

CS 188: Artificial Intelligence

Fall 2007

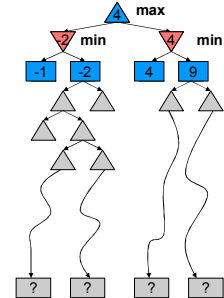
Lecture 8: Expectimax Search

9/20/2007

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 Many slides over the course adapted from either Stuart Russell or Andrew Moore

Truncated Minimax

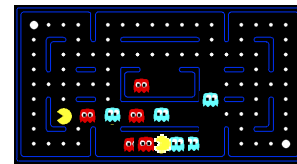
- Minimax tree
- Leaves marked with utility
- Cannot search to leaves
- Limited search
 - Instead, search a limited depth
 - Replace terminal utilities with an eval function for non-terminal positions
- Evaluation function is critical!



What Makes a Good Player?

- Two main ways to make a player good
 - Deep search (pruning, move ordering, etc)
 - Good evaluation functions
- How to make good evaluation functions?
 - Inventing features brings domain knowledge about what aspects of the game state are important (i.e. food good, ghost bad, pellets better when ghosts are near)
 - But, balancing the weights on these features is quite hard to do with intuition
 - Better to let the agent play and tune based on experience
 - Good application for machine learning (coming up!)

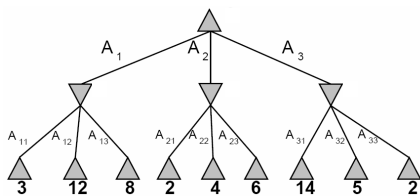
Evaluation for Pacman



[DEMO4: evalFunction, thrashing]

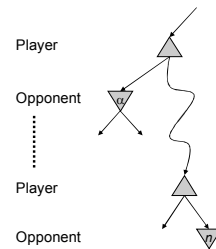
$$Eval(s) = w_1f_1(s) + w_2f_2(s) + \dots + w_nf_n(s)$$

α - β Pruning Example



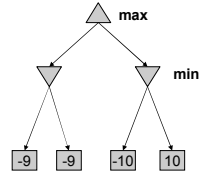
α - β Pruning

- General configuration
 - α is the best value the Player can get at any choice point along the current path
 - If n is worse than α , MAX will avoid it, so prune n 's branch
 - Define β similarly for MIN



Non-Optimal Opponents

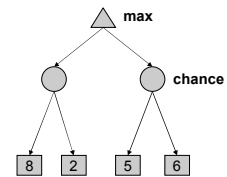
- In minimax search:
 - Max plans assuming that Min will act according to a minimax calculation
 - Min's calculation depends on Min's belief that Max is a minimax player, and so on
 - Max and Min's thought processes interweave and we can sort the whole process out in one tree
- What if instead we think the opponent is random?
 - Or, reasonably smart but not minimax?



[DEMO1:
minVsExp2]

Expectimax Search Trees

- What if we don't know what the result of an action will be? E.g.,
 - In solitaire, next card is unknown
 - In minesweeper, don't know where the mines are
 - In pacman, the ghosts act randomly
- Can do **expectimax search**
 - Chance nodes, like min nodes, except the outcome is uncertain
 - Calculate **expected utilities**
 - Max nodes as in minimax search
 - Chance nodes take average (expectation) of value of children
- Later, we'll learn how to formalize the underlying problem as a **Markov Decision Process**



Maximum Expected Utility

- Why should we average utilities? Why not minimax?
- Principle of maximum expected utility: an agent should choose the action which **maximizes its expected utility, given its knowledge**
- General principle for decision making
- Often taken as the definition of rationality
- We'll see this idea over and over in this course!
- Let's decompress this definition...

Reminder: Probabilities

- A **random variable** represents an event whose outcome is unknown
- A **probability distribution** is an assignment of weights to outcomes
- Example: traffic on freeway?
 - Random variable: T = whether there's traffic
 - Outcomes: T in {none, light, heavy}
 - Distribution: $P(T=none) = 0.25$, $P(T=light) = 0.55$, $P(T=heavy) = 0.20$
- Some laws of probability (more later):
 - Probabilities are always non-negative
 - Probabilities over all possible outcomes sum to one
- As we get more evidence, probabilities may change:
 - $P(T=heavy) = 0.20$, $P(T=heavy | Hour=8am) = 0.60$
 - We'll talk about methods for reasoning and updating probabilities later

What are Probabilities?

- Objectivist / frequentist answer:**
 - Averages over repeated *experiments*
 - E.g. empirically estimating $P(\text{rain})$ from historical observation
 - Assertion about how future experiments will go (in the limit)
 - New evidence changes the *reference class*
 - Makes one think of *inherently random* events, like rolling dice
- Subjectivist / Bayesian answer:**
 - Degrees of belief about unobserved variables
 - E.g. an agent's belief that it's raining, given the temperature
 - E.g. pacman's belief that the ghost will turn left, given the state
 - Often *estimate* probabilities from past experience (more later)
 - New evidence *updates beliefs* (more later)

Uncertainty Everywhere

- Not just for games of chance!**
 - I'm snuffling: am I sick?
 - Email contains "FREE!": is it spam?
 - Tooth hurts: have cavity?
 - Safe to cross street?
 - 60 min enough to get to the airport?
 - Robot rotated wheel three times, how far did it advance?
- Why can a random variable have uncertainty?**
 - Inherently random process (dice, etc)
 - Insufficient or weak evidence
 - Ignorance of underlying processes
 - Unmodeled variables
 - The world's just noisy!
- Compare to *fuzzy logic*, which has *degrees of truth*, or rather than just *degrees of belief*

Reminder: Expectations

- Often a quantity of interest depends on a random variable
- The expected value of a function is its average output, weighted by a given distribution over inputs
- Example: How late if I leave 60 min before my flight?
 - Lateness is a function of traffic:
 - L(none) = -10, L(light) = -5, L(heavy) = 15
 - What is my expected lateness?
 - Need to specify some belief over T to weight the outcomes
 - Say P(T) = {none: 2/5, light: 2/5, heavy: 1/5}
 - The expected lateness:

$$E_{P(T)}[L(T)] = \frac{2}{5} \times (-10) + \frac{2}{5} \times (-5) + \frac{1}{5} \times (15)$$

$$P(\text{none})L(\text{none}) + P(\text{light})L(\text{light}) + P(\text{heavy})L(\text{heavy})$$

Expectations

- Real valued functions of random variables:

$$f : X \rightarrow R$$
- Expectation of a function of a random variable

$$E_{P(X)}[f(X)] = \sum_x f(x)P(x)$$

- Example: Expected value of a fair die roll

X	P	f
1	1/6	1
2	1/6	2
3	1/6	3
4	1/6	4
5	1/6	5
6	1/6	6

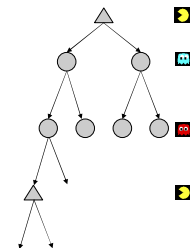
$$1 \times \frac{1}{6} + 2 \times \frac{1}{6} + 3 \times \frac{1}{6} + 4 \times \frac{1}{6} + 5 \times \frac{1}{6} + 6 \times \frac{1}{6} = 3.5$$

Utilities

- Utilities are functions from outcomes (states of the world) to real numbers that describe an agent's preferences
- Where do utilities come from?
 - In a game, may be simple (+1/-1)
 - Utilities summarize the agent's goals
 - Theorem: any set of preferences between outcomes can be summarized as a utility function (provided the preferences meet certain conditions)
- In general, we hard-wire utilities and let actions emerge (why don't we let agents decide their own utilities?)
- More on utilities soon...

Expectimax Search

- In expectimax search, we have a probabilistic model of how the opponent (or environment) will behave in any state
 - Model could be a simple uniform distribution (roll a die)
 - Model could be sophisticated and require a great deal of computation
 - We have a node for every outcome out of our control: opponent or environment
 - The model can predict that smart actions are likely!
- For now, assume for any state we magically have a distribution to assign probabilities to opponent actions / environment outcomes



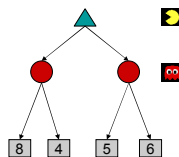
Having a probabilistic belief about an agent's action does not mean that agent is flipping any coins!

Expectimax Pseudocode

```
def value(s)
  if s is a max node return maxValue(s)
  if s is an exp node return expValue(s)
  if s is a terminal node return evaluation(s)
```

```
def maxValue(s)
  values = [value(s') for s' in successors(s)]
  return max(values)
```

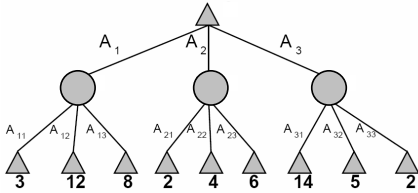
```
def expValue(s)
  values = [value(s') for s' in successors(s)]
  weights = [probability(s, s') for s' in successors(s)]
  return expectation(values, weights)
```



Expectimax for Pacman

- Notice that we've gotten away from thinking that the ghosts are trying to minimize pacman's score
- Instead, they are now a part of the environment
- Pacman has a belief (distribution) over how they will act
- Quiz: Can we see minimax as a special case of expectimax?
- Quiz: what would pacman's computation look like if we assumed that the ghosts were doing 1-ply minimax and taking the result 80% of the time, otherwise moving randomly?
- If you take this further, you end up calculating belief distributions over your opponents' belief distributions over your belief distributions, etc...
 - Can get unmanageable very quickly!

Expectimax Pruning?



Expectimax Evaluation

- For minimax search, evaluation function insensitive to monotonic transformations
 - We just want better states to have higher evaluations (get the ordering right)
- For expectimax, we need the magnitudes to be meaningful as well
 - E.g. must know whether a 50% / 50% lottery between A and B is better than 100% chance of C
 - 100 or -10 vs 0 is different than 10 or -100 vs 0

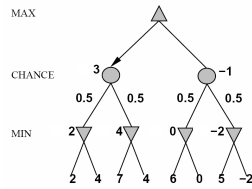
Mixed Layer Types

- E.g. Backgammon
- Expectiminimax

- Environment is an extra player that moves after each agent
- Chance nodes take expectations, otherwise like minimax

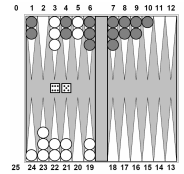
```

if state is a MAX node then
    return the highest EXPECTIMINIMAX-VALUE of SUCCESSORS(state)
if state is a MIN node then
    return the lowest EXPECTIMINIMAX-VALUE of SUCCESSORS(state)
if state is a chance node then
    return average of EXPECTIMINIMAX-VALUE of SUCCESSORS(state)
    
```



Stochastic Two-Player

- Dice rolls increase b : 21 possible rolls with 2 dice
 - Backgammon ≈ 20 legal moves
 - Depth 4 = $20 \times (21 \times 20)^3 = 1.2 \times 10^9$
- As depth increases, probability of reaching a given node shrinks
 - So value of lookahead is diminished
 - So limiting depth is less damaging
 - But pruning is less possible...
- TDGammon uses depth-2 search + very good eval function + reinforcement learning: world-champion level play



Non-Zero-Sum Games

- Similar to minimax:
 - Utilities are now tuples
 - Each player maximizes their own entry at each node
 - Propagate (or back up) nodes from children
 - Can give rise to cooperation and competition dynamically...

