CS 188: Artificial Intelligence
Fall 2007

Lecture 12: Reinforcement Learning

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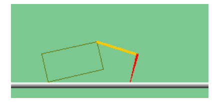
Reinforcement Learning

- Reinforcement learning:
  - Still have an MDP:
    - A set of states $s \in S$
    - A set of actions (per state) A
    - A model T(s,a,s')
    - A reward function R(s,a,s')
  - Still looking for a policy $\pi(s)$
  - New twist: don't know T or R
    - I.e. don't know which states are good or what the actions do
    - Must actually try actions and states out to learn



[DEMO]

Example: Animal Learning

- RL studied experimentally for more than 60 years in psychology
  - Rewards: food, pain, hunger, drugs, etc.
  - Mechanisms and sophistication debated
- Example: foraging
  - Bees learn near-optimal foraging plan in field of artificial flowers with controlled nectar supplies
  - Bees have a direct neural connection from nectar intake measurement to motor planning area

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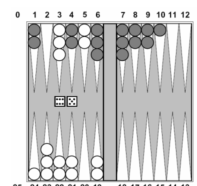
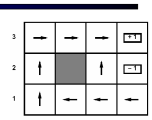
Example: Backgammon

- Reward only for win / loss in terminal states, zero otherwise
- TD-Gammon learns a function approximation to V(s) using a neural network
- Combined with depth 3 search, one of the top 3 players in the world
- You could imagine training Pacman this way…
- … but it's tricky!



Passive Learning

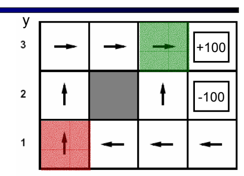
- Simplified task
  - You don't know the transitions T(s,a,s')
  - You don't know the rewards R(s,a,s')
  - You are given a policy $\pi(s)$
  - Goal: learn the state values (and maybe the model)
- In this case:
  - No choice about what actions to take
  - Just execute the policy and learn from experience
  - We'll get to the general case soon



Example: Direct Estimation

- Episodes:

(1,1) up -1	(1,1) up -1
(1,2) up -1	(1,2) up -1
(1,2) up -1	(1,3) right -1
(1,3) right -1	(2,3) right -1
(2,3) right -1	(3,3) right -1
(3,3) right -1	(3,2) up -1
(3,2) up -1	(4,2) exit -100
(3,3) right -1	(done)
(4,3) exit +100	
(done)	



γ = 1, R = -1

U(1,1) ~ (92 + -106) / 2 = -7

U(3,3) ~ (99 + 97 + -102) / 3 = 31.3

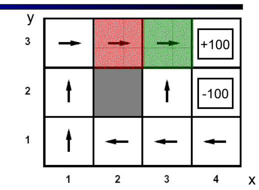
Model-Based Learning

- Idea:
  - Learn the model empirically (rather than values)
  - Solve the MDP as if the learned model were correct
- Empirical model learning
  - Simplest case:
    - Count outcomes for each s,a
    - Normalize to give estimate of T(s,a,s')
    - Discover R(s,a,s') the first time we experience (s,a,s')
  - More complex learners are possible (e.g. if we know that all squares have related action outcomes, e.g. "stationary noise")

Example: Model-Based Learning

- Episodes:

(1,1) up -1	(1,1) up -1
(1,2) up -1	(1,2) up -1
(1,2) up -1	(1,3) right -1
(1,3) right -1	(2,3) right -1
(2,3) right -1	(3,3) right -1
(3,3) right -1	(3,2) up -1
(3,2) up -1	(4,2) exit -100
(3,3) right -1	(done)
(4,3) exit +100	
(done)	



$\gamma = 1$

T(<3,3>, right, <4,3>) = 1 / 3

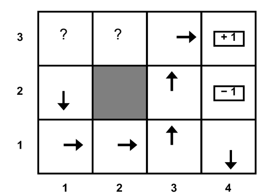
T(<2,3>, right, <3,3>) = 2 / 2

Model-Based Learning

- In general, want to learn the optimal policy, not evaluate a fixed policy
- Idea: adaptive dynamic programming
  - Learn an initial model of the environment:
  - Solve for the optimal policy for this model (value or policy iteration)
  - Refine model through experience and repeat
  - Crucial: we have to make sure we actually learn about all of the model

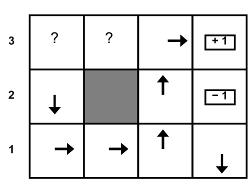
Example: Greedy ADP

- Imagine we find the lower path to the good exit first
- Some states will never be visited following this policy from (1,1)
- We'll keep re-using this policy because following it never collects the regions of the model we need to learn the optimal policy



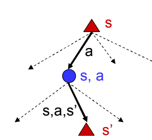
What Went Wrong?

- Problem with following optimal policy for current model:
  - Never learn about better regions of the space if current policy neglects them
- Fundamental tradeoff: exploration vs. exploitation
  - Exploration: must take actions with suboptimal estimates to discover new rewards and increase eventual utility
  - Exploitation: once the true optimal policy is learned, exploration reduces utility
  - Systems must explore in the beginning and exploit in the limit



Model-Free Learning

- Big idea: why bother learning T?
  - Update V each time we experience a transition
  - Frequent outcomes will contribute more updates (over time)
- Temporal difference learning (TD)
  - Policy still fixed!
  - Move values toward value of whatever successor occurs



$$V^{\pi}(s) \leftarrow \sum_{s'} T(s, \pi(s), s')[R(s, a, s') + \gamma V^{\pi}(s')]$$

$$sample = R(s, a, s') + \gamma V^{\pi}(s')$$

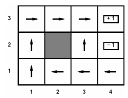
$$V^{\pi}(s) \leftarrow (1 - \alpha)V^{\pi}(s) + (\alpha)sample$$

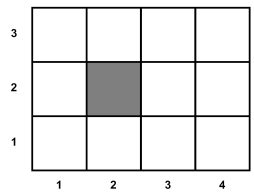
Example: Passive TD

$$V^{\pi}(s) \leftarrow (1-\alpha)V^{\pi}(s) + (\alpha)\left[R(s,a,s') + \gamma V^{\pi}(s')\right]$$

(1,1) up -1	(1,1) up -1
(1,2) up -1	(1,2) up -1
(1,2) up -1	(1,3) right -1
(1,3) right -1	(2,3) right -1
(2,3) right -1	(3,3) right -1
(3,3) right -1	(3,2) up -1
(3,2) up -1	(4,2) exit -100
(3,3) right -1	(done)
(4,3) exit +100	
(done)	

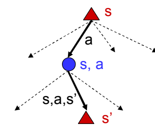
Take $\gamma = 1$, $\alpha = 0.5$





Problems with TD Value Learning

- TD value leaning is model-free for policy evaluation
- However, if we want to turn our value estimates into a policy, we're sunk:



$$\pi(s) = \arg\max_a Q^*(s,a)$$

$$Q^*(s,a) = \sum_{s'} T(s,a,s')\left[R(s,a,s') + \gamma V^*(s')\right]$$

- Idea: learn Q-values directly
- Makes action selection model-free too!

Q-Learning

- Learn Q*(s,a) values
  - Receive a sample (s,a,s',r)
  - Consider your old estimate: $Q(s,a)$
  - Consider your new sample estimate:

$$Q^*(s,a) = \sum_{s'} T(s,a,s')\left[R(s,a,s') + \gamma V^*(s')\right]$$

$$sample = R(s,a,s') + \gamma \max_{a'} Q(s',a')$$

  - Nudge the old estimate towards the new sample:

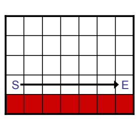
$$Q(s,a) \leftarrow (1-\alpha)Q(s,a) + (\alpha)\left[sample\right]$$

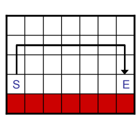
Q-Learning Example

- [DEMO]

Q-Learning Properties

- Will converge to optimal policy
  - If you explore enough
  - If you make the learning rate small enough
- Neat property: does not learn policies which are optimal in the presence of action selection noise





Exploration / Exploitation

- Several schemes for forcing exploration
  - Simplest: random actions (ε greedy)
    - Every time step, flip a coin
    - With probability ε, act randomly
    - With probability 1-ε, act according to current policy
  - Problems with random actions?
    - You do explore the space, but keep thrashing around once learning is done
    - One solution: lower ε over time
    - Another solution: exploration functions

Exploration Functions

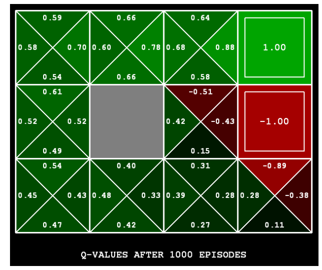
- When to explore
  - Random actions: explore a fixed amount
  - Better idea: explore areas whose badness is not (yet) established
- Exploration function
  - Takes a value estimate and a count, and returns an optimistic utility, e.g. $f(u, n) = u + k/n$ (exact form not important)

$$Q_{i+1}(s,a) \leftarrow_\alpha R(s,a,s') + \gamma \max_{a'} Q_i(s',a')$$

$$Q_{i+1}(s,a) \leftarrow_\alpha R(s,a,s') + \gamma \max_{a'} f(Q_i(s',a'), N(s',a'))$$

Q-Learning

- Q-learning produces tables of q-values:

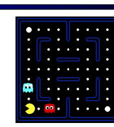


Q-Learning

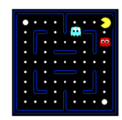
- In realistic situations, we cannot possibly learn about every single state!
  - Too many states to visit them all in training
  - Too many states to even hold the q-tables in memory
- Instead, we want to generalize:
  - Learn about some small number of training states from experience
  - Generalize that experience to new, similar states
  - This is a fundamental idea in machine learning, and we'll see it over and over again

Example: Pacman

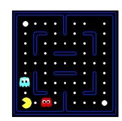
- Let's say we discover through experience that this state is bad:



- In naïve q learning, we know nothing about this state or its q states:

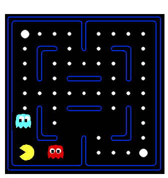


- Or even this one!



Feature-Based Representations

- Solution: describe a state using a vector of features
  - Features are functions from states to real numbers (often 0/1) that capture important properties of the state
  - Example features:
    - Distance to closest ghost
    - Distance to closest dot
    - Number of ghosts
    - 1 / (dist to dot)^2
    - Is Pacman in a tunnel? (0/1)
    - …… etc.
  - Can also describe a q-state (s, a) with features (e.g. action moves closer to food)



Linear Feature Functions

- Using a feature representation, we can write a q function (or value function) for any state using a few weights:

$$V(s) = w_1 f_1(s) + w_2 f_2(s) + \ldots + w_n f_n(s)$$

$$Q(s,a) = w_1 f_1(s,a) + w_2 f_2(s,a) + \ldots + w_n f_n(s,a)$$

- Advantage: our experience is summed up in a few powerful numbers
- Disadvantage: states may share features but be very different in value!

Function Approximation

$$Q(s,a) = w_1 f_1(s,a) + w_2 f_2(s,a) + \ldots + w_n f_n(s,a)$$

- Q-learning with linear q-functions:

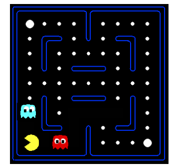
$$Q(s,a) \leftarrow Q(s,a) + \alpha\,[error]$$

$$w_i \leftarrow w_i + \alpha\,[error]\, f_i(s,a)$$

- Intuitive interpretation:
  - Adjust weights of active features
  - E.g. if something unexpectedly bad happens, disprefer all states with that state's features
- Formal justification: online least squares (much later)

Example: Q-Pacman

$$Q(s,a) = 4.0 f_{DOT}(s,a) - 1.0 f_{GST}(s,a)$$



$$f_{DOT}(s, \text{NORTH}) = 0.5$$

$$f_{GST}(s, \text{NORTH}) = 1.0$$

$$Q(s,a) = +1$$

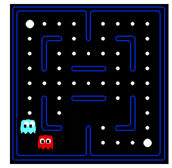
$$R(s,a,s') = -500$$

$$error = -501$$

$$w_{DOT} \leftarrow 4.0 + \alpha\,[-501]\, 0.5$$

$$w_{GST} \leftarrow -1.0 + \alpha\,[-501]\, 1.0$$

$$Q(s,a) = 3.0 f_{DOT}(s,a) - 3.0 f_{GST}(s,a)$$



Policy Search

Policy Search

- Problem: often the feature-based policies that work well aren't the ones that approximate V / Q best
  - E.g. your value functions from project 2 were probably horrible estimates of future rewards, but they still produced good decisions
  - We'll see this distinction between modeling and prediction again later in the course
- Solution: learn the policy that maximizes rewards rather than the value that predicts rewards
- This is the idea behind policy search, such as what controlled the upside-down helicopter

Policy Search

- Simplest policy search:
  - Start with an initial linear value function or q-function
  - Nudge each feature weight up and down and see if your policy is better than before
- Problems:
  - How do we tell the policy got better?
  - Need to run many sample episodes!
  - If there are a lot of features, this can be impractical

Policy Search*

- Advanced policy search:
  - Write a stochastic (soft) policy:

$$\pi_w(s) \propto e^{\sum_i w_i f_i(s,a)}$$

  - Turns out you can efficiently approximate the derivative of the returns with respect to the parameters w (details in the book, but you don't have to know them)
  - Take uphill steps, recalculate derivatives, etc.

Take a Deep Breath…

- We're done with search and planning!
- Next, we'll look at how to reason with probabilities
  - Diagnosis
  - Tracking objects
  - Speech recognition
  - Robot mapping
  - … lots more!
- Last part of course: machine learning