Question 1 (Class)

Variables – T_1, T_2, T_3, T_4 where $T_i \in \{S|D|R|T, F|B|A|P\}$

Constraints -

1. Tony and his three best friends (Steven, Donna, and Randy) were each at a different table.

2. There's only one game (foosball, billiards, air hockey, ping-pong) at each table.

3. The game at table 1 is foosball, and it is not being played by Steven.

4. The billiards table is somewhere to the left of the air hockey table.
$\rightarrow$ *Billiards can not be at table* 4, *air hockey can not be at table* 1

5. Randy is playing the game at table 2 or table 4.

6. The ping-pong table is two tables to the left of where Donna is playing.
$\rightarrow$ *Ping Pong cannot be at table* 3 *or* 4, *Donna cannot be at table* 1 *or* 2

1. Forward Checking

Table 1	*Table 2*	*Table 3*	*Table 4*
{S,D,R,T}	{S,D,R,T}	{S,D,R,T}	{S,D,R,T}
{F,B,A,P}	{F,B,A,P}	{F,B,A,P}	{F,B,A,P}

After rule 3

Table 1	*Table 2*	*Table 3*	*Table 4*
{D,R,T}	{S,D,R,T}	{S,D,R,T}	{S,D,R,T}
{F}	{F,B,A,P}	{F,B,A,P}	{F,B,A,P}

After rule 4

Table 1	*Table 2*	*Table 3*	*Table 4*
{D,R,T}	{S,D,R,T}	{S,D,R,T}	{S,D,R,T}
{F}	{B,P}	{B,A,P}	{A,P}

After rule 5

Table 1	*Table 2*	*Table 3*	*Table 4*
{D,T}	*{S,D,R,T}*	*{S,D,T}*	*{S,D,R,T}*
{F}	*{B,P}*	*{B,A,P}*	*{A,P}*

After rule 6

Table 1	*Table 2*	*Table 3*	*Table 4*
{T}	*{S,R,T}*	*{S,D,T}*	*{S,D,R,T}*
{F}	*{B,P}*	*{B,A}*	*{A}*

Running backtracking search—we can set that $T_1 = \{T, F\}$ by MRV.

We choose Table 4 next—we can set $T_4 = \{S|D|R, A\}$.

We choose Table 3 next—we can set $T_3 = \{S|D, B\}$

We choose Table 2 next—we can set $T_2 = \{S|R, P\}$

1. *By MRV again—we choose T_2 or T_3. We choose the LCV which is S—thus $T_2 = \{S, P\}$.*

We choose $T_3 = \{D, B\}$

This violates our constraints so we backtrack to 1.

We choose the other value and set $T_2 = \{R, P\}$

Then $T_3 = \{S, B\}$

Then $T_4 = \{D, A\}$

All constraints are still valid so we have a solution.

2. ***Arc Consistency (Use back)***

Start with table from forward checking.

Table 1	*Table 2*	*Table 3*	*Table 4*
{D,T}	*{S,R,T}*	*{S,D,T}*	*{S,D,R,T}*
{F}	*{B,P}*	*{B,A}*	*{A}*

Donna cannot be at table three due to table 1 being foosball.

Table 1	Table 2	Table 3	Table 4
{T}	{S,R,T}	{S,T}	{S,D,R,T}
{F}	{B,P}	{B,A}	{A}

Donna must be at table 4, and air hockey must be at table 4

Table 1	Table 2	Table 3	Table 4
{T}	{S,R,T}	{S}	{D }
{F}	{B,P}	{B}	{A}

Table 2 must be Randy and ping pong to be consistent.

Table 1	Table 2	Table 3	Table 4
{T}	{R}	{S}	{D }
{F}	{P}	{B}	{A}

Now we run backtracking search—we simply choose each value for each variable until we arrive at the solution.

Question 2 (Class)

a. *Each search state is a 5×5 array taking values in {black, white}. The starting state has all cells taking values in white. Our operators in each state are to flip a given cell, which changes the color of that cell as well as the four adjacent cells. Our goal test is whether or not all cells are black.*

b. *A valid heuristic for this problem is to round down the number of white cells divided by five. More formally,*

$$h(state) = [\# \frac{\{(i,j): state[i][j] = white\}}{5}]$$

This is valid since a given step can change the color of at most five cells (itself and its four neighbors). So we need at least this many steps to convert all the cells to black.

c. *Two properties of Fiver are that (i) executing the same moves in a different order yields the same result and (ii) executing the same move twice in a row returns the board to its original state (prior to those two moves). Now, consider a solution of that involves making the same move twice. If we reorder the moves in that solution so that the two identical moves appear last, then this is also a solution by (i) and we would have completed the puzzle before making these final two moves by (ii). Thus, a lower-cost solution exists.*

d. *CSP formulation:*

Variables: $X_{i,j}$ *for each position corresponds to how many times that position is selected in the solution. Note that the order in which we select is irrelevant.*

Domains: {0, 1} Note that we can ignore solutions with values > 1 because of the property in (c).

Constraints: For each position (i, j), $X_{i,j} + X_{i-1,j} + X_{i,j-1} + X_{i+1,j} + X_{i,j+1} \ mod \ 2$ *= 1*

e. *Even with a reasonably tight heuristic, the branching factor of* n^2 *that arises in A* search could lead to a very inefficient search of the solution space. Also, A* has no method to detect when it has made a mistake (chosen a local set of moves that guarantee a white square or a repeated move). The heuristics for CSPs would constrain the search to explore solutions to local parts of the problem (MRV, in particular). Note, however, that the CSP formulation is not guaranteed to generate an optimal solution.*

Question 1 (Homework)

(a) What are the unary constraint(s) (list them explicitly)

$$X_1 \neq i$$

(b) What are the binary constraint(s) (list them explicitly)

$$(X1, X2) \in \{(i,m),(m,i),(g,m),(m,g)\}$$
$$(X2, X3) \in \{(i,m),(m,i),(g,m),(m,g)\}$$
$$(X1, X3) \in \{(i,m),(m,i),(g,m),(m,g),(i,g),(g,i)\}$$

(c) Which variable will be assigned first by the MRV heuristic?

$$X_1$$

(d) If we assign X3 = i, show the domains of the remaining variables after forward checking.

$$X1 \in \{g,m\}$$
$$X2 \in \{m\}$$
$$X3 = i$$

(e) If no variables are assigned, show the initial domains after running arc consistency.

arc consistency has no effect.

$$X1 \in \{g,m\}$$
$$X2 \in \{i,g,m\}$$
$$X3 \in \{i,g,m\}$$

(f) If it's a cool day, and we drop the requirement that the ice swan cannot be nearest the door, what are the initial domains after running arc consistency?

arc consistency has no effect.

$$X1 \in \{i,g,m\}$$
$$X2 \in \{i,g,m\}$$
$$X3 \in \{i,g,m\}$$