

## Question 1 (Class) – Value of Information

### [Adapted from problem 16.11 in Russell & Norvig]

A used-car buyer can decide to carry out various tests with various costs (e.g., kick the tires, take the car to a qualified mechanic) and then, depending on the outcome of the tests, decide which car to buy. We will assume that the buyer is deciding whether to buy car  $c$  and that there is time to carry out at most one test which costs \$50 and which can help to figure out the quality of the car.

A car can be in good shape (of good quality  $Q=q$ ) or in bad shape (of bad quality  $Q=\sim q$ ), and the test might help to indicate what shape the car is in. There are only two outcomes for the test  $T$ : pass ( $T=\text{pass}$ ) or fail ( $T=\text{fail}$ ). Car  $c$  costs \$1,500, and its market value is \$2,000 if it is in good shape; if not, \$700 in repairs will be needed to make it in good shape. The buyer's estimate is that  $c$  has 70% chance of being in good shape.

*a) Draw the decision network that represents this problem. (The only action is whether to buy or not the car).*

*b) Calculate the expected net gain from buying car  $c$ , given no test.*

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*c) Tests can be described by the probability that the car will pass or fail the test given that the car is in good or bad shape. We have the following information:*

$$P(T=\text{pass} \mid Q=q) = 0.9$$

$$P(T=\text{pass} \mid Q=\sim q) = 0.2$$

**[Note that the numbers were changed from the book. It is also a useful exercise to redo this problem with the numbers from the book.]**

*Calculate the probability that the car will pass (or fail) its test, and then the probability that it is in good (or bad) shape given each possible test outcome [Hint: Bayes' theorem could come handy here.]*

*d) Calculate the optimal decisions given either a pass or a fail, and their expected utilities.*

*e) Calculate the value of (perfect) information of the test. Should the buyer pay for a test?*

## Question 2 (Homework) – Properties of VPI

a) Prove that if the optimal action to do is not changed for any value of the new evidence  $E'$ , then the value of information is 0.

[Use the formula for VPI from the slides of lecture 19]

b) Prove that the value of perfect information is always non-negative (i.e.  $VPE_e(E_j) \geq 0$  for any  $j$  and  $e$ ).

[Hint: use the fact that the sum of maximums is always greater or equal than the maximum of a sum.]

### Question 3 (Homework) – HMM and particle filtering

You sometimes get colds, which make you sneeze. You also get allergies, which make you sneeze. Sometimes you are well, which doesn't make you sneeze. You decide to model the process using the following HMM, with hidden states  $X \in \{well, allergy, cold\}$  and observations  $E \in \{well, allergy, cold\}$ :

$$P(X_1)$$

<i>well</i>	1
<i>allergy</i>	0
<i>cold</i>	0

$$P(X_t | X_{t-1} = well)$$

<i>well</i>	0.7
<i>allergy</i>	0.2
<i>cold</i>	0.1

$$P(X_t | X_{t-1} = allergy)$$

<i>well</i>	0.6
<i>allergy</i>	0.3
<i>cold</i>	0.1

$$P(X_t | X_{t-1} = cold)$$

<i>well</i>	0.2
<i>allergy</i>	0.2
<i>cold</i>	0.6

Transitions

$$P(E_t | X_t = well)$$

<i>quiet</i>	1.0
<i>sneeze</i>	0.0

$$P(E_t | X_t = allergy)$$

<i>quiet</i>	0.0
<i>sneeze</i>	1.0

$$P(E_t | X_t = cold)$$

<i>quiet</i>	0.0
<i>sneeze</i>	1.0

Emissions

Note that colds are “stickier” in that you tend to have them for multiple days, while allergies come and go on a quicker time scale. However, allergies are more frequent. Assume that on the first day, you are well.

a) Imagine that you observe the sequence quiet, sneeze, sneeze. What is the probability that you were well all three days and observed these effects?

b) What is the posterior distribution over your state on day 2 ( $X_2$ ) if  $E_1 = quiet$ ,  $E_2 = sneeze$ ?

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c) What is the posterior distribution over your state on day 3 ( $X_3$ ) if  $E_1 = \text{quiet}$ ,  $E_2 = \text{sneeze}$ ,  $E_3 = \text{sneeze}$ ?

d) What is the Viterbi (most likely) sequence for the observation sequence quiet, sneeze, sneeze, sneeze, quiet, quiet, sneeze, quiet, quiet? [Hint: you should not have to do extensive calculations.]

Imagine you are monitoring your state using the particle filtering algorithm, and on a given day you have 5 particles on *well*, 2 on *cold*, and 3 on *allergy* before making an observation on that day.

e) If you observe sneeze, what weight will each of your particles have?

f) After resampling, what is the expected number of particles you will have on cold?