CS 188: Artificial Intelligence
Fall 2007

Lecture 7: Adversarial Search
9/18/2007

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Many slides over the course adapted from either Stuart Russell or Andrew Moore

Announcements

- Project 2 is up (Multi-Agent Pacman)
- SVN groups coming, watch web page
- Dan's office hours Tuesday moving to Friday 1-2pm

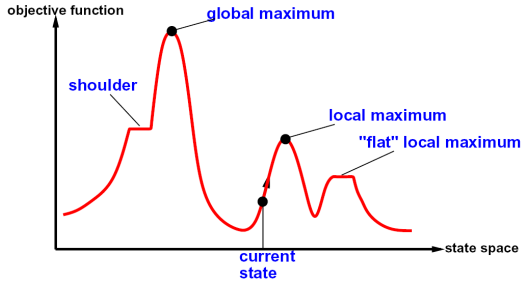
Local Search

- Queue-based algorithms keep fallback options (backtracking)
- Local search: improve what you have until you can't make it better
- Generally much more efficient (but incomplete)

Hill Climbing

- Simple, general idea:
  - Start wherever
  - Always choose the best neighbor
  - If no neighbors have better scores than current, quit
- Why can this be a terrible idea?
  - Complete?
  - Optimal?
- What's good about it?

Hill Climbing Diagram



- Random restarts?
- Random sideways steps?

Simulated Annealing

- Idea: Escape local maxima by allowing downhill moves
  - But make them rarer as time goes on

```
function SIMULATED-ANNEALING(problem, schedule) returns a solution state
  inputs: problem, a problem
          schedule, a mapping from time to "temperature"
  local variables: current, a node
                   next, a node
                   T, a "temperature" controlling prob. of downward steps
  current ← MAKE-NODE(INITIAL-STATE[problem])
  for t ← 1 to ∞ do
      T ← schedule[t]
      if T = 0 then return current
      next ← a randomly selected successor of current
      ΔE ← VALUE[next] – VALUE[current]
      if ΔE > 0 then current ← next
      else current ← next only with probability e^(Δ E/T)
```

Simulated Annealing

- Theoretical guarantee:
  - Stationary distribution: $p(x) \propto e^{\frac{E(x)}{kT}}$
  - If T decreased slowly enough, will converge to optimal state!
- Is this an interesting guarantee?
- Sounds like magic, but reality is reality:
  - The more downhill steps you need to escape, the less likely you are to every make them all in a row
  - People think hard about *ridge operators* which let you jump around the space in better ways

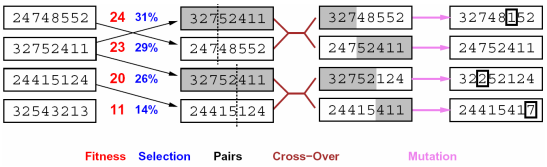
Beam Search

- Like hill-climbing search, but keep K states at all times:

Greedy Search Beam Search

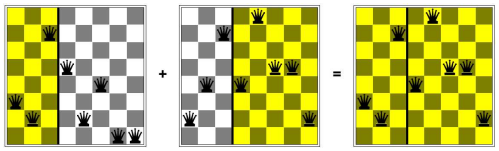
- Variables: beam size, encourage diversity?
- The best choice in MANY practical settings
- Complete? Optimal?
- Why do we still need optimal methods?

Genetic Algorithms



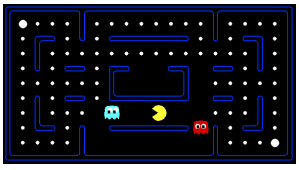
- Genetic algorithms use a natural selection metaphor
- Like beam search (selection), but also have pairwise crossover operators, with optional mutation
- Probably the most misunderstood, misapplied (and even maligned) technique around!

Example: N-Queens



- Why does crossover make sense here?
- When wouldn't it make sense?
- What would mutation be?
- What would a good fitness function be?

Adversarial Search



[DEMO 1]

Game Playing State-of-the-Art

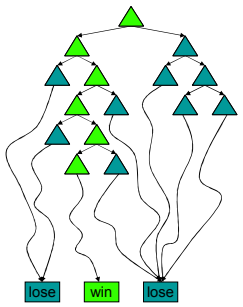
- **Checkers:** Chinook ended 40-year-reign of human world champion Marion Tinsley in 1994. Used an endgame database defining perfect play for all positions involving 8 or fewer pieces on the board, a total of 443,748,401,247 positions. Checkers is now solved!
- **Chess:** Deep Blue defeated human world champion Gary Kasparov in a six-game match in 1997. Deep Blue examined 200 million positions per second, used very sophisticated evaluation and undisclosed methods for extending some lines of search up to 40 ply.
- **Othello:** human champions refuse to compete against computers, which are too good.
- **Go:** human champions refuse to compete against computers, which are too bad. In go, b > 300, so most programs use pattern knowledge bases to suggest plausible moves.
- **Pacman:** unknown

Game Playing

- Axes:
  - Deterministic or stochastic?
  - One, two or more players?
  - Perfect information (can you see the state)?
- Want algorithms for calculating a strategy (policy) which recommends a move in each state

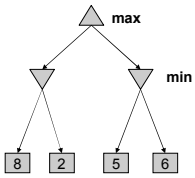
Deterministic Single-Player?

- Deterministic, single player, perfect information:
  - Know the rules
  - Know what actions do
  - Know when you win
  - E.g. Freecell, 8-Puzzle, Rubik's cube
- … it's just search!
- Slight reinterpretation:
  - Each node stores the best outcome it can reach
  - This is the maximal outcome of its children
  - Note that we don't store path sums as before
- After search, can pick move that leads to best node

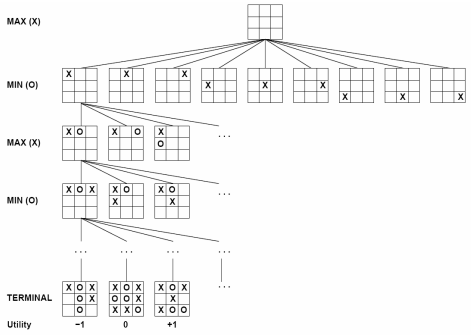


Deterministic Two-Player

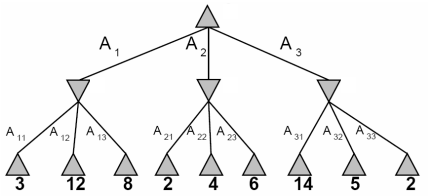
- E.g. tic-tac-toe, chess, checkers
- **Minimax search**
  - A state-space search tree
  - Players alternate
  - Each layer, or ply, consists of a round of moves
  - Choose move to position with highest minimax value = best achievable utility against best play
- Zero-sum games
  - One player maximizes result
  - The other minimizes result



Tic-tac-toe Game Tree



Minimax Example



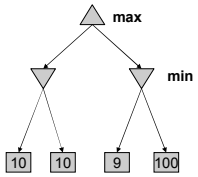
Minimax Search

function MAX-VALUE(*state*) **returns** *a utility value*
  if TERMINAL-TEST(*state*) **then return** UTILITY(*state*)
  $v \leftarrow -\infty$
  for *a*, *s* in SUCCESSORS(*state*) **do** $v \leftarrow$ MAX(v, MIN-VALUE(s))
  return *v*

function MIN-VALUE(*state*) **returns** *a utility value*
  if TERMINAL-TEST(*state*) **then return** UTILITY(*state*)
  $v \leftarrow \infty$
  for *a*, *s* in SUCCESSORS(*state*) **do** $v \leftarrow$ MIN(v, MAX-VALUE(s))
  return *v*

Minimax Properties

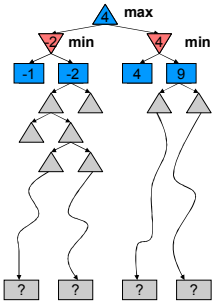
- Optimal against a perfect player. Otherwise?
- Time complexity?
  - $O(b^m)$
- Space complexity?
  - O(bm)
- For chess, b ≈ 35, m ≈ 100
  - Exact solution is completely infeasible
  - But, do we need to explore the whole tree?



[DEMO2: minVsExp]

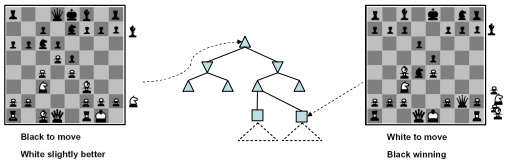
Resource Limits

- Cannot search to leaves
- Limited search
  - Instead, search a limited depth of the tree
  - Replace terminal utilities with an eval function for non-terminal positions
- Guarantee of optimal play is gone
- More plies makes a BIG difference
  - [DEMO 3: limitedDepth]
- Example:
  - Suppose we have 100 seconds, can explore 10K nodes / sec
  - So can check 1M nodes per move
  - α-β reaches about depth 8 – decent chess program



Evaluation Functions

- Function which scores non-terminals

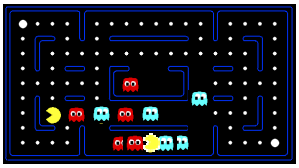


- Ideal function: returns the utility of the position
- In practice: typically weighted linear sum of features:

$$Eval(s) = w_1 f_1(s) + w_2 f_2(s) + \ldots + w_n f_n(s)$$

- e.g. $f_1(s)$ = (num white queens – num black queens), etc.

Evaluation for Pacman



[DEMO4: evalFunction, thrashing]

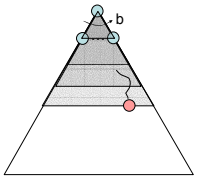
$$Eval(s) = w_1 f_1(s) + w_2 f_2(s) + \ldots + w_n f_n(s)$$

Iterative Deepening

Iterative deepening uses DFS as a subroutine:

1. Do a DFS which only searches for paths of length 1 or less. (DFS gives up on any path of length 2)
2. If "1" failed, do a DFS which only searches paths of length 2 or less.
3. If "2" failed, do a DFS which only searches paths of length 3 or less.

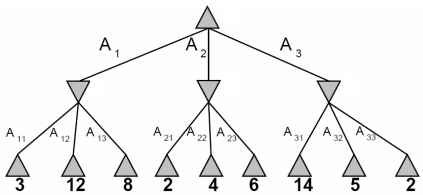
   ….and so on.



This works for single-agent search as well!

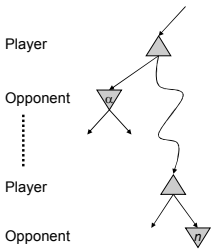
Why do we want to do this for multiplayer games?

α-β Pruning Example



α-β Pruning

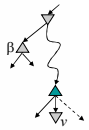
- General configuration
  - α is the best value the MAX can get at any choice point along the current path
  - If *n* is worse than α, MAX will avoid it, so prune *n*'s branch
  - Define β similarly for MIN



α-β Pruning Pseudocode

```
function MAX-VALUE(state) returns a utility value
  if TERMINAL-TEST(state) then return UTILITY(state)
  v ← −∞
  for a, s in SUCCESSORS(state) do v ← MAX(v, MIN-VALUE(s))
  return v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  inputs: state, current state in game
          α, the value of the best alternative for MAX along the path to state
          β, the value of the best alternative for MIN along the path to state
  if TERMINAL-TEST(state) then return UTILITY(state)
  v ← −∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then return v
    α ← MAX(α, v)
  return v
```

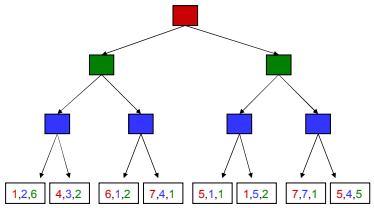


α-β Pruning Properties

- Pruning has no effect on final result
- Good move ordering improves effectiveness of pruning
- With "perfect ordering":
  - Time complexity drops to $O(b^{m/2})$
  - Doubles solvable depth
  - Full search of, e.g. chess, is still hopeless!
- A simple example of metareasoning, here reasoning about which computations are relevant

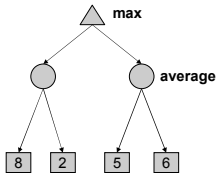
Non-Zero-Sum Games

- Similar to minimax:
  - Utilities are now tuples
  - Each player maximizes their own entry at each node
  - Propagate (or back up) nodes from children



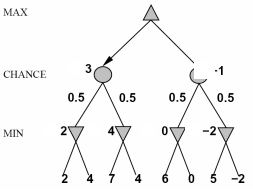
Stochastic Single-Player

- What if we don't know what the result of an action will be? E.g.,
  - In solitaire, shuffle is unknown
  - In minesweeper, don't know where the mines are
- Can do **expectimax search**
  - Chance nodes, like actions except the environment controls the action chosen
  - Calculate utility for each node
  - Max nodes as in search
  - Chance nodes take average (expectation) of value of children
- Later, we'll learn how to formalize this as a **Markov Decision Process**



Stochastic Two-Player

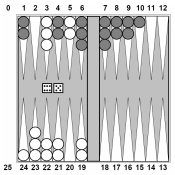
- E.g. backgammon
- Expectiminimax (!)
  - Environment is an extra player that moves after each agent
  - Chance nodes take expectations, otherwise like minimax




```
if state is a MAX node then
    return the highest EXPECTIMINIMAX-VALUE of SUCCESSORS(state)
if state is a MIN node then
    return the lowest EXPECTIMINIMAX-VALUE of SUCCESSORS(state)
if state is a chance node then
    return average of EXPECTIMINIMAX-VALUE of SUCCESSORS(state)
```

Stochastic Two-Player

- Dice rolls increase *b*: 21 possible rolls with 2 dice
  - Backgammon ≈ 20 legal moves
  - Depth 4 = 20 x (21 x 20)^3 1.2 x 10^9
- As depth increases, probability of reaching a given node shrinks
  - So value of lookahead is diminished
  - So limiting depth is less damaging
  - But pruning is less possible…
- TDGammon uses depth-2 search + very good eval function + reinforcement learning: world-champion level play



What's Next?

- Make sure you know what:
  - Probabilities are
  - Expectations are
- Next topics:
  - Dealing with uncertainty
  - How to learn evaluation functions
  - Markov Decision Processes