

CS 188 Section Handout: Probability Basics

Let $D \in \{d_0, d_1, d_2, d_3\}$ be one of four dice, where d_0 is fair. Let $R \in [1, 6]$ be the outcome of a die roll.

1 Joint probability assembly

A casino employee informs you (the inspector) about a dealer, Angelo: “He cheats a third of the time, using loaded die d_1 for which the six-dotted side shows up five times as often as each of the other sides.”

D	$P(D)$
d_0	
d_1	

R	D	$P(R D)$
1	d_0	
2	d_0	
3	d_0	
4	d_0	
5	d_0	
6	d_0	
1	d_1	
2	d_1	
3	d_1	
4	d_1	
5	d_1	
6	d_1	

R	D	$P(R, D)$
1	d_0	
2	d_0	
3	d_0	
4	d_0	
5	d_0	
6	d_0	
1	d_1	
2	d_1	
3	d_1	
4	d_1	
5	d_1	
6	d_1	

2 Estimation

Your informant reports back with the following statistics about another dealer, Bert: “He uses two dice, both loaded. Using d_2 , I watched him roll 15 sixes, 10 fives, 5 fours, 5 threes, 5 twos, and no ones. With d_3 , I observed 5 sixes, 10 fives, 15 fours, 15 threes, 15 twos, and 20 ones.”

R	D	$c(R, D)$
1	d_2	
2	d_2	
3	d_2	
4	d_2	
5	d_2	
6	d_2	
1	d_3	
2	d_3	
3	d_3	
4	d_3	
5	d_3	
6	d_3	

R	D	$P(R, D)$
1	d_2	
2	d_2	
3	d_2	
4	d_2	
5	d_2	
6	d_2	
1	d_3	
2	d_3	
3	d_3	
4	d_3	
5	d_3	
6	d_3	

3 Inference by enumeration

- What is $P(R = 6 \mid \text{dealer} = A)$?
- What is $P(R = 6 \mid \text{dealer} = B)$?
- What is $P(D = d_0 \mid R = 6, \text{dealer} = A)$?
- What is $P(D = d_0 \mid \text{dealer} = B)$?
- What is $P(D = d_2 \mid \text{dealer} = B)$?

Suppose you know that A works some five nights per week and B works on some other two nights.

- What is $P(D)$?

D	$P(D)$
d_0	
d_1	
d_2	
d_3	

- One night, you observe a dealer (A or B) roll a six. What is the probability that the die is loaded?

4 Sequence of independent events

You confront the suspected dealer and ask him to roll his current die three times in a row. He rolls the sequence $S = (6, 2, 6)$

Calculate the following likelihood probabilities:

- $P(S \mid D = d_0) =$
- $P(S \mid D = d_1) =$
- $P(S \mid D = d_2) =$
- $P(S \mid D = d_3) =$

5 Bayes' Rule

Given the evidence you acquired previously, determine which die was rolled:

$$\hat{d} = \arg \max_d P(D = d \mid S)$$

Is the die loaded? Which dealer are you arresting?