CS 188: Artificial Intelligence
Spring 2006

Lecture 7: CSPs II
2/7/2006

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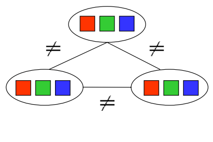
Many slides from either Stuart Russell or Andrew Moore

Today

- More CSPs
  - Applications
  - Tree Algorithms
  - Cutset Conditioning
- Local Search

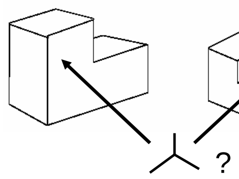
Reminder: CSPs

- CSPs:
  - Variables
  - Domains
  - Constraints
    - Implicit (provide code to compute)
    - Explicit (provide a subset of the possible tuples)
- Unary Constraints
- Binary Constraints
- N   ay Constraints



Example: The Waltz Algorithm

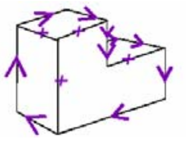
- The Waltz algorithm is for interpreting line drawings of solid polyhedra
- An early example of a computation posed as a CSP



- Look at all intersections
- Adjacent intersections impose constraints on each other

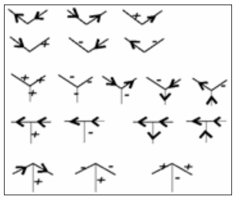
Waltz on Simple Scenes

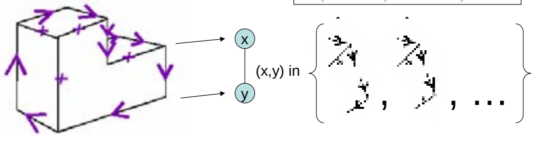
- Assume all objects:
  - Have no shadows or cracks
  - Three-faced vertices
  - “General position”: no junctions change with small movements of the eye.
- Then each line on image is one of the following:
  - Boundary line (edge of an object) (→) with right hand of arrow denoting “solid” and left hand denoting “space”
  - Interior convex edge (+)
  - Interior concave edge (-)



Legal Junctions

- Only certain junctions are physically possible
- How can we formulate a CSP to label an image?
- Variables: vertices
- Domains: junction labels
- Constraints: both ends of a line should have the same label





Example: Boolean Satisfiability

- Given a Boolean expression, is it satisfiable?
- Very basic problem in computer science

$$p_1 \wedge (p_2 \rightarrow p_3) \wedge ((\neg p_1 \wedge \neg p_3) \rightarrow \neg p_2) \wedge (p_1 \vee p_3)$$

- Turns out you can always express in 3-CNF

$$(p_1) \wedge (\neg p_2 \vee p_3) \wedge (p_1 \vee p_3 \vee \neg p_2) \wedge (p_1 \vee p_2 \vee p_3)$$

- 3-SAT: find a satisfying truth assignment

Example: 3-SAT

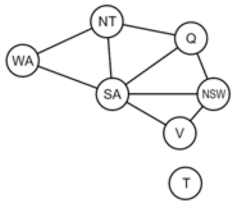
- Variables: $p_1, p_2, \ldots p_n$
- Domains: $\{\mathsf{true}, \mathsf{false}\}$
- Constraints:

$$\left.\begin{array}{r} p_i \vee p_j \vee p_k \\ \neg p_{i'} \vee p_{j'} \vee p_{k'} \\ \vdots \\ p_{i''} \vee \neg p_{j''} \vee \neg p_{k''} \end{array}\right\}$$

Implicitly conjoined (all clauses must be satisfied)

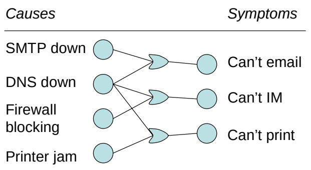
CSPs: Queries

- Types of queries:
  - Legal assignment (last class)
  - All assignments
  - Possible values of some query variable(s) given some evidence (partial assignments)



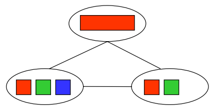
Example: Fault Diagnosis

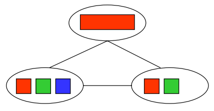
- Fault networks:
  - Variables?
  - Domains?
  - Constraints?
- Various ways to query, given symptoms
  - Some cause (abduction)
  - Simplest cause
  - All possible causes
  - What test is most useful?
  - Prediction: cause to effect
- We'll see this idea again with Bayes' nets



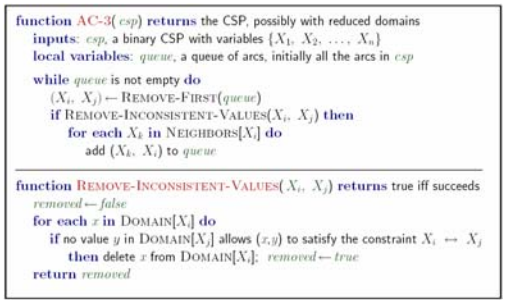
Reminder: Consistency

- Basic solution: DFS / backtracking
  - Add a new assignment
  - Check for violations
- Forward checking:
  - Pre-filter unassigned domains after every assignment
  - Only remove values which conflict with current assignments
- Arc consistency
  - We only defined it for binary CSPs
  - Check for impossible values on all pairs of variables
  - Run (or not) after each assignment before recursing





Arc Consistency



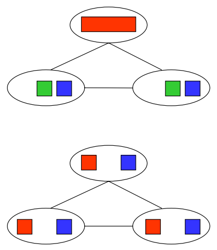
```
function AC-3( $csp$ ) returns the CSP, possibly with reduced domains
  inputs: $csp$, a binary CSP with variables $\{X_1, X_2, \ldots, X_n\}$
  local variables: $queue$, a queue of arcs, initially all the arcs in $csp$

  while $queue$ is not empty do
    $(X_i, X_j) \leftarrow$ REMOVE-FIRST($queue$)
    if REMOVE-INCONSISTENT-VALUES($X_i, X_j$) then
      for each $X_k$ in NEIGHBORS[$X_i$] do
        add $(X_k, X_i)$ to $queue$

function REMOVE-INCONSISTENT-VALUES( $X_i, X_j$ ) returns true iff succeeds
  $removed \leftarrow false$
  for each $x$ in DOMAIN[$X_i$] do
    if no value $y$ in DOMAIN[$X_j$] allows $(x,y)$ to satisfy the constraint $X_i \leftrightarrow X_j$
      then delete $x$ from DOMAIN[$X_i$]; $removed \leftarrow true$
  return $removed$
```

Limitations of Arc Consistency

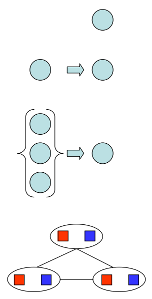
- After running arc consistency:
  - Can have one solution left
  - Can have multiple solutions left
  - Can have no solutions left (and not know it)



What went wrong here?

K-Consistency

- Increasing degrees of consistency
  - 1-Consistency (Node Consistency): Each single node's domain has a value which meets that node's unary constraints
  - 2-Consistency (Arc Consistency): For each pair of nodes, any consistent assignment to one can be extended to the other
  - K-Consistency: For each k nodes, any consistent assignment to k-1 can be extended to the kth node.



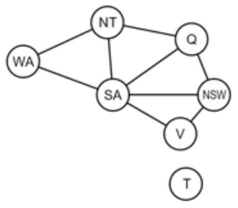
- Higher k more expensive to compute

Strong K-Consistency

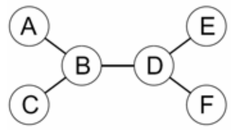
- Strong k-consistency: also k-1, k-2, … 1 consistent
- Claim: strong n-consistency means we can solve without backtracking!
- Why?
  - Choose any assignment to any variable
  - Choose a new variable
  - By 2-consistency, there is a choice consistent with the first
  - Choose a new variable
  - By 3-consistency, there is a choice consistent with the first 2
  - …
- Lots of middle ground between arc consistency and n-consistency! (e.g. path consistency)

Problem Structure

- Tasmania and mainland are independent subproblems
- Identifiable as connected components of constraint graph
- Suppose each subproblem has c variables out of n total
  - Worst-case solution cost is $O((n/c)(d^c))$, linear in n
  - E.g., n = 80, d = 2, c =20
  - 2^{80} = 4 billion years at 10 million nodes/sec
  - $(4)(2^{20})$ = 0.4 seconds at 10 million nodes/sec



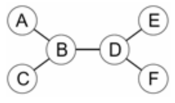
Tree-Structured CSPs

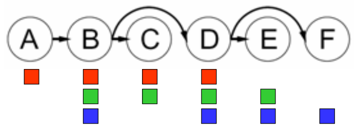


- Theorem: if the constraint graph has no loops, the CSP can be solved in $O(n\ d^2)$ time
  - Compare to general CSPs, where worst-case time is $O(d^n)$
- This property also applies to logical and probabilistic reasoning: an important example of the relation between syntactic restrictions and the complexity of reasoning.

Tree-Structured CSPs

- Choose a variable as root, order variables from root to leaves such that every node's parent precedes it in the ordering

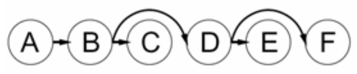




- For i = n : 2, apply RemoveInconsistent($Parent(X_i),X_i$)
- For i = 1 : n, assign X_i consistently with $Parent(X_i)$
- Runtime: $O(n\ d^2)$ (why?)

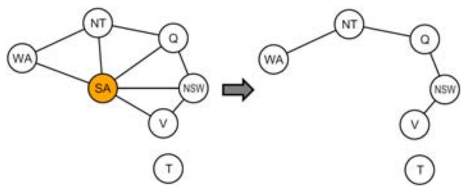
Tree-Structured CSPs

- Why does this work?
- Claim: After each node is processed leftward, all nodes to the right can be assigned in any way consistent with their parent.
- Proof: Induction on position



- Why doesn't this algorithm work with loops?
- Note: we'll see this basic idea again with Bayes' nets and call it belief propagation

Nearly Tree-Structured CSPs

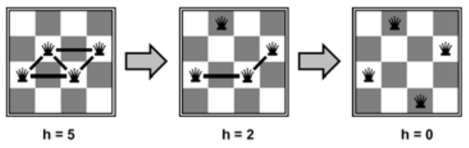


- Conditioning: instantiate a variable, prune its neighbors' domains
- Cutset conditioning: instantiate (in all ways) a set of variables such that the remaining constraint graph is a tree
- Cutset size c gives runtime O((d^c) (n-c) d^2), very fast for small c

Iterative Algorithms for CSPs

- Hill-climbing, simulated annealing typically work with "complete" states, i.e., all variables assigned
- To apply to CSPs:
  - Allow states with unsatisfied constraints
  - Operators *reassign* variable values
- Variable selection: randomly select any conflicted variable
- Value selection by min-conflicts heuristic:
  - Choose value that violates the fewest constraints
  - I.e., hillclimb with h(n) = total number of violated constraints

Example: 4-Queens

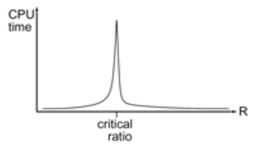


- States: 4 queens in 4 columns (4^4 = 256 states)
- Operators: move queen in column
- Goal test: no attacks
- Evaluation: h(n) = number of attacks

Performance of Min-Conflicts

- Given random initial state, can solve n-queens in almost constant time for arbitrary n with high probability (e.g., n = 10,000,000)
- The same appears to be true for any randomly-generated CSP *except* in a narrow range of the ratio

$$R = \frac{\text{number of constraints}}{\text{number of variables}}$$



Summary

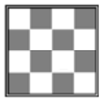
- CSPs are a special kind of search problem:
  - States defined by values of a fixed set of variables
  - Goal test defined by constraints on variable values
- Backtracking = depth-first search with one legal variable assigned per node
- Variable ordering and value selection heuristics help significantly
- Forward checking prevents assignments that guarantee later failure
- Constraint propagation (e.g., arc consistency) does additional work to constrain values and detect inconsistencies
- The constraint graph representation allows analysis of problem structure
- Tree-structured CSPs can be solved in linear time
- Iterative min-conflicts is usually effective in practice

Local Search Methods

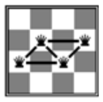
- Queue-based algorithms keep fallback options (backtracking)
- Local search: improve what you have until you can't make it better
- Generally much more efficient (but incomplete)

Types of Problems

- Planning problems:
  - We want a path to a solution (examples?)
  - Usually want an optimal path
  - *Incremental formulations*



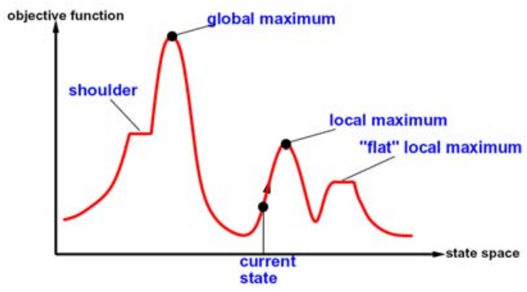
- Identification problems:
  - We actually just want to know what the goal is (examples?)
  - Usually want an optimal goal
  - *Complete-state formulations*
  - Iterative improvement algorithms



Hill Climbing

- Simple, general idea:
  - Start wherever
  - Always choose the best neighbor
  - If no neighbors have better scores than current, quit
- Why can this be a terrible idea?
  - Complete?
  - Optimal?
- What's good about it?

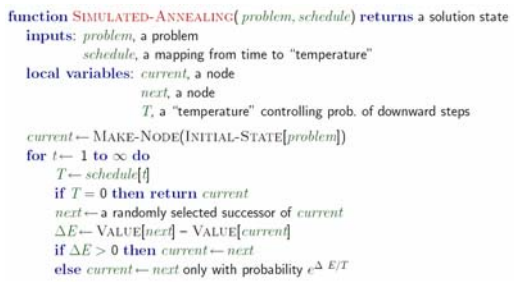
Hill Climbing Diagram



- Random restarts?
- Random sideways steps?

Simulated Annealing

- Idea: Escape local maxima by allowing downhill moves
  - But make them rarer as time goes on



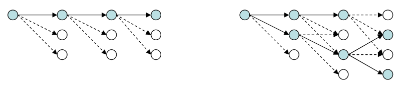
```
function SIMULATED-ANNEALING( problem, schedule) returns a solution state
    inputs: problem, a problem
            schedule, a mapping from time to "temperature"
    local variables: current, a node
                     next, a node
                     T, a "temperature" controlling prob. of downward steps
    current ← MAKE-NODE(INITIAL-STATE[problem])
    for t ← 1 to ∞ do
        T ← schedule[t]
        if $T = 0$ then return current
        next ← a randomly selected successor of current
        $\Delta E$ ← VALUE[next] – VALUE[current]
        if $\Delta E > 0$ then current ← next
        else current ← next only with probability $e^{\Delta E/T}$
```

Simulated Annealing

- Theoretical guarantee:
  - Stationary distribution: $p(x) \propto e^{\frac{E(x)}{kT}}$
  - If T decreased slowly enough, will converge to optimal state!
- Is this an interesting guarantee?
- Sounds like magic, but reality is reality:
  - The more downhill steps you need to escape, the less likely you are to every make them all in a row
  - People think hard about *ridge operators* which let you jump around the space in better ways

Beam Search

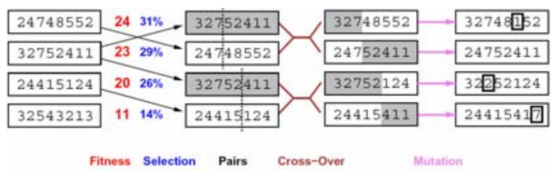
- Like greedy search, but keep K states at all times:



Greedy Search   Beam Search

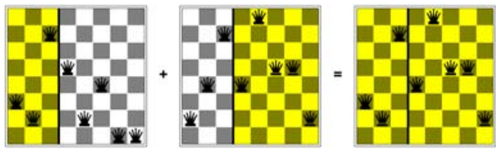
- Variables: beam size, encourage diversity?
- The best choice in MANY practical settings
- Complete? Optimal?
- Why do we still need optimal methods?

Genetic Algorithms



- Genetic algorithms use a natural selection metaphor
- Like beam search (selection), but also have pairwise crossover operators, with optional mutation
- Probably the most misunderstood, misapplied (and even maligned) technique around!

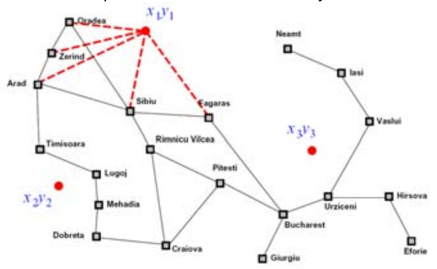
Example: N-Queens



- Why does crossover make sense here?
- When wouldn't it make sense?
- What would mutation be?
- What would a good fitness function be?

Continuous Problems

- Placing airports in Romania
  - States: $(x_1,y_1,x_2,y_2,x_3,y_3)$
  - Cost: sum of squared distances to closest city



Gradient Methods

- How to deal with continous (therefore infinite) state spaces?
- Discretization: bucket ranges of values
  - E.g. force integral coordinates
- Continuous optimization
  - E.g. gradient ascent

$$\nabla f = \left(\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial y_1}, \frac{\partial f}{\partial x_2}, \frac{\partial f}{\partial y_2}, \frac{\partial f}{\partial x_3}, \frac{\partial f}{\partial y_3}\right)$$

$$x \leftarrow x + \alpha \nabla f(x)$$

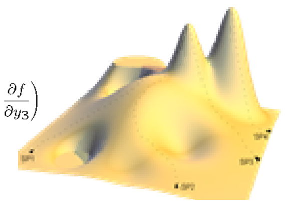
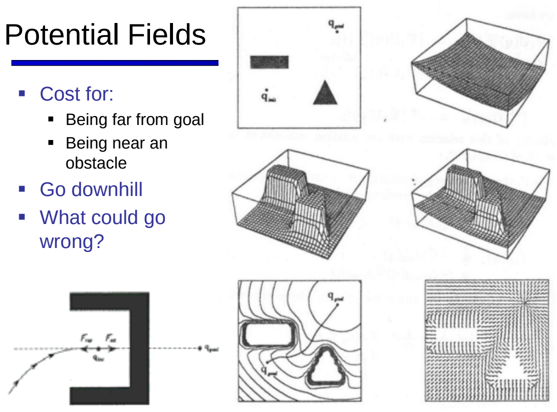


Image from vias.org

Potential Fields

- Cost for:
  - Being far from goal
  - Being near an obstacle
- Go downhill
- What could go wrong?



Potential Field Methods

Define a function $u\left(\underset{\sim}{q}\right)$

u : Configurations $\to \Re$

Such that

$u \to$ huge as you move towards an obstacle

$u \to$ small as you move towards the goal

Write $d_g\left(\underset{\sim}{q}\right) =$ distance from $\underset{\sim}{q}$ to $\underset{\sim}{q}$ goal

$d_i\left(\underset{\sim}{q}\right) =$ distance from $\underset{\sim}{q}$ to nearest obstacle

---

One definition of u : $u(q) = d_i(q) - d_g(q)$

Preferred definition : $u(q) = \frac{1}{2}\sum\left(d_g(q)\right)^2 + \frac{1}{2}\eta\frac{1}{d_i(q)^2}$

SIMPLE MOTION PLANNER:

Gradient descent on *u*

Next Time

- Probabilities (chapter 13)
  - Basis of most of the rest of the course
  - You might want to read up in advance!