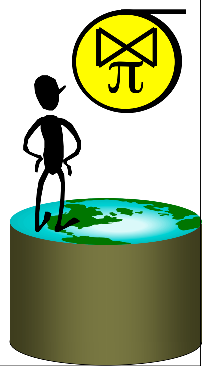
# Relational Algebra

**R & G, Chapter 4**

By relieving the brain of all unnecessary work, a good notation sets it free to concentrate on more advanced problems, and, in effect, *increases the mental power of the race.*

**-- Alfred North Whitehead (1861 - 1947)**



## Relational Query Languages

- ***Query languages*: Allow manipulation and retrieval of data from a database.**
- **Relational model supports simple, powerful QLs:**
  - Strong formal foundation based on logic.
  - Allows for much optimization.
- **Query Languages != programming languages!**
  - QLs not expected to be "Turing complete".
  - QLs not intended to be used for complex calculations.
  - QLs support easy, efficient access to large data sets.

## Formal Relational Query Languages

**Two mathematical Query Languages form the basis for "real" languages (e.g. SQL), and for implementation:**

***Relational Algebra*: More operational, very useful for representing execution plans.**

***Relational Calculus*: Lets users describe what they want, rather than how to compute it. (Non-procedural, *declarative*.)**

☛ *Understanding Algebra & Calculus is key to understanding SQL, query processing!*

## Preliminaries

- **A query is applied to *relation instances*, and the result of a query is also a relation instance.**
  - *Schemas* of input relations for a query are fixed (but query will run over any legal instance)
  - The schema for the *result* of a given query is also fixed. It is determined by the definitions of the query language constructs.
- **Positional vs. named-field notation:**
  - Positional notation easier for formal definitions, named-field notation more readable.
  - Both used in SQL
    - Though positional notation is not encouraged

## Relational Algebra: 5 Basic Operations

- *Selection* ( σ ) Selects a subset of ***rows*** from relation (horizontal).
- *Projection* ( π ) Retains only wanted ***columns*** from relation (vertical).
- *Cross-product* ( × ) Allows us to combine two relations.
- *Set-difference* ( — ) Tuples in r1, but not in r2.
- *Union* ( ∪ ) Tuples in r1 or in r2.

Since each operation returns a relation, operations can be *composed!* (Algebra is "closed".)

## Example Instances

*R1*

| sid | bid | day |
|---|---|---|
| 22 | 101 | 10/10/96 |
| 58 | 103 | 11/12/96 |

*S1*

| sid | sname | rating | age |
|---|---|---|---|
| 22 | dustin | 7 | 45.0 |
| 31 | lubber | 8 | 55.5 |
| 58 | rusty | 10 | 35.0 |

*Boats*

| bid | bname | color |
|---|---|---|
| 101 | Interlake | blue |
| 102 | Interlake | red |
| 103 | Clipper | green |
| 104 | Marine | red |

*S2*

| sid | sname | rating | age |
|---|---|---|---|
| 28 | yuppy | 9 | 35.0 |
| 31 | lubber | 8 | 55.5 |
| 44 | guppy | 5 | 35.0 |
| 58 | rusty | 10 | 35.0 |

## Projection ($\pi$)

- Examples: $\pi_{age}(S2)$ ; $\pi_{sname,rating}(S2)$
- Retains only attributes that are in the "*projection list*".
- *Schema* of result:
  - exactly the fields in the projection list, with the same names that they had in the input relation.
- Projection operator has to *eliminate duplicates* (How do they arise? Why remove them?)
  - Note: real systems typically don't do duplicate elimination unless the user explicitly asks for it. (Why not?)

## Projection ($\pi$)

| sid | sname | rating | age |
|---|---|---|---|
| 28 | yuppy | 9 | 35.0 |
| 31 | lubber | 8 | 55.5 |
| 44 | guppy | 5 | 35.0 |
| 58 | rusty | 10 | 35.0 |

**S2**

| sname | rating |
|---|---|
| yuppy | 9 |
| lubber | 8 |
| guppy | 5 |
| rusty | 10 |

$\pi_{sname,rating}(S2)$

| age |
|---|
| 35.0 |
| 55.5 |

$\pi_{age}(S2)$

## Selection ($\sigma$)

- Selects rows that satisfy *selection condition*.
- Result is a relation.
  ***Schema*** of result is same as that of the input relation.
- Do we need to do duplicate elimination?

| sid | sname | rating | age |
|---|---|---|---|
| 28 | yuppy | 9 | 35.0 |
| ~~31~~ | ~~lubber~~ | ~~8~~ | ~~55.5~~ |
| ~~44~~ | ~~guppy~~ | ~~5~~ | ~~35.0~~ |
| 58 | rusty | 10 | 35.0 |

$\sigma_{rating>8}(S2)$

| sname | rating |
|---|---|
| yuppy | 9 |
| rusty | 10 |

$\pi_{sname,rating}(\sigma_{rating>8}(S2))$

## Union and Set-Difference

- Both of these operations take two input relations, which must be ***union-compatible***:
  - Same number of fields.
  - `Corresponding' fields have the same type.
- For which, if any, is duplicate elimination required?

## Union

| sid | sname | rating | age |
|---|---|---|---|
| 22 | dustin | 7 | 45.0 |
| 31 | lubber | 8 | 55.5 |
| 58 | rusty | 10 | 35.0 |

**S1**

| sid | sname | rating | age |
|---|---|---|---|
| 28 | yuppy | 9 | 35.0 |
| 31 | lubber | 8 | 55.5 |
| 44 | guppy | 5 | 35.0 |
| 58 | rusty | 10 | 35.0 |

**S2**

| sid | sname | rating | age |
|---|---|---|---|
| 22 | dustin | 7 | 45.0 |
| 31 | lubber | 8 | 55.5 |
| 58 | rusty | 10 | 35.0 |
| 44 | guppy | 5 | 35.0 |
| 28 | yuppy | 9 | 35.0 |

$S1 \cup S2$

## Set Difference

| sid | sname | rating | age |
|---|---|---|---|
| 22 | dustin | 7 | 45.0 |
| 31 | lubber | 8 | 55.5 |
| 58 | rusty | 10 | 35.0 |

**S1**

| sid | sname | rating | age |
|---|---|---|---|
| 28 | yuppy | 9 | 35.0 |
| 31 | lubber | 8 | 55.5 |
| 44 | guppy | 5 | 35.0 |
| 58 | rusty | 10 | 35.0 |

**S2**

| sid | sname | rating | age |
|---|---|---|---|
| 22 | dustin | 7 | 45.0 |

$S1 - S2$

| sid | sname | rating | age |
|---|---|---|---|
| 28 | yuppy | 9 | 35.0 |
| 44 | guppy | 5 | 35.0 |

$S2 - S1$

## Cross-Product

- S1 × R1: Each row of S1 paired with each row of R1.
- Q: How many rows in the result?
- *Result schema* has one field per field of S1 and R1, with field names `inherited' if possible.
  - *May have a naming conflict*: Both S1 and R1 have a field with the same name.
  - In this case, can use the *renaming operator*:

$$\rho\,(C(1 \rightarrow sid1, 5 \rightarrow sid2),\ S1 \times R1)$$

## Cross Product Example

| sid | bid | day |
|---|---|---|
| 22 | 101 | 10/10/96 |
| 58 | 103 | 11/12/96 |

**R1**

| sid | sname | rating | age |
|---|---|---|---|
| 22 | dustin | 7 | 45.0 |
| 31 | lubber | 8 | 55.5 |
| 58 | rusty | 10 | 35.0 |

**S1**

**R1 X S1 =**

| (sid) | sname | rating | age | (sid) | bid | day |
|---|---|---|---|---|---|---|
| 22 | dustin | 7 | 45.0 | 22 | 101 | 10/10/96 |
| 22 | dustin | 7 | 45.0 | 58 | 103 | 11/12/96 |
| 31 | lubber | 8 | 55.5 | 22 | 101 | 10/10/96 |
| 31 | lubber | 8 | 55.5 | 58 | 103 | 11/12/96 |
| 58 | rusty | 10 | 35.0 | 22 | 101 | 10/10/96 |
| 58 | rusty | 10 | 35.0 | 58 | 103 | 11/12/96 |

## Compound Operator: Intersection

- In addition to the 5 basic operators, there are several additional "Compound Operators"
  - These add no computational power to the language, but are useful shorthands.
  - Can be expressed solely with the basic ops.
- Intersection takes two input relations, which must be ***union-compatible***.
- Q: How to express it using basic operators?

$$R \cap S = R - (R - S)$$

## Intersection

| sid | sname | rating | age |
|---|---|---|---|
| 22 | dustin | 7 | 45.0 |
| 31 | lubber | 8 | 55.5 |
| 58 | rusty | 10 | 35.0 |

**S1**

| sid | sname | rating | age |
|---|---|---|---|
| 28 | yuppy | 9 | 35.0 |
| 31 | lubber | 8 | 55.5 |
| 44 | guppy | 5 | 35.0 |
| 58 | rusty | 10 | 35.0 |

**S2**

| sid | sname | rating | age |
|---|---|---|---|
| 31 | lubber | 8 | 55.5 |
| 58 | rusty | 10 | 35.0 |

$S1 \cap S2$

## Compound Operator: Join

- Joins are compound operators involving cross product, selection, and (sometimes) projection.
- Most common type of join is a "*natural join*" (often just called "join"). R ⋈ S conceptually is:
  - Compute R × S
  - Select rows where attributes that appear in both relations have equal values
  - Project all unique atttributes and one copy of each of the common ones.
- Note: Usually done much more efficiently than this.

## Natural Join Example

| sid | bid | day |
|---|---|---|
| 22 | 101 | 10/10/96 |
| 58 | 103 | 11/12/96 |

**R1**

| sid | sname | rating | age |
|---|---|---|---|
| 22 | dustin | 7 | 45.0 |
| 31 | lubber | 8 | 55.5 |
| 58 | rusty | 10 | 35.0 |

**S1**

**S1 ⋈ R1 =**

| sid | sname | rating | age | bid | day |
|---|---|---|---|---|---|
| 22 | dustin | 7 | 45.0 | 101 | 10/10/96 |
| 58 | rusty | 10 | 35.0 | 103 | 11/12/96 |

## Other Types of Joins

- ***Condition Join (or "theta-join")*:**

$$R \bowtie_c S = \sigma_c(R \times S)$$

| (sid) | sname | rating | age | (sid) | bid | day |
|---|---|---|---|---|---|---|
| 22 | dustin | 7 | 45.0 | 58 | 103 | 11/12/96 |
| 31 | lubber | 8 | 55.5 | 58 | 103 | 11/12/96 |

$$S1 \bowtie_{S1.sid < R1.sid} R1$$

- *Result schema* same as that of cross-product.
- May have fewer tuples than cross-product.
- ***Equi-Join*: Special case: condition *c* contains only conjunction of *equalities*.**

## Compound Operator: Division

- Useful for expressing "for all" queries like: *Find sids of sailors who have reserved all boats.*
- For A/B, attributes of B must be subset of attrs of A.
  - May need to "project" to make this happen.
- E.g., let *A* have 2 fields, *x* and *y*; *B* have only field *y*:

$$A/B = \{\langle x \rangle \mid \forall \langle y \rangle \in B (\exists \langle x, y \rangle \in A)\}$$

***A/B* contains all tuples (x) such that for *every y* tuple in *B*, there is an *xy* tuple in *A*.**

## Examples of Division A/B

Q for intuition: What is (R/S)×S?

*A*

| sno | pno |
|---|---|
| s1 | p1 |
| s1 | p2 |
| s1 | p3 |
| s1 | p4 |
| s2 | p1 |
| s2 | p2 |
| s3 | p2 |
| s4 | p2 |
| s4 | p4 |

*B1*

| pno |
|---|
| p2 |

*B2*

| pno |
|---|
| p2 |
| p4 |

*B3*

| pno |
|---|
| p1 |
| p2 |
| p4 |

*A/B1*

| sno |
|---|
| s1 |
| s2 |
| s3 |
| s4 |

*A/B2*

| sno |
|---|
| s1 |
| s4 |

*A/B3*

| sno |
|---|
| s1 |

## Expressing A/B Using Basic Operators

- **Division is not essential op; just a useful shorthand.**
  - (Also true of joins, but joins are so common that systems implement joins specially.)
- ***Idea*: For *A(x,y)/B(y)*, compute all *x* values that are not `disqualified' by some *y* value in *B*.**
  - *x* value is *disqualified* if by attaching *y* value from *B*, we obtain an *xy* tuple that is not in *A*.

Disqualified *x* values: $\pi_x((\pi_x(A) \times B) - A)$

*A/B*: $\pi_x(A)$ − Disqualified *x* values

## Examples

*Reserves*

| sid | bid | day |
|---|---|---|
| 22 | 101 | 10/10/96 |
| 58 | 103 | 11/12/96 |

*Sailors*

| sid | sname | rating | age |
|---|---|---|---|
| 22 | dustin | 7 | 45.0 |
| 31 | lubber | 8 | 55.5 |
| 58 | rusty | 10 | 35.0 |

*Boats*

| bid | bname | color |
|---|---|---|
| 101 | Interlake | Blue |
| 102 | Interlake | Red |
| 103 | Clipper | Green |
| 104 | Marine | Red |

## Find names of sailors who've reserved boat #103

- **Solution 1:** $\pi_{sname}((\sigma_{bid=103} Reserves) \bowtie Sailors)$
- **Solution 2:** $\pi_{sname}(\sigma_{bid=103}(Reserves \bowtie Sailors))$

## Find names of sailors who've reserved a red boat

- **Information about boat color only available in Boats; so need an extra join:**

$$\pi_{sname}((\sigma_{color='red'} Boats) \bowtie Reserves \bowtie Sailors)$$

❖ A more efficient solution:

$$\pi_{sname}(\pi_{sid}((\pi_{bid}\sigma_{color='red'} Boats) \bowtie Res) \bowtie Sailors)$$

☛ *A query optimizer can find this given the first solution!*

## Find sailors who've reserved a red or a green boat

- **Can identify all red or green boats, then find sailors who've reserved one of these boats:**

$$\rho\ (Tempboats, (\sigma_{color='red' \vee color='green'} Boats))$$

$$\pi_{sname}(Tempboats \bowtie Reserves \bowtie Sailors)$$

## Find sailors who've reserved a red <u>and</u> a green boat

- **Cut-and-paste previous slide?**

$$\rho\ (Tempboats, (\sigma_{color='red' \wedge color='green'} Boats))$$

$$\pi_{sname}(Tempboats \bowtie Reserves \bowtie Sailors)$$

## Find sailors who've reserved a red <u>and</u> a green boat

- **Previous approach won't work! Must identify sailors who've reserved red boats, sailors who've reserved green boats, then find the intersection (note that *sid* is a key for Sailors):**

$$\rho\ (Tempred, \pi_{sid}((\sigma_{color='red'} Boats) \bowtie Reserves))$$

$$\rho\ (Tempgreen, \pi_{sid}((\sigma_{color='green'} Boats) \bowtie Reserves))$$

$$\pi_{sname}((Tempred \cap Tempgreen) \bowtie Sailors)$$

## Find the names of sailors who've reserved all boats

- **Uses division; schemas of the input relations to / must be carefully chosen:**

$$\rho\ (Tempsids, (\pi_{sid,bid} Reserves) / (\pi_{bid} Boats))$$

$$\pi_{sname}(Tempsids \bowtie Sailors)$$

❖ To find sailors who've reserved all 'Interlake' boats:

$$\ldots\ /\ \pi_{bid}(\sigma_{bname='Interlake'} Boats)$$

## Summary

- **Relational Algebra: a small set of operators mapping relations to relations**
  - Operational, in the sense that you specify the explicit order of operations
  - A *closed* set of operators! Can mix and match.
- **Basic ops include:** σ, π, ×, ∪, —
- **Important compound ops:** ∩, ⋈, /