

Question 1 (Homework) - More Bayes Nets

In this problem, you will build a small model of the performance of a CS 188 student on the final. A student can either pass or fail (T). This is influenced by his exam readiness (R) and the difficulty of the final (D). His readiness in turn depends on whether he attends lecture (A).

Assume all variables in are binary (e.g. $D = \{d, \neg d\}$).

a) Draw the Bayes net for this model.

What conditional probability tables (CPTs) describe this model?

b) Consider the following probability tables.

Table 1

| T | A | D | |
|------|----------|----------|------|
| pass | a | d | 0.15 |
| pass | a | $\neg d$ | 0.01 |
| pass | $\neg a$ | d | 0.05 |
| pass | $\neg a$ | $\neg d$ | 0.09 |
| fail | a | d | 0.1 |
| fail | a | $\neg d$ | 0.3 |
| fail | $\neg a$ | d | 0.27 |
| fail | $\neg a$ | $\neg d$ | 0.03 |

Table 2

| T | A | D | |
|------|----------|----------|------|
| pass | a | d | 0.2 |
| pass | a | $\neg d$ | 0.15 |
| pass | $\neg a$ | d | 0.6 |
| pass | $\neg a$ | $\neg d$ | 0.5 |
| fail | a | d | 0.8 |
| fail | a | $\neg d$ | 0.85 |
| fail | $\neg a$ | d | 0.4 |
| fail | $\neg a$ | $\neg d$ | 0.5 |

Table 3

| T | A | D | |
|------|----------|----------|-------|
| pass | a | d | 0.018 |
| pass | a | $\neg d$ | 0.008 |
| pass | $\neg a$ | d | 0.27 |
| pass | $\neg a$ | $\neg d$ | 0.216 |
| fail | a | d | 0.042 |
| fail | a | $\neg d$ | 0.032 |
| fail | $\neg a$ | d | 0.27 |
| fail | $\neg a$ | $\neg d$ | 0.144 |

CS188 - Introduction to Artificial Intelligence

Section Handout #10 - Probability

Klein, Fall 2007 - Week of Nov 12th

Which of these tables can represent the joint distribution on test performance, lecture attendance, and exam difficulty $P(T,A,D)$? Which can model test performance **given** attendance and exam difficulty $P(T|A,D)$? For now, do not consider whether they respect any particular Bayes net.

c) Of the tables you identified as modeling the joint distribution $P(T,A,D)$, which can represent the joint distribution for the model represented by the Bayes net in (a)?

d) Suppose the following table represents the joint probability over all variables $P(T,DA,R)$.

| T | D | A | R | $P(T,D,A,R)$ |
|-------------|----------|----------|----------|--------------|
| <i>pass</i> | <i>d</i> | <i>a</i> | <i>r</i> | 0.3024 |
| <i>pass</i> | <i>d</i> | <i>a</i> | $\neg r$ | 0.0324 |
| <i>pass</i> | <i>d</i> | $\neg a$ | <i>r</i> | 0.0024 |
| <i>pass</i> | <i>d</i> | $\neg a$ | $\neg r$ | 0.0114 |
| <i>pass</i> | $\neg d$ | <i>a</i> | <i>r</i> | 0.2394 |
| <i>pass</i> | $\neg d$ | <i>a</i> | $\neg r$ | 0.054 |
| <i>pass</i> | $\neg d$ | $\neg a$ | <i>r</i> | 0.0019 |
| <i>pass</i> | $\neg d$ | $\neg a$ | $\neg r$ | 0.019 |
| <i>fail</i> | <i>d</i> | <i>a</i> | <i>r</i> | 0.0756 |
| <i>fail</i> | <i>d</i> | <i>a</i> | $\neg r$ | 0.1296 |
| <i>fail</i> | <i>d</i> | $\neg a$ | <i>r</i> | 0.0006 |
| <i>fail</i> | <i>d</i> | $\neg a$ | $\neg r$ | 0.0456 |
| <i>fail</i> | $\neg d$ | <i>a</i> | <i>r</i> | 0.0126 |
| <i>fail</i> | $\neg d$ | <i>a</i> | $\neg r$ | 0.054 |
| <i>fail</i> | $\neg d$ | $\neg a$ | <i>r</i> | 0.0001 |
| <i>fail</i> | $\neg d$ | $\neg a$ | $\neg r$ | 0.019 |

For each of conditional probability tables (CPTs) in the Bayes net from part (a), fill in the values of the probabilities such that the joint probability over all variables is given by the table. (You can use the back of the sheet to do this).

CS188 - Introduction to Artificial Intelligence

Section Handout #10 - Probability

Klein, Fall 2007 - Week of Nov 12th

e) Suppose we wish to do sampling on this Bayes net to calculate $P(T=\text{pass}|A=a)$. Suppose further that you are given 3 sets of numbers generated randomly on $[0,1]$, and that each set contains one random number for each variable in the Bayes net. Describe how you would use these numbers and the CPTs from part (d) to generate 3 samples according to the joint distribution of the Bayes net. (Hint: samples produced by rejection sampling **before** rejection are sampled according to the joint distribution of a Bayes net).

f) Now suppose you are given the following sets of random numbers.

Set 1: {T: 0.45, A: 0.6, D: 0.7, R:0.02}

Set 2: {T: 0.1, A:0.95, D:0.5, R:0.5}

Set 3: {T: 0.9, A: 0.8, D: 0.1, R: 0.8}

What samples would you produce under rejection sampling? What about likelihood weighting?

CS188 - Introduction to Artificial Intelligence

Section Handout #10 - Probability

Klein, Fall 2007 - Week of Nov 12th

g) Now suppose that we want to model the performance of a student on both the the first project, the midterm, and the final. This time, we model a student's performance ($T=\{\text{pass},\text{fail}\}$) as depending only on their test readiness ($R=\{r,\neg r\}$). However, this time, we believe that the amount of preparation a student does for each task depends on the amount of preparation done for the previous task. Draw a Bayes net for this particular model of student performance. (The order of events here is project (p), midterm (m), and final and (f). You may denote, for example, the student's readiness for the midterm as R_m .

h) Viewing this model as an HMM, what are the hidden states? What are the observations?

i) Suppose we know that the student passed all three tasks. Write an expression for the forward probability that the student was ready for the final. Your answer should have the form $P(\text{[some variables = some values]} \mid \text{[other variables = other values]})$

j) Write an expression for the same forward probability as in (i), but this time, in terms of the forward probability at the midterm, the probability $P(T_f \mid R_f)$, and the probability $P(R_f \mid R_m)$

Question 2 (Homework) - Even more Bayes nets

You've been hired to build a quality control system to decide whether car engines coming off an assembly line are *bad* or *ok*. However, this decision must be based on three noisy boolean observations: the engine may be *wobbly* (motion sensor), *rumbly* (sound sensor), or *hot* (heat sensor). Each sensor gives a Boolean observation: *true* or *false*.

You formalize the problem using the following variables and domains:

- B = {*bad*, *ok*}
- W = {*true*, *false*}
- R = {*true*, *false*}
- H = {*true*, *false*}

(a) Draw a Bayes net for a model of these variables in which the sensor readings are independent of each other for a given engine quality.

(b) We would like to use this model to determine the probability that an engine is *ok* given the sensor observations. Write down an expression for the probability $P(B=ok | W=true, R=false, H=true)$ in terms of the conditional probabilities that describe the Bayes net you drew in part (a).

(c) Consider the following conditional probability tables for this Bayes net.

P(B)

| | |
|------------|-------|
| <i>bad</i> | 0.375 |
| <i>ok</i> | 0.625 |

P(R|B)

| | | |
|--------------|------------|-------|
| <i>true</i> | <i>bad</i> | 0.25 |
| <i>false</i> | <i>bad</i> | 0.125 |
| <i>true</i> | <i>ok</i> | 0.25 |
| <i>false</i> | <i>ok</i> | 0.375 |

P(W|B)

| | | |
|--------------|------------|-------|
| <i>true</i> | <i>bad</i> | 0.25 |
| <i>false</i> | <i>bad</i> | 0.125 |
| <i>true</i> | <i>ok</i> | 0.125 |
| <i>false</i> | <i>ok</i> | 0.5 |

P(H|B)

| | | |
|--------------|------------|-------|
| <i>true</i> | <i>bad</i> | 0.125 |
| <i>false</i> | <i>bad</i> | 0.25 |
| <i>true</i> | <i>ok</i> | 0.125 |
| <i>false</i> | <i>ok</i> | 0.5 |

CS188 - Introduction to Artificial Intelligence

Section Handout #10 - Probability

Klein, Fall 2007 - Week of Nov 12th

Suppose that you are given the sensor readings for a new engine with $W=\text{false}$, $R=\text{true}$, $H=\text{false}$. Using the (estimated) CPTs from (c), what value of B does your model predict?

(d) What is the posterior probability of that prediction (i.e. what is $P(B|W=\text{false},R=\text{true},H=\text{false})$ for whatever value of B your model predicts)?

(e) The company informs you that, according to their utility function, the utility of rejecting an engine is -1 , regardless of whether or not it is actually bad. However, the utility of accepting an engine is 2 for ok engines and -20 for bad engines. Given these utilities, what is the minimum posterior probability your agent needs to have for $B = \text{ok}$ before accepting an engine becomes the rational action?