CS 188: Artificial Intelligence
Fall 2007

Lecture 3: A* Search

9/4/2007

Dan Klein – UC Berkeley

Many slides over the course adapted from either Stuart Russell or Andrew Moore

Announcements

- Sections:
  - New section 106: Tu 5-6pm
  - You can go to any section, if there's space
  - Sections start this week
- Homework
  - Project 1 on the web, due 9/12
  - New written homework format:
    - One or two questions handed out end of section (and online)
    - Due the next week in section, graded check / no check
    - Each assignment 1% of grade, cap of 10%, so can skip at least one week, depends on how many there are
    - Solve in groups of any size, write up alone

Today

- A* Search
- Heuristic Design
- Local Search

Recap: Search

- Search problems:
  - States (configurations of the world)
  - Successor functions, costs, start and goal tests
- Search trees:
  - Nodes: represent paths / plans
  - Paths have costs (sum of action costs)

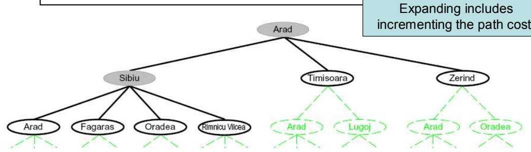
$$g(n) = \sum_{x \rightarrow y \in n} cost(x \rightarrow y)$$

  - Strategies differ (only) in fringe management

General Tree Search

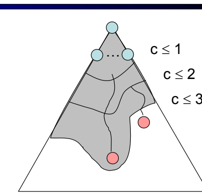
```
function TREE-SEARCH( problem, strategy) returns a solution, or failure
   initialize the search tree using the initial state of problem
   loop do
      if there are no candidates for expansion then return failure
      choose a leaf node for expansion according to strategy
      if the node contains a goal state then return the corresponding solution
      else expand the node and add the resulting nodes to the search tree
   end
```

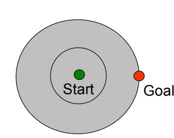
Expanding includes incrementing the path cost!



Uniform Cost

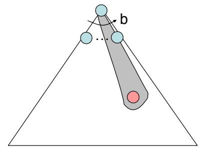
- Strategy: expand lowest path cost
- The good: UCS is complete and optimal!
- The bad:
  - Explores options in every "direction"
  - No information about goal location

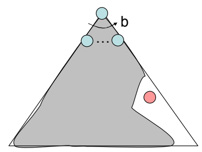




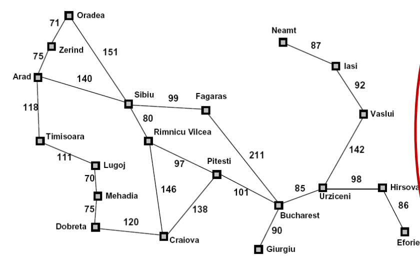
Best First

- Strategy: expand nodes which appear closest to goal
  - Heuristic: function which maps states to distance
- A common case:
  - Best-first takes you straight to the (wrong) goal
- Worst case: like a badly guided DFS





Example: Heuristic Function

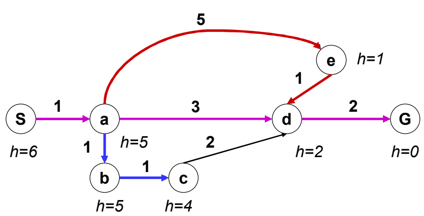


Straight−line distance to Bucharest	
Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	178
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	98
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

h(x)

Combining UCS and Greedy

- Uniform-cost orders by path cost, or *backward cost* g(n)
- Best-first orders by goal proximity, or *forward cost* h(n)

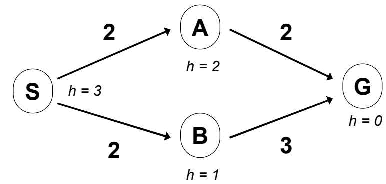


- A* Search orders by the sum: f(n) = g(n) + h(n)

Example: Teg Grenager

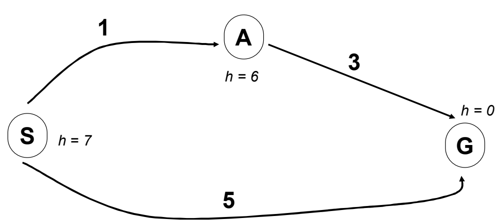
When should A* terminate?

- Should we stop when we enqueue a goal?



- No: only stop when we dequeue a goal

Is A* Optimal?



- What went wrong?
- Actual bad goal cost > estimated good goal cost
- We need estimates to be less than actual costs!

Admissible Heuristics

- A heuristic is ***admissible*** (optimistic) if:

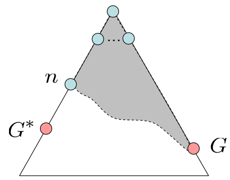
$$h(n) \leq h^*(n)$$

where $h^*(n)$ is the true cost to a nearest goal

- E.g. Euclidean distance on a map problem
- Coming up with admissible heuristics is most of what's involved in using A* in practice.

Optimality of A*: Blocking

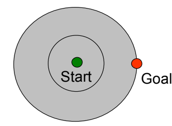
- Proof:
  - What could go wrong?
  - We'd have to have to pop a suboptimal goal G off the fringe before G*
  - This can't happen:
    - Imagine a suboptimal goal G is on the queue
    - Some node *n* which is a subpath of G* must be on the fringe (why?)
    - *n* will be popped before G



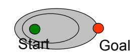
$$f(n) \leq g(G^*)$$
$$g(G^*) < g(G)$$
$$g(G) = f(G)$$
$$f(n) < f(G)$$

UCS vs A* Contours

- Uniform-cost expanded in all directions

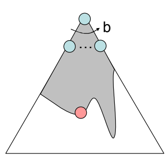


- A* expands mainly toward the goal, but does hedge its bets to ensure optimality

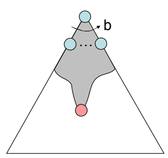


Properties of A*

Uniform Cost



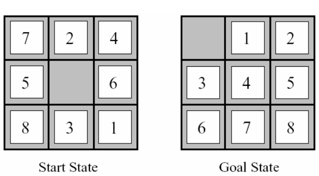
A*



Admissible Heuristics

- Most of the work is in coming up with admissible heuristics
- Inadmissible heuristics are often quite effective (especially when you have no choice)
- Very common hack: use α x h(n) for admissible h, α > 1 to generate a faster but less optimal inadmissible h' from admissible h

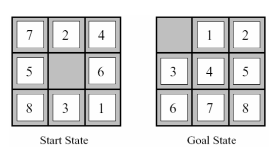
Example: 8 Puzzle



- What are the states?
- What are the actions?
- What states can I reach from the start state?
- What should the costs be?

8 Puzzle I

- Number of tiles misplaced?
- Why is it admissible?
- h(start) = 8
- This is a relaxed problem heuristic



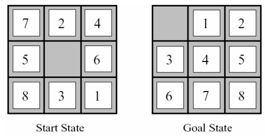
	Average nodes expanded when optimal path has length…		
	…4 steps	…8 steps	…12 steps
ID	112	6,300	3.6×10^6
TILES	13	39	227

8 Puzzle II

- What if we had an easier 8-puzzle where any tile could slide any direction at any time, ignoring other tiles?
- Total *Manhattan* distance
- Why admissible?
- h(start) =

  3 + 1 + 2 + …

  = 18



	Average nodes expanded when optimal path has length…		
	…4 steps	…8 steps	…12 steps
TILES	13	39	227
MAN-HATTAN	12	25	73

8 Puzzle III

- How about using the *actual cost* as a heuristic?
  - Would it be admissible?
  - Would we save on nodes?
  - What's wrong with it?
- With A*: a trade-off between quality of estimate and work per node!

Trivial Heuristics, Dominance

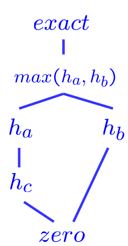
- Dominance: $h_a \geq h_c$ if

  $$\forall n : h_a(n) \geq h_c(n)$$

- Heuristics form a semi-lattice:
  - Max of admissible heuristics is admissible

  $$h(n) = max(h_a(n), h_b(n))$$

- Trivial heuristics
  - Bottom of lattice is the zero heuristic (what does this give us?)
  - Top of lattice is the exact heuristic



Course Scheduling

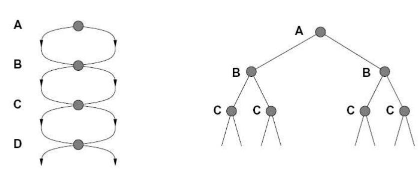
- From the university's perspective:
  - Set of courses $\{c_1, c_2, \ldots c_n\}$
  - Set of room / times $\{r_1, r_2, \ldots r_n\}$
  - Each pairing (c_k, r_m) has a cost w_{km}
  - What's the best assignment of courses to rooms?
- States: list of pairings
- Actions: add a legal pairing
- Costs: cost of the new pairing
- Admissible heuristics?
- (Who can think of a cs170 answer to this problem?)

Other A* Applications

- Pathing / routing problems
- Resource planning problems
- Robot motion planning
- Language analysis
- Machine translation
- Speech recognition
- …

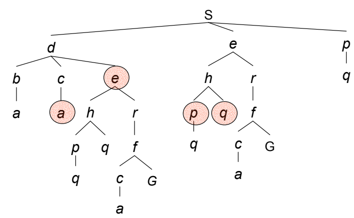
Tree Search: Extra Work?

- Failure to detect repeated states can cause exponentially more work. Why?



Graph Search

- In BFS, for example, we shouldn't bother expanding the circled nodes (why?)



Graph Search

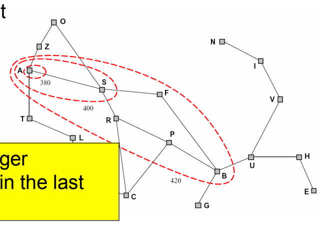
- Very simple fix: never expand a state twice

```
function GRAPH-SEARCH( problem, fringe) returns a solution, or failure
   closed ← an empty set
   fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
   loop do
       if fringe is empty then return failure
       node ← REMOVE-FRONT(fringe)
       if GOAL-TEST(problem, STATE[node]) then return node
       if STATE[node] is not in closed then
           add STATE[node] to closed
           fringe ← INSERTALL(EXPAND(node, problem), fringe)
   end
```

- Can this wreck completeness? Optimality?

Optimality of A* Graph Search

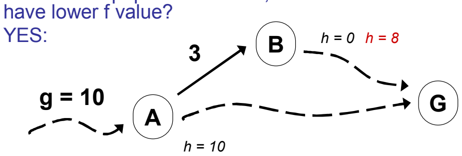
- Consider what A* does:
  - Expands nodes in increasing total f value (f-contours)
  - Proof idea: optimal goals have lower f value, so get expanded first



We made a stronger assumption than in the last proof… What?

Consistency

- Wait, how do we know we expand in increasing f value?
- Couldn't we pop some node *n*, and find its child *n'* to have lower f value?
- YES:



- What can we assume to prevent these inversions?
- Consistency: $c(n, a, n') \geq h(n) - h(n')$
- Real cost always exceeds reduction in heuristic

Optimality

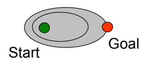
- Tree search:
  - A* optimal if heuristic is admissible (and non-negative)
  - UCS is a special case (h = 0)
- Graph search:
  - A* optimal if heuristic is consistent
  - UCS optimal (h = 0 is consistent)
- In general, natural admissible heuristics tend to be consistent

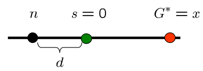
Summary: A*

- A* uses both backward costs and (estimates of) forward costs
- A* is optimal with admissible heuristics
- Heuristic design is key: often use relaxed problems

Large Scale Problems

- What states get expanded?
  - All states with f-cost less than optimal goal cost
  - How far "in every direction" will this be?
    - Intuition: depth grows like the heuristic "gap": h(start) – g(goal)
    - Gap usually at least linear in problem size
    - Work exponential in depth
- In huge problems, often A* isn't enough
  - State space just too big
  - Can't visit all states with f less than optimal
  - Often, can't even store the entire fringe
- Solutions
  - Better heuristics
  - Beam search (limited fringe size)
  - Greedy hill-climbing (fringe size = 1)





assume $h(n) = \beta|x - n|$

$$f(G^*) = x$$

$$f(n) = g(n) + h(n)$$
$$= d + \beta(x + d)$$

$$f(G^*) = f(n) \Rightarrow x = d + \beta(x + d)$$

$$\Rightarrow d = \left(\frac{1 - \beta}{1 + \beta}\right) x$$

Limited Memory Options

- Hill-Climbing Search:
  - Only "best" node kept around, no fringe!
  - Usually prioritize successor choice by h (greedy hill climbing)
  - Compare to greedy backtracking, which still has fringe
- Beam Search (Limited Memory Search)
  - In between: keep K nodes in fringe
  - Dump lowest priority nodes as needed
  - Can prioritize by h alone (greedy beam search), or h+g (limited memory A*)
  - Why not applied to UCS?
  - We'll return to beam search later…
- No guarantees once you limit the fringe size!