Bayes Net: Independence and Inference

CS 188 Staff

1 Conditional Independence

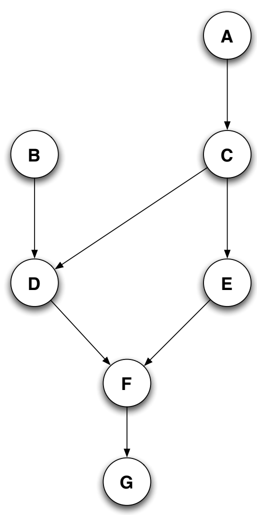

Figure 1:

For the bayes net in Figure, determine which of the following conditional independence statements hold:

- $B \perp\!\!\!\perp C$
- $B \perp\!\!\!\perp C | A, G$
- $A \perp\!\!\!\perp D | B, C, F$
- $D \perp\!\!\!\perp E | B$
- $A \perp\!\!\!\perp F | C, D$

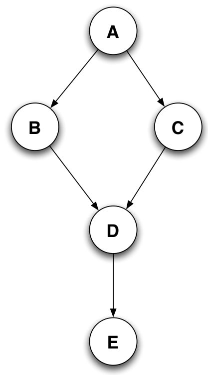

Figure 2: Bayes nets for Variable Elimination

2 Variable Elimination by Factors

Consider the bayes net in figure 2, where each of the variables are assumed to be binary 0-1 valued. Perform variable elimination by factors to obtain $P(B|C=1, E=0)$. See the next section for a review of Variable Elimination by factors.

Appendix: Variable Elimination via Factors Review

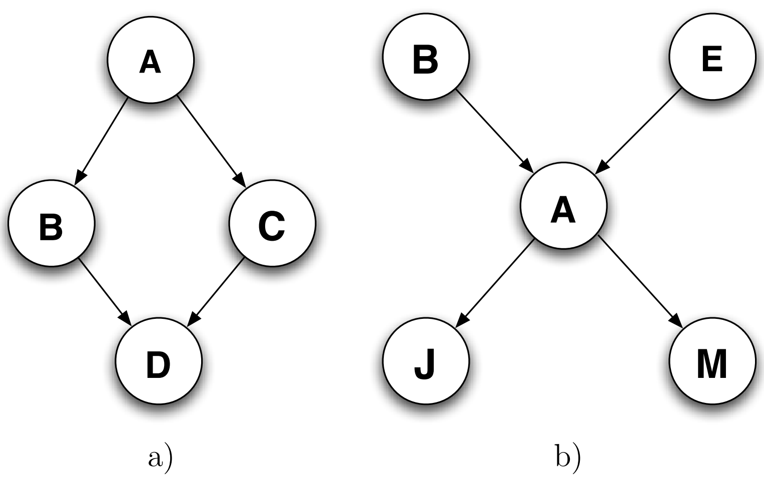

Figure 3:

In this problem we'll review Variable Elimination by factors. We'll develop this method by working through the example in Figure 3a). Suppose we want to find the marginal distribution on D, $P(D)$. First, we associate with each variable a *factor* which is the function $P(X|Pa(X))$ which depends on the values of X and $Pa(X)$. Initially we have the following factors:

$$\underbrace{f_A(A)}_{P(A)} \quad \underbrace{f_{B,A}(B,A)}_{P(B|A)} \quad \underbrace{f_{C,A}(C,A)}_{P(C|A)} \quad \underbrace{f_{D,B,C}(D,B,C)}_{P(D|B,C)}$$

Now let's eliminate variable B first.[1] In order to do this, we gather all the factors that involve B, $f_{B,A}(B,A)$, $f_{D,B,C}(D,B,C)$, and take the entry-wise product of them:

$$f_{A,B,C,D}(A,B,C,D) = f_{B,A}(B,A) f_{D,B,C}(D,B,C)$$

This operation is called a *join* after the corresponding database operation of the same name. Next we marginalize out B by summing out values of B from $f_{A,B,C,D}$:

$$f_{A,\overline{B},C,D}(A,C,D) = \sum_B f_{A,B,CD,}(A,B,C,D)$$

The new factor is indexed by $\overline{B}$ to indicate it has been marginalized. After eliminating B, we are left with the following factors:

$$\underbrace{f_A(A)}_{P(A)} \quad \underbrace{f_{C,A}(C,A)}_{P(C|A)} \quad \underbrace{f_{A,\overline{B},C,D}(A,C,D)}_{\sum_B P(B|A)P(D|BC)}$$

Note that our new factor $f_{A,\overline{B},C,D}$ doesn't have a probabilistic interpretation which will typically be the case. We continue to eliminate variables by performing *join* and *marginalize* operations on factors until we are left with only factors that involve only B. We then join these factors and re-normalize the factors to obtain the marginal distribution of B. This procedure is guaranteed to finish since we reduce the number of factors every time we perform an elimination by at least 1 (Why?).

[1] This obviously isn't the optimal order of elimination.