CS 188: Artificial Intelligence
Fall 2007

Lecture 24: Perceptrons
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General Naïve Bayes

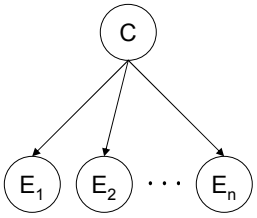
- A general *naive Bayes* model:

|C| x |E|^n parameters

$$P(\text{Cause}, \text{Effect}_1 \ldots \text{Effect}_n) = P(\text{Cause}) \prod_i P(\text{Effect}_i|\text{Cause})$$

|C| parameters

n x |E| x |C| parameters



- We only specify how each feature depends on the class
- Total number of parameters is *linear* in n

Example: Spam Filtering

- Model: $P(C, W_1 \dots W_n) = P(C) \prod_i P(W_i|C)$
- Parameters:

$P(C)$

```
ham : 0.66
spam: 0.33
```

$P(W|\text{spam})$

```
the     : 0.016
to      : 0.015
and     : 0.012
...
free    : 0.001
click   : 0.001
...
morally : 0.001
nicely  : 0.001
...
```

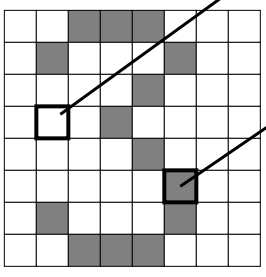
$P(W|\text{ham})$

```
the     : 0.021
to      : 0.013
and     : 0.011
...
free    : 0.005
click   : 0.004
...
screens : 0.000
minute  : 0.000
...
```

Example: OCR

$P(C)$

1	0.1
2	0.1
3	0.1
4	0.1
5	0.1
6	0.1
7	0.1
8	0.1
9	0.1
0	0.1



$P(F_{3,1} = on|C)$

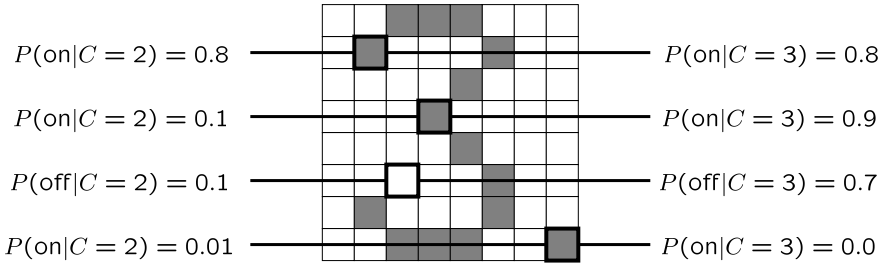
1	0.01
2	0.05
3	0.05
4	0.30
5	0.80
6	0.90
7	0.05
8	0.60
9	0.50
0	0.80

$P(F_{5,5} = on|C)$

1	0.05
2	0.01
3	0.90
4	0.80
5	0.90
6	0.90
7	0.25
8	0.85
9	0.60
0	0.80

Example: Overfitting

$P(\text{features}, C = 2)$ $P(\text{features}, C = 3)$



2 wins!!

Generalization and Overfitting

- Relative frequency parameters will overfit the training data!
  - Unlikely that every occurrence of "minute" is 100% spam
  - Unlikely that every occurrence of "seriously" is 100% ham
  - What about all the words that don't occur in the training set?
  - In general, we can't go around giving unseen events zero probability
- As an extreme case, imagine using the entire email as the only feature
  - Would get the training data perfect (if deterministic labeling)
  - Wouldn't *generalize* at all
  - Just making the bag-of-words assumption gives us some generalization, but isn't enough
- To generalize better: we need to smooth or regularize the estimates

Estimation: Smoothing

- Problems with maximum likelihood estimates:
  - If I flip a coin once, and it's heads, what's the estimate for P(heads)?
  - What if I flip 10 times with 8 heads?
  - What if I flip 10M times with 8M heads?
- Basic idea:
  - We have some prior expectation about parameters (here, the probability of heads)
  - Given little evidence, we should skew towards our prior
  - Given a lot of evidence, we should listen to the data

Estimation: Smoothing

- Relative frequencies are the maximum likelihood estimates

$$\theta_{ML} = \arg\max_{\theta} P(\mathbf{X}|\theta) = \arg\max_{\theta} \prod_i P_\theta(X_i) \quad \Rightarrow \quad \hat{P}(x) = \frac{\text{count}(x)}{\text{total samples}}$$

- In Bayesian statistics, we think of the parameters as just another random variable, with its own distribution

$$\theta_{MAP} = \arg\max_{\theta} P(\theta|\mathbf{X}) = \arg\max_{\theta} P(\mathbf{X}|\theta)P(\theta)/P(\mathbf{X}) = \arg\max_{\theta} P(\mathbf{X}|\theta)P(\theta)$$

⇒ ????

Estimation: Laplace Smoothing

- Laplace's estimate:
  - Pretend you saw every outcome once more than you actually did

$$P_{LAP}(x) = \frac{c(x)+1}{\sum_x [c(x)+1]}$$

$$= \frac{c(x)+1}{N+|X|}$$

$P_{ML}(X) =$

$P_{LAP}(X) =$

  - Can derive this as a MAP estimate with *Dirichlet priors* (see cs281a)

Estimation: Laplace Smoothing

- Laplace's estimate (extended):
  - Pretend you saw every outcome k extra times

$$P_{LAP,k}(x) = \frac{c(x)+k}{N+k|X|}$$

$P_{LAP,0}(X) =$

$P_{LAP,1}(X) =$

$P_{LAP,100}(X) =$

  - What's Laplace with k = 0?
  - k is the strength of the prior
- Laplace for conditionals:
  - Smooth each condition independently:

$$P_{LAP,k}(x|y) = \frac{c(x,y)+k}{c(y)+k|X|}$$

Estimation: Linear Interpolation

- In practice, Laplace often performs poorly for P(X|Y):
  - When |X| is very large
  - When |Y| is very large
- Another option: linear interpolation
  - Also get P(X) from the data
  - Make sure the estimate of P(X|Y) isn't too different from P(X)

$$P_{LIN}(x|y) = \alpha \hat{P}(x|y) + (1.0 - \alpha)\hat{P}(x)$$

  - What if α is 0? 1?
- For even better ways to estimate parameters, as well as details of the math see cs281a, cs294

Real NB: Smoothing

- For real classification problems, smoothing is critical
- New odds ratios:

$$\frac{P(W|\text{ham})}{P(W|\text{spam})}$$

```
helvetica : 11.4
seems     : 10.8
group     : 10.2
ago       :  8.4
areas     :  8.3
...
```

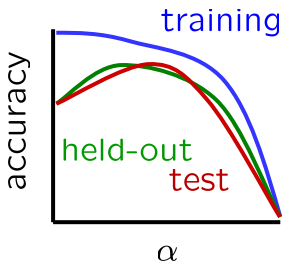
$$\frac{P(W|\text{spam})}{P(W|\text{ham})}$$

```
verdana : 28.8
Credit  : 28.4
ORDER   : 27.2
<FONT>  : 26.9
money   : 26.5
...
```

Do these make more sense?

Tuning on Held-Out Data

- Now we've got two kinds of unknowns
  - Parameters: the probabilities P(Y|X), P(Y)
  - Hyperparameters, like the amount of smoothing to do: k, α
- Where to learn?
  - Learn parameters from training data
  - Must tune hyperparameters on different data
    - Why?
  - For each value of the hyperparameters, train and test on the held-out data
  - Choose the best value and do a final test on the test data



Baselines

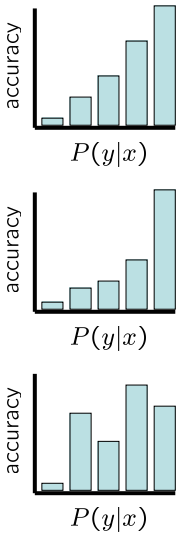
- First task: get a baseline
  - Baselines are very simple "straw man" procedures
  - Help determine how hard the task is
  - Help know what a "good" accuracy is
- Weak baseline: most frequent label classifier
  - Gives all test instances whatever label was most common in the training set
  - E.g. for spam filtering, might label everything as ham
  - Accuracy might be very high if the problem is skewed
- For real research, usually use previous work as a (strong) baseline

Confidences from a Classifier

- The confidence of a probabilistic classifier:
  - Posterior over the top label

$$\text{confidence}(x) = \arg\max_y P(y|x)$$

  - Represents how sure the classifier is of the classification
  - Any probabilistic model will have confidences
  - No guarantee confidence is correct
- Calibration
  - Weak calibration: higher confidences mean higher accuracy
  - Strong calibration: confidence predicts accuracy rate
  - What's the value of calibration?



Generative vs. Discriminative

- Generative classifiers:
  - E.g. naïve Bayes
  - We build a causal model of the variables
  - We then query that model for causes, given evidence
- Discriminative classifiers:
  - E.g. perceptron (next)
  - No causal model, no Bayes rule, often no probabilities
  - Try to predict output directly
  - Loosely: mistake driven rather than model driven

Errors, and What to Do

- Examples of errors

> Dear GlobalSCAPE Customer,
>
> GlobalSCAPE has partnered with ScanSoft to offer you the latest version of OmniPage Pro, for just $99.99* - the regular list price is $499! The most common question we've received about this offer is - Is this genuine? We would like to assure you that this offer is authorized by ScanSoft, is genuine and valid. You can get the . . .

> . . . To receive your $30 Amazon.com promotional certificate, click through to
>
> http://www.amazon.com/apparel
>
> and see the prominent link for the $30 offer. All details are there. We hope you enjoyed receiving this message. However, if you'd rather not receive future e-mails announcing new store launches, please click . . .

What to Do About Errors?

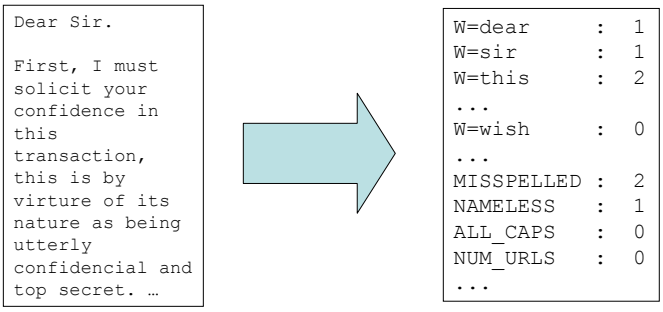
- Need more features– words aren't enough!
  - Have you emailed the sender before?
  - Have 1K other people just gotten the same email?
  - Is the sending information consistent?
  - Is the email in ALL CAPS?
  - Do inline URLs point where they say they point?
  - Does the email address you by (your) name?
- Naïve Bayes models can incorporate a variety of features, but tend to do best in homogeneous cases (e.g. all features are word occurrences)

Features

- A feature is a function which signals a property of the input
- Examples:
  - ALL_CAPS: value is 1 iff email in all caps
  - HAS_URL: value is 1 iff email has a URL
  - NUM_URLS: number of URLs in email
  - VERY_LONG: 1 iff email is longer than 1K
  - SUSPICIOUS_SENDER: 1 iff reply-to domain doesn't match originating server
- Features are anything you can think of code to evaluate on an input
  - Some cheap, some very very expensive to calculate
  - Can even be the output of another classifier
  - Domain knowledge goes here!
- In naïve Bayes, how did we encode features?

Feature Extractors

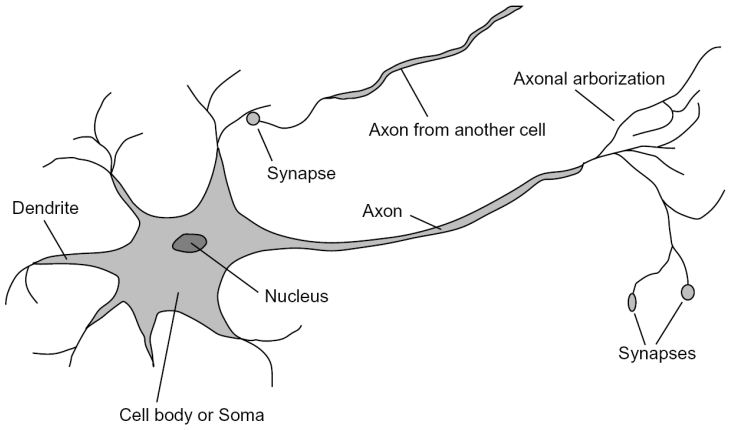
- A feature extractor maps inputs to feature vectors



- Many classifiers take feature vectors as inputs
- Feature vectors usually very sparse, use sparse encodings (i.e. only represent non-zero keys)

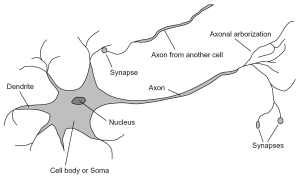
Some (Vague) Biology

- Very loose inspiration: human neurons



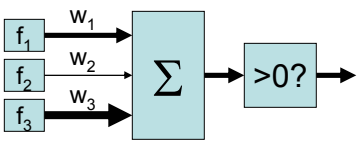
The Binary Perceptron

- Inputs are features
- Each feature has a weight
- Sum is the activation



$$\text{activation}_w(x) = \sum_i w_i \cdot f_i(x)$$

- If the activation is:
  - Positive, output 1
  - Negative, output 0



Example: Spam

- Imagine 4 features:
  - Free (number of occurrences of “free”)
  - Money (occurrences of “money”)
  - BIAS (always has value 1)

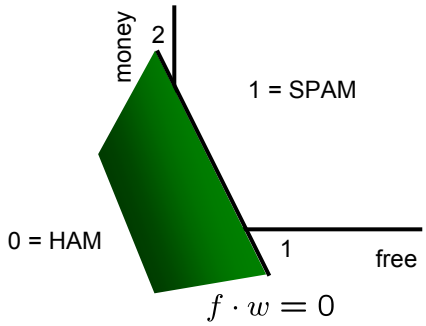
x	$f(x)$	w	$\sum_i w_i \cdot f_i(x)$
“free money”	BIAS : 1 free : 1 money : 1 the : 0 ...	BIAS : -3 free : 4 money : 2 the : 0 ...	$(1)(-3) +$ $(1)(4) +$ $(1)(2) +$ $(0)(0) +$ ... $= 3$

Binary Decision Rule

- In the space of feature vectors
  - Any weight vector is a hyperplane
  - One side will be class 1
  - Other will be class 0

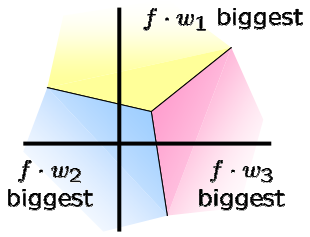
w

```
BIAS  : -3
free  :  4
money :  2
the   :  0
...
```



The Multiclass Perceptron

- If we have more than two classes:
  - Have a weight vector for each class
  - Calculate an activation for each class



$$\text{activation}_w(x, c) = \sum_i w_{c,i} \cdot f_i(x)$$

  - Highest activation wins

$$c = \arg\max_c \ (\text{activation}_w(x, c))$$

Example

"win the vote"

```
BIAS  :  1
win   :  1
game  :  0
vote  :  1
the   :  1
...
```

w_{SPORTS}

```
BIAS  : -2
win   :  4
game  :  4
vote  :  0
the   :  0
...
```

$w_{POLITICS}$

```
BIAS  :  1
win   :  2
game  :  0
vote  :  4
the   :  0
...
```

w_{TECH}

```
BIAS  :  2
win   :  0
game  :  2
vote  :  0
the   :  0
...
```

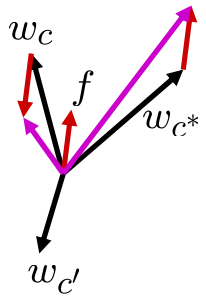
The Perceptron Update Rule

- Start with zero weights
- Pick up training instances one by one
- Try to classify

$$c = \arg\max_c \; w_c \cdot f(x)$$
$$= \arg\max_c \; \sum_i w_{c,i} \cdot f_i(x)$$

- If correct, no change!
- If wrong: lower score of wrong answer, raise score of right answer

$$w_c = w_c - f(x)$$
$$w_{c^*} = w_{c^*} + f(x)$$



Example

"win the vote"

"win the election"

"win the game"

w_{SPORTS}

```
BIAS  :
win   :
game  :
vote  :
the   :
...
```

$w_{POLITICS}$

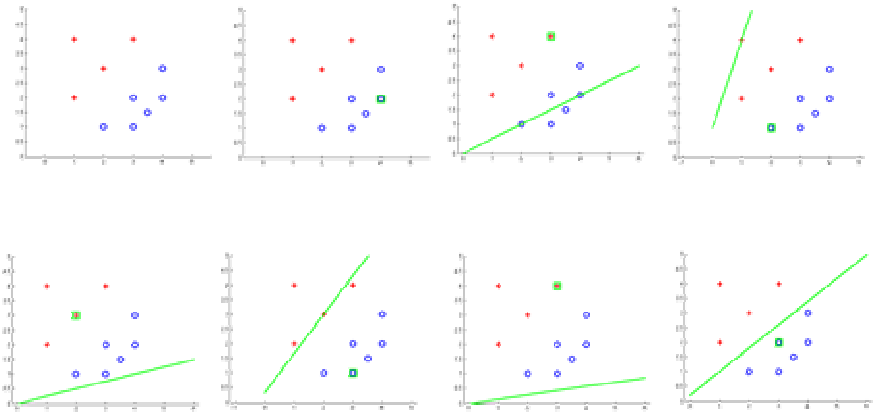
```
BIAS  :
win   :
game  :
vote  :
the   :
...
```

w_{TECH}

```
BIAS  :
win   :
game  :
vote  :
the   :
...
```

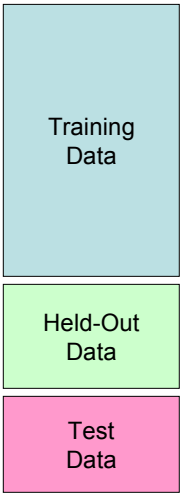
Examples: Perceptron

- Separable Case



Mistake-Driven Classification

- In naïve Bayes, parameters:
  - From data statistics
  - Have a causal interpretation
  - One pass through the data
- For the perceptron parameters:
  - From reactions to mistakes
  - Have a discriminative interpretation
  - Go through the data until held-out accuracy maxes out

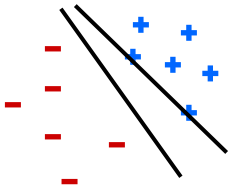


Properties of Perceptrons

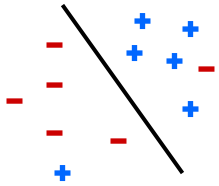
- Separability: some parameters get the training set perfectly correct
- Convergence: if the training is separable, perceptron will eventually converge (binary case)
- Mistake Bound: the maximum number of mistakes (binary case) related to the *margin* or degree of separability

$$\text{mistakes} < \frac{1}{\delta^2}$$

Separable

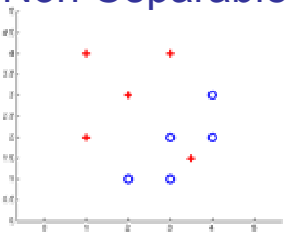


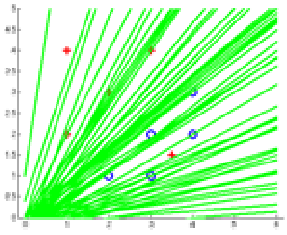
Non-Separable

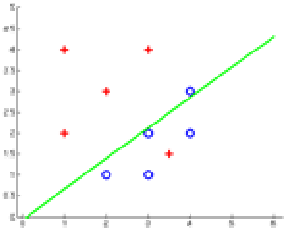


Examples: Perceptron

- Non-Separable Case

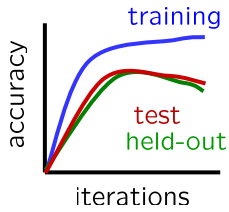


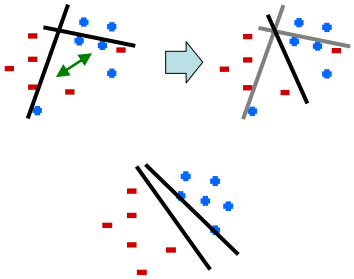




Issues with Perceptrons

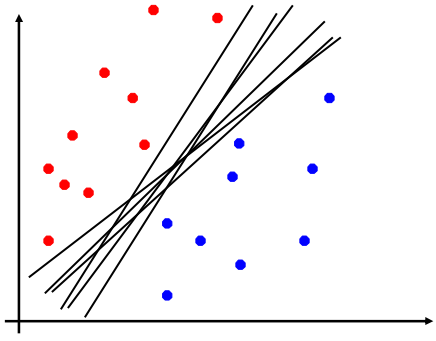
- Overtraining: test / held-out accuracy usually rises, then falls
  - Overtraining isn't quite as bad as overfitting, but is similar
- Regularization: if the data isn't separable, weights might thrash around
  - Averaging weight vectors over time can help (averaged perceptron)
- Mediocre generalization: finds a "barely" separating solution





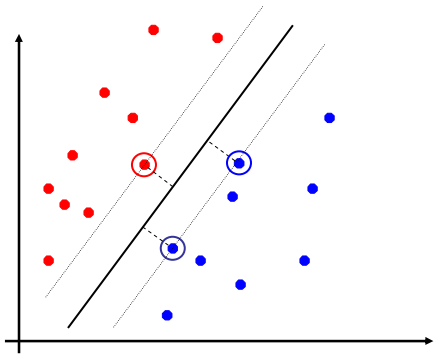
Linear Separators

- Which of these linear separators is optimal?



Support Vector Machines

- Maximizing the margin: good according to intuition and PAC theory.
- Only support vectors matter; other training examples are ignorable.
- Support vector machines (SVMs) find the separator with max margin
- Mathematically, gives a quadratic program to solve
- Basically, SVMs are perceptrons with smarter update counts!



Summary

- Naïve Bayes
  - Build classifiers using model of training data
  - Smoothing estimates is important in real systems
  - Classifier confidences are useful, when you can get them
- Perceptrons:
  - Make less assumptions about data
  - Mistake-driven learning
  - Multiple passes through data