector **p** represent the probability of being in a particular state at a particular time. **p** is initialized to have a 1 in the position corresponding to the $(1,1)$ (start) cell in the grid. Now consider the 12 by 12 transition matrices T_{up} and T_{right}.

$T_{up}(i,j)$ = probability of landing in cell i when moving Up from cell j.

$T_{right}(i,j)$ = probability of landing in cell i when moving Right from cell j.

The probability of being in each square after the [Up,Up,Right,Right,Right] sequence is ${T_{right}}^3 {T_{up}}^2 \mathbf{p} =$

0.0252	0.0622	0.1799	0.3279
0.1805	0	0.0444	0.0137
0.0246	0.0282	0.0263	0.0869

Value Iteration

0.7948	0.8436	0.8912	0.9382	0.8912
0.8050	0.8625	0.9149	1.0000	0.9149
0.7550	0	0.6577	−1.0000	0.6577
0.6987	0.6487	0.6053	0.3947	0.5840