CS 188: Artificial Intelligence
Fall 2007

Lecture 13: Reinforcement Learning
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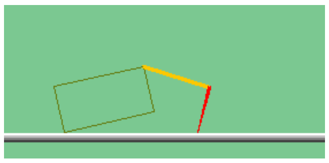
Reinforcement Learning

- Reinforcement learning:
  - Still have an MDP:
    - A set of states $s \in S$
    - A set of actions (per state) A
    - A model T(s,a,s')
    - A reward function R(s,a,s')
  - Still looking for a policy $\pi(s)$
  - New twist: don't know T or R
    - I.e. don't know which states are good or what the actions do
    - Must actually try actions and states out to learn
  - Quantities: V(s), Q(s,a) are expected future returns



Q-Learning

- Learn Q*(s,a) values
  - Receive a sample (s,a,s',r)
  - Consider your old estimate: $Q(s,a)$
  - Consider your new sample estimate:

$$Q^*(s,a) = \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma \max_{a'} Q^*(s',a')\right]$$

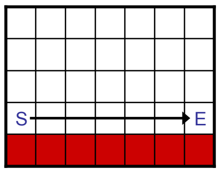
$$sample = R(s,a,s') + \gamma \max_{a'} Q(s',a')$$

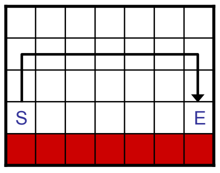
  - Nudge the old estimate towards the new sample:

$$Q(s,a) \leftarrow (1-\alpha)Q(s,a) + (\alpha)\,[sample]$$

Q-Learning Properties

- Will converge to optimal policy
  - If you explore enough
  - If you make the learning rate small enough
  - But not decrease it too quickly!
- Neat property: learns optimal q-values regardless of action selection noise (some caveats)





Exploration / Exploitation

- Several schemes for forcing exploration
  - Simplest: random actions (ε greedy)
    - Every time step, flip a coin
    - With probability ε, act randomly
    - With probability 1-ε, act according to current policy
  - Problems with random actions?
    - You do explore the space, but keep thrashing around once learning is done
    - One solution: lower ε over time
    - Another solution: exploration functions

Exploration Functions

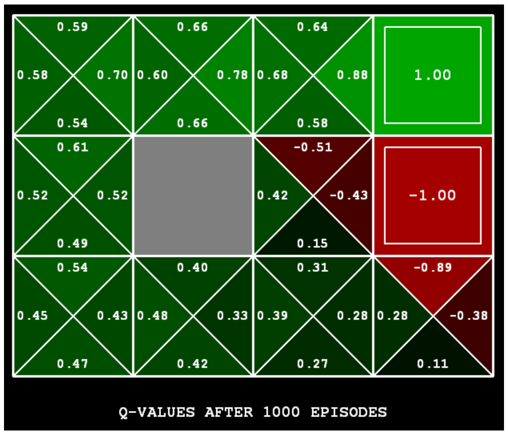
- When to explore
  - Random actions: explore a fixed amount
  - Better idea: explore areas whose badness is not (yet) established
- Exploration function
  - Takes a value estimate and a count, and returns an optimistic utility, e.g. $f(u,n) = u + k/n$ (exact form not important)

$$Q_{i+1}(s,a) \leftarrow_{\alpha} R(s,a,s') + \gamma \max_{a'} Q_i(s',a')$$

$$Q_{i+1}(s,a) \leftarrow_{\alpha} R(s,a,s') + \gamma \max_{a'} f(Q_i(s',a'), N(s',a'))$$

Q-Learning

- Q-learning produces tables of q-values:

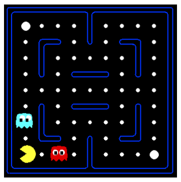


Q-Learning

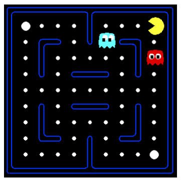
- In realistic situations, we cannot possibly learn about every single state!
  - Too many states to visit them all in training
  - Too many states to hold the q-tables in memory
- Instead, we want to generalize:
  - Learn about some small number of training states from experience
  - Generalize that experience to new, similar states
  - This is a fundamental idea in machine learning, and we’ll see it over and over again

Example: Pacman

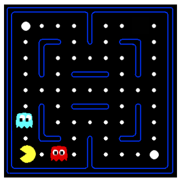
- Let’s say we discover through experience that this state is bad:



- In naïve q learning, we know nothing about this state or its q states:

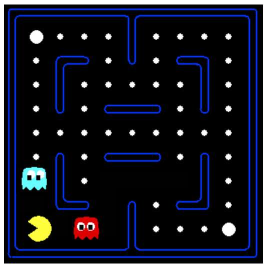


- Or even this one!



Feature-Based Representations

- Solution: describe a state using a vector of features
  - Features are functions from states to real numbers (often 0/1) that capture important properties of the state
  - Example features:
    - Distance to closest ghost
    - Distance to closest dot
    - Number of ghosts
    - 1 / (dist to dot)^2
    - Is Pacman in a tunnel? (0/1)
    - …… etc.
  - Can also describe a q-state (s, a) with features (e.g. action moves closer to food)



Linear Feature Functions

- Using a feature representation, we can write a q function (or value function) for any state using a few weights:

$$V(s) = w_1 f_1(s) + w_2 f_2(s) + \ldots + w_n f_n(s)$$

$$Q(s,a) = w_1 f_1(s,a) + w_2 f_2(s,a) + \ldots + w_n f_n(s,a)$$

- Advantage: our experience is summed up in a few powerful numbers
- Disadvantage: states may share features but be very different in value!

Function Approximation

$$Q(s,a) = w_1 f_1(s,a) + w_2 f_2(s,a) + \ldots + w_n f_n(s,a)$$

- Q-learning with linear q-functions:

$$Q(s,a) \leftarrow Q(s,a) + \alpha \, [error]$$

$$w_i \leftarrow w_i + \alpha \, [error] \, f_i(s,a)$$

- Intuitive interpretation:
  - Adjust weights of active features
  - E.g. if something unexpectedly bad happens, disprefer all states with that state's features
- Formal justification: online least squares

Example: Q-Pacman

$Q(s,a) = 4.0 f_{DOT}(s,a) - 1.0 f_{GST}(s,a)$

$f_{DOT}(s, \text{NORTH}) = 0.5$

$f_{GST}(s, \text{NORTH}) = 1.0$

$Q(s,a) = +1$

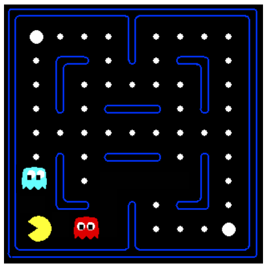
$R(s,a,s') = -500$

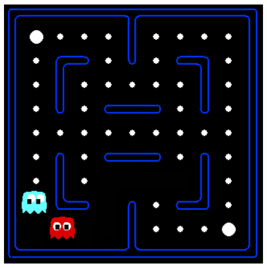
$error = -501$

$w_{DOT} \leftarrow 4.0 + \alpha\,[-501]\,0.5$

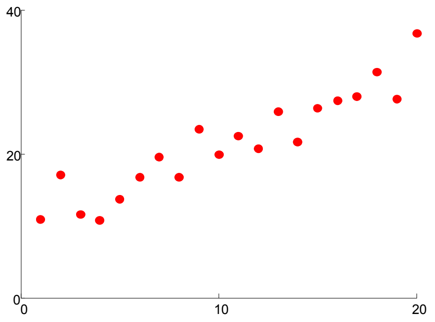
$w_{GST} \leftarrow -1.0 + \alpha\,[-501]\,1.0$

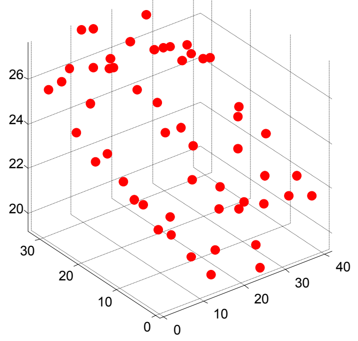
$Q(s,a) = 3.0 f_{DOT}(s,a) - 3.0 f_{GST}(s,a)$





Linear regression

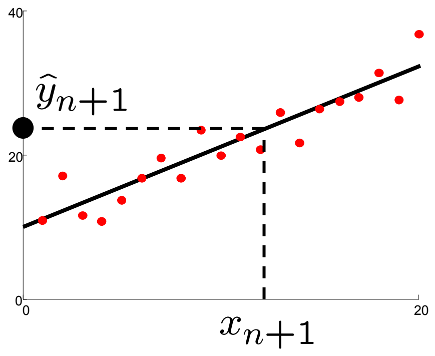


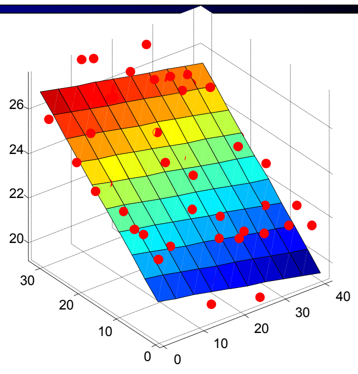


Given examples $(x_i, y_i)_{i=1\ldots n}$

Predict y_{n+1} given a new point x_{n+1}

Linear regression





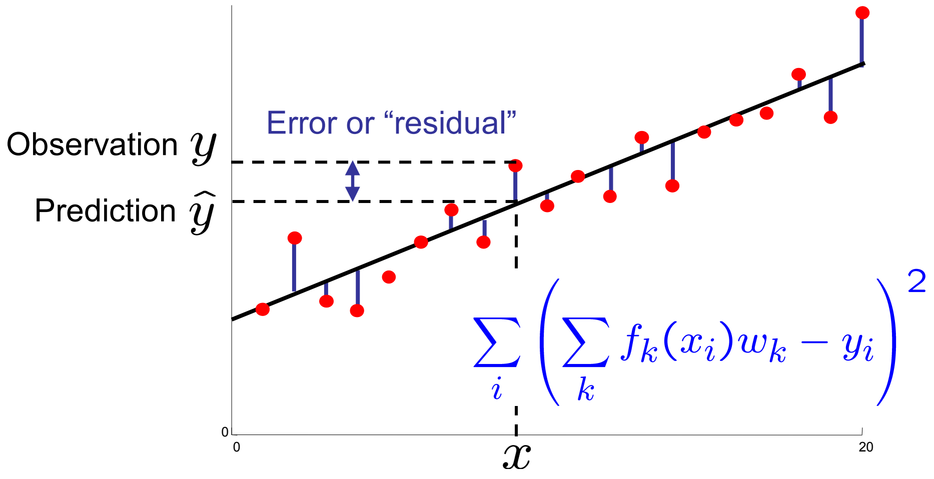
Prediction

$\widehat{y}_i = w_0 + w_1 x_i$

Prediction

$\widehat{y}_i = w_0 + w_1 x_{i,1} + w_2 x_{i,2}$

Ordinary Least Squares (OLS)



$$\sum_i \left(\sum_k f_k(x_i) w_k - y_i \right)^2$$

Minimizing Error

$$E(w) = \frac{1}{2}\sum_i \left(\sum_k f_k(x_i)w_k - y_i\right)^2$$

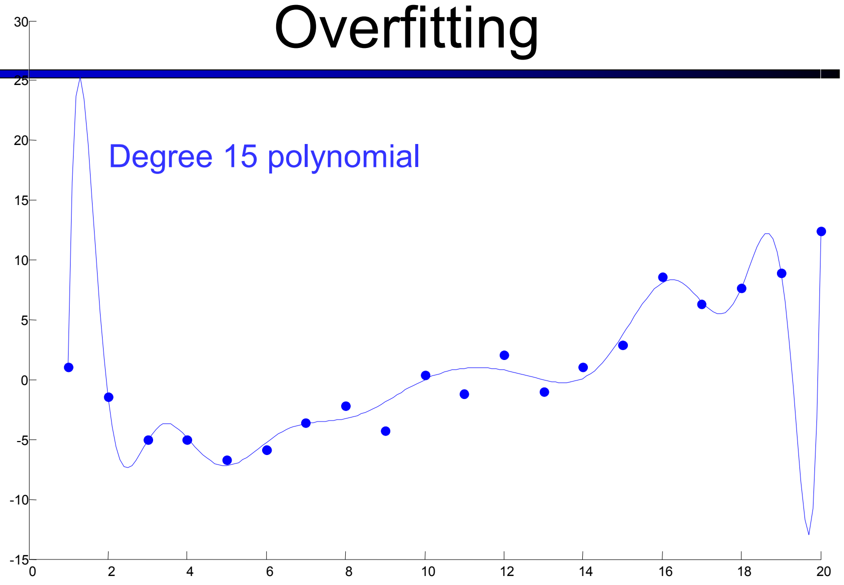
$$\frac{\partial E}{\partial w_m} = \sum_i \left(\sum_k f_k(x_i)w_k - y_i\right) f_m(x_i)$$

$$E \leftarrow E + \alpha \sum_i \left(\sum_k f_k(x_i)w_k - y_i\right) f_m(x_i)$$

Value update explained:

$$w_i \leftarrow w_i + \alpha\, [error]\, f_i(s,a)$$

Overfitting



[DEMO]

Policy Search

Policy Search

- Problem: often the feature-based policies that work well aren't the ones that approximate V / Q best
  - E.g. your value functions from project 2 were probably horrible estimates of future rewards, but they still produced good decisions
  - We'll see this distinction between modeling and prediction again later in the course
- Solution: learn the policy that maximizes rewards rather than the value that predicts rewards
- This is the idea behind policy search, such as what controlled the upside-down helicopter

Policy Search

- Simplest policy search:
  - Start with an initial linear value function or q-function
  - Nudge each feature weight up and down and see if your policy is better than before
- Problems:
  - How do we tell the policy got better?
  - Need to run many sample episodes!
  - If there are a lot of features, this can be impractical

Policy Search*

- Advanced policy search:
  - Write a stochastic (soft) policy:

$$\pi_w(s) \propto e^{\sum_i w_i f_i(s,a)}$$

  - Turns out you can efficiently approximate the derivative of the returns with respect to the parameters w (details in the book, but you don't have to know them)
  - Take uphill steps, recalculate derivatives, etc.

Take a Deep Breath…

- We're done with search and planning!

- Next, we'll look at how to reason with probabilities
  - Diagnosis
  - Tracking objects
  - Speech recognition
  - Robot mapping
  - … lots more!

- Last part of course: machine learning