CS 188: Artificial Intelligence
Fall 2007

Lecture 17: Bayes Nets III
10/25/2007

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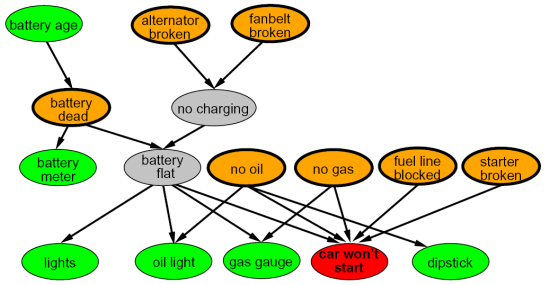
Representing Knowledge

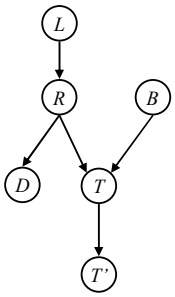


Properties of BNs

- Bayes' nets:
  - Specify complex joint distributions using simple local conditional distributions
  - Conditional independence makes this possible
  - The graph structure of a BN guarantees certain conditional independences
- Questions:
  - What independences does a BN have?
  - How to compute quantities we want from quantities we have

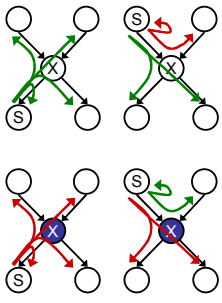
Independence?

- Recipe: shade evidence nodes
- Attempt 1: if two nodes are connected by an undirected path not blocked by a shaded node, they are conditionally independent
- Almost works, but not quite
  - Where does it break?
  - Answer: the v-structure at T doesn't count as a link in a path unless shaded



Reachability (the Bayes' Ball)

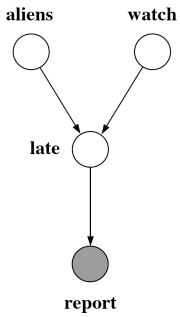
- Correct algorithm:
  - Shade in evidence
  - Start at source node
  - Try to reach target by search
  - States: pair of (node X, previous state S)
  - Successor function:
    - X unobserved:
      - To any child
      - To any parent if coming from a child
    - X observed:
      - From parent to parent
  - If you can't reach a node, it's conditionally independent of the start node given evidence



Example

$A \perp\!\!\!\perp W$ *Yes*

$A \perp\!\!\!\perp W \mid R$



Example

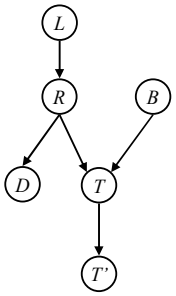
$L \perp\!\!\!\perp T' | T$ Yes

$L \perp\!\!\!\perp B$ Yes

$L \perp\!\!\!\perp B | T$

$L \perp\!\!\!\perp B | T'$

$L \perp\!\!\!\perp B | T, R$ Yes



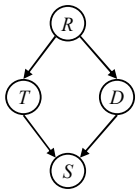
Example

- Variables:
  - R: Raining
  - T: Traffic
  - D: Roof drips
  - S: I'm sad
- Questions:

$T \perp\!\!\!\perp D$

$T \perp\!\!\!\perp D | R$ Yes

$T \perp\!\!\!\perp D | R, S$



Causality?

- When Bayes' nets reflect the true causal patterns:
  - Often simpler (nodes have fewer parents)
  - Often easier to think about
  - Often easier to elicit from experts
- BNs need not actually be causal
  - Sometimes no causal net exists over the domain
  - E.g. consider the variables *Traffic* and *Drips*
  - End up with arrows that reflect correlation, not causation
- What do the arrows really mean?
  - Topology may happen to encode causal structure
  - Topology only guaranteed to encode conditional independencies

Example: Traffic

- Basic traffic net
- Let's multiply out the joint

$P(R)$

r	1/4
¬r	3/4

$P(T|R)$

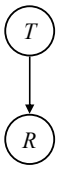
r	t	3/4
	¬t	1/4
¬r	t	1/2
	¬t	1/2

$P(T, R)$

r	t	3/16
r	¬t	1/16
¬r	t	6/16
¬r	¬t	6/16

Example: Reverse Traffic

- Reverse causality?



$P(T)$

t	9/16
¬t	7/16

$P(R|T)$

t	r	1/3
	¬r	2/3
¬t	r	1/7
	¬r	6/7

$P(T, R)$

r	t	3/16
r	¬t	1/16
¬r	t	6/16
¬r	¬t	6/16

Example: Coins

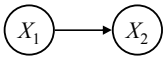
- Extra arcs don't prevent representing independence, just allow non-independence

$P(X_1)$

h	0.5
t	0.5

$P(X_2)$

h	0.5
t	0.5



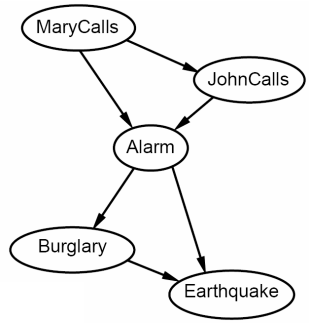
$P(X_1)$

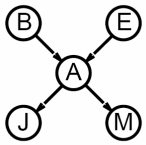
h	0.5
t	0.5

$P(X_2|X_1)$

h | h	0.5
t | h	0.5
h | t	0.5
t | t	0.5

Alternate BNs





Summary

- Bayes nets compactly encode joint distributions
- Guaranteed independencies of distributions can be deduced from BN graph structure
- A Bayes' net may have other independencies that are not detectable until you inspect its specific distribution
- The Bayes' ball algorithm (aka d-separation) tells us when an observation of one variable can change belief about another variable

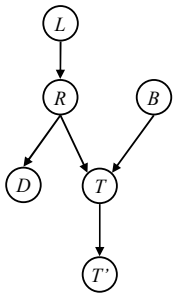
Inference

- Inference: calculating some statistic from a joint probability distribution
- Examples:
  - Posterior probability:

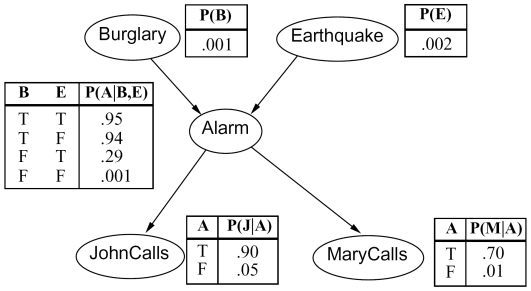
$$P(Q|E_1 = e_1, \ldots E_k = e_k)$$

  - Most likely explanation:

$$\text{argmax}_q \; P(Q = q|E_1 = e_1 \ldots)$$



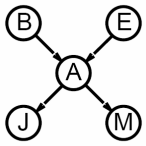
Reminder: Alarm Network



Inference by Enumeration

- Given unlimited time, inference in BNs is easy
- Recipe:
  - State the marginal probabilities you need
  - Figure out ALL the atomic probabilities you need
  - Calculate and combine them
- Example:

$$P(b|j,m) = \frac{P(b,j,m)}{P(j,m)}$$

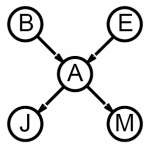


Example

$$P(b|j,m) = \frac{P(b,j,m)}{P(j,m)}$$

$$\begin{aligned} P(b,j,m) = \; & P(b,e,a,j,m) + \\ & P(b,\bar{e},a,j,m) + \\ & P(b,e,\bar{a},j,m) + \\ & P(b,\bar{e},\bar{a},j,m) \end{aligned}$$

$$= \sum_{e,a} P(b,e,a,j,m)$$



Where did we use the BN structure?

We didn't!

Example

- In this simple method, we only need the BN to synthesize the joint entries

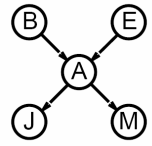
$$P(b,j,m) = P(b)P(e)P(a|b,e)P(j|a)P(m|a) + P(b)P(e)P(\bar{a}|b,e)P(j|\bar{a})P(m|\bar{a}) + P(b)P(\bar{e})P(a|b,\bar{e})P(j|a)P(m|a) + P(b)P(\bar{e})P(\bar{a}|b,\bar{e})P(j|\bar{a})P(m|\bar{a})$$

Normalization Trick

$$P(B|j,m) = \frac{P(B,j,m)}{P(j,m)}$$

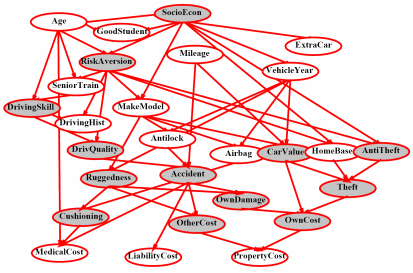
$$P(b,j,m) = \sum_{e,a} P(b,e,a,j,m)$$

$$P(\bar{b},j,m) = \sum_{e,a} P(\bar{b},e,a,j,m)$$



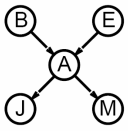
$$\begin{pmatrix} P(b,j,m) \\ P(\bar{b},j,m) \end{pmatrix} \xrightarrow{\text{Normalize}} \begin{pmatrix} P(b|j,m) \\ P(\bar{b}|j,m) \end{pmatrix}$$

Inference by Enumeration?



Nesting Sums

- Atomic inference is extremely slow!
- Slightly clever way to save work:
  - Move the sums as far right as possible
  - Example:



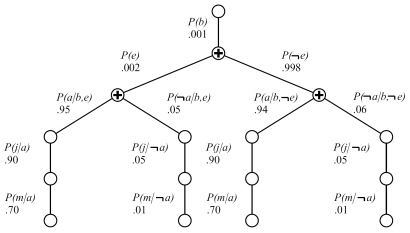
$$P(b,j,m) = \sum_{e,a} P(b,e,a,j,m)$$

$$= \sum_{e,a} P(b)P(e)P(a|b,e)P(j|a)P(m|a)$$

$$= P(b)\sum_{e} P(e)\sum_{a} P(a|b,e)P(j|a)P(m|a)$$

Evaluation Tree

- View the nested sums as a computation tree:

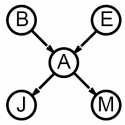


- Still repeated work: calculate P(m | a) P(j | a) twice, etc.

Variable Elimination: Idea

- Lots of redundant work in the computation tree
- We can save time if we cache all partial results
- This is the basic idea behind variable elimination

Basic Objects



- Track objects called factors
- Initial factors are local CPTs

$\underbrace{P(B)}_{f_B(B)}$ $\underbrace{P(J|A)}_{f_J(A,J)}$ $\underbrace{P(A|B,E)}_{f_A(A,B,E)}$

- During elimination, create new factors
- Anatomy of a factor:

4 numbers, one for each value of D and E

$$f_{A\bar{B}\bar{C}D}(D,E)$$

Variables introduced

Variables summed out

Argument variables, always non-evidence variables

Basic Operations

- First basic operation: join factors
- Combining two factors:
  - Just like a database join
  - Build a factor over the union of the domains
- Example:

$$f_1(A,B) \times f_2(B,C) \Rightarrow f_3(A,B,C)$$

$$f_3(a,b,c) = f_1(a,b) \cdot f_2(b,c)$$

$$\text{“}P(a,b|c) = P(a|b) \cdot P(b|c)\text{”}$$

Basic Operations

- Second basic operation: marginalization
- Take a factor and sum out a variable
  - Shrinks a factor to a smaller one
  - A projection operation
- Example:

$$f_{\bar{A}B}(b) = \sum_a f_{AB}(a,b)$$

$$\text{“}P(b) = \sum_a P(a,b)\text{”}$$

Example

$$P(b,j,m)$$

$$= \underbrace{P(b)}_{B} \textstyle\sum_e \underbrace{P(e)}_{E} \sum_a \underbrace{P(a|b,e)}_{A} \underbrace{P(j|a)}_{J} \underbrace{P(m|a)}_{M}$$

$$= f_B(b) \sum_e f_E(e) \sum_a f_A(a,b,e) f_J(a) f_M(a)$$

$$= f_B(b) \sum_e f_E(e) \sum_a f_{AJM}(a,b,e)$$

$$= f_B(b) \sum_e f_E(e) f_{\bar{A}JM}(b,e)$$

Example

$$P(b,j,m)$$

$$= f_B(b) \sum_e f_E(e) f_{\bar{A}JM}(b,e)$$

$$= f_B(b) \sum_e f_{\bar{A}EJM}(b,e)$$

$$= f_B(b) f_{\bar{A}\bar{E}JM}(b)$$

$$= f_{\bar{A}B\bar{E}JM}(b)$$

Variable Elimination

- What you need to know:
  - VE caches intermediate computations
  - Polynomial time for tree-structured graphs!
  - Saves time by marginalizing variables ask soon as possible rather than at the end
- We will see special cases of VE later
  - You'll have to implement the special cases
- Approximations
  - Exact inference is slow, especially when you have a lot of hidden nodes
  - Approximate methods give you a (close) answer, faster