CS-184: Computer Graphics
Lecture #20: Fluid Simulation I

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Fluids

Kim, Thuerey, James, and Gross, 2008

Fluids

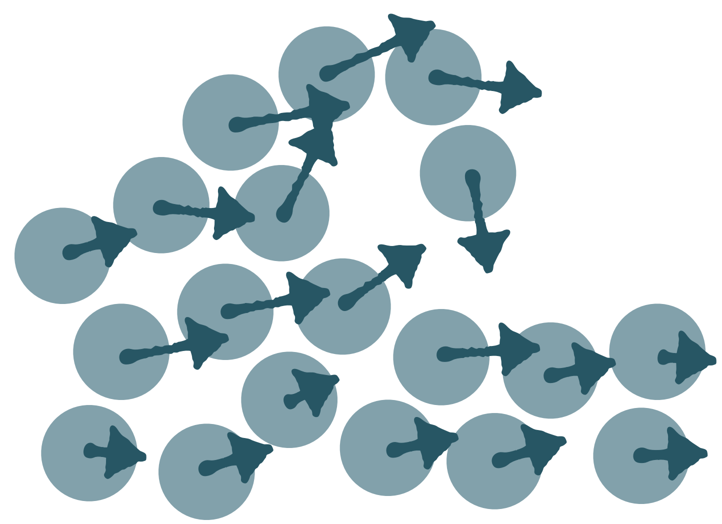
Losasso, Talton, Kwatra, and Fedkiw, 2008

Fluids

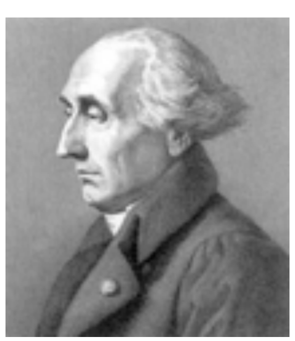
Feldman, O’Brien, and Arikan, 2003

Two ways of representing flow

Two ways of representing flow

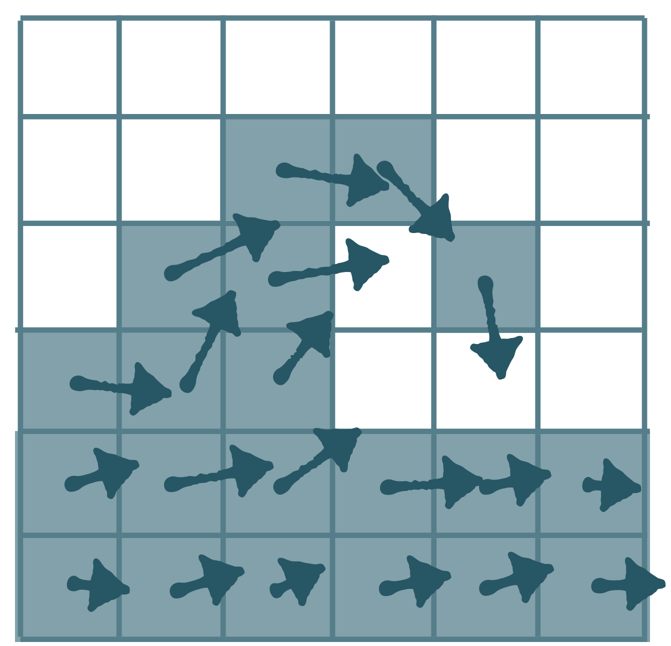


Particles
"Lagrangian"



J.-L. Lagrange
(dead now)

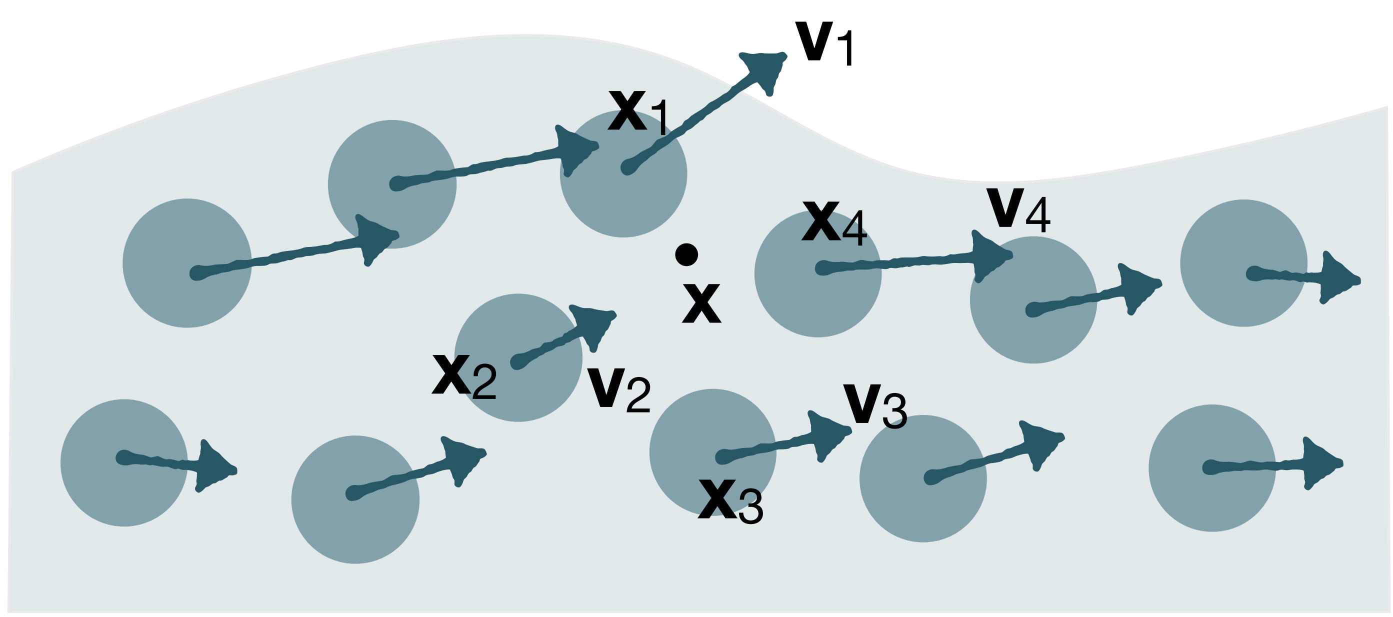
L. Euler
(also dead)



Grid
"Eulerian"

Smoothed particle hydrodynamics (SPH)

- Each particle has mass, position, velocity
- Particles represent **samples** of continuous underlying scalar/vector fields (density, velocity, *etc.*)



$\mathbf{v}(\mathbf{x}) = ?$

SPH interpolation

- Evaluate the field anywhere by weighted averaging

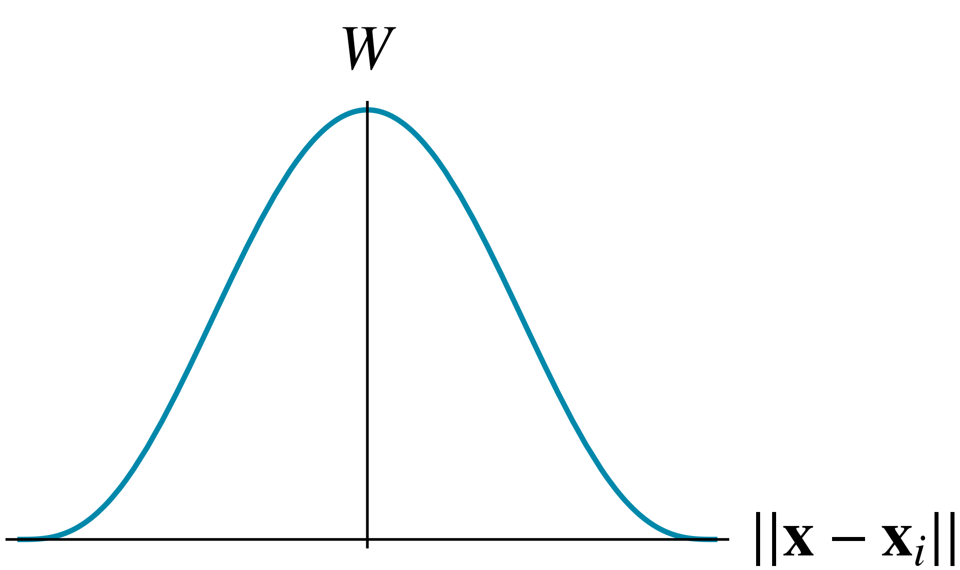
$$A(\mathbf{x}) = \sum_i A_i \frac{m_i}{\rho_i} W(\mathbf{x} - \mathbf{x}_i)$$

Value at point **x**

Value at particle *i*

"Volume" of particle *i*

Smoothing kernel



Density:

$$\rho_i := \rho(\mathbf{x}_i) = \sum_j m_j W(\mathbf{x}_i - \mathbf{x}_j)$$

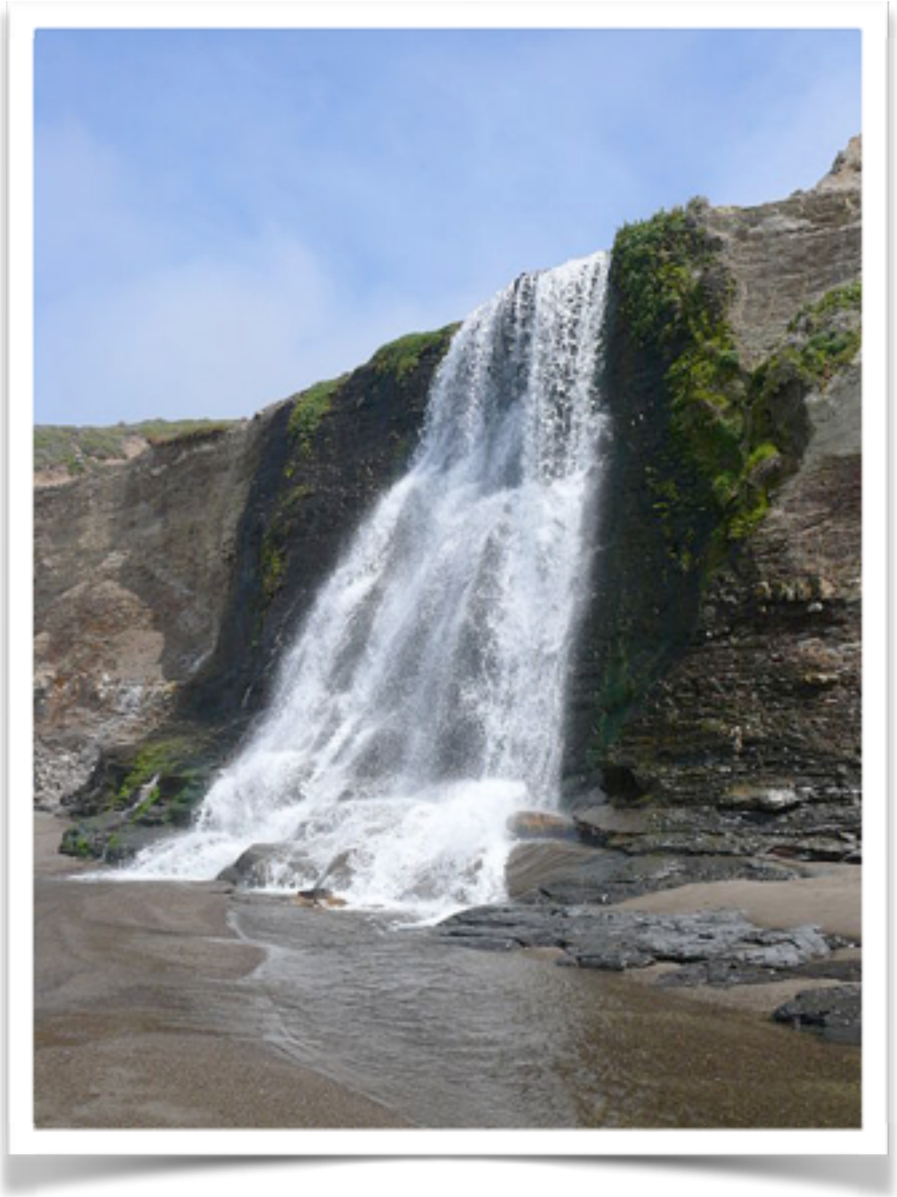
Particle-based fluids

- Each particle has a velocity
- Each time step:
  - Compute acceleration **a** of each particle
  - Update velocities: $\mathbf{v} = \mathbf{v} + \mathbf{a}\,\mathrm{d}t$
  - Update positions: $\mathbf{x} = \mathbf{x} + \mathbf{v}\,\mathrm{d}t$

Forces?

Leapfrog integration

Gravity



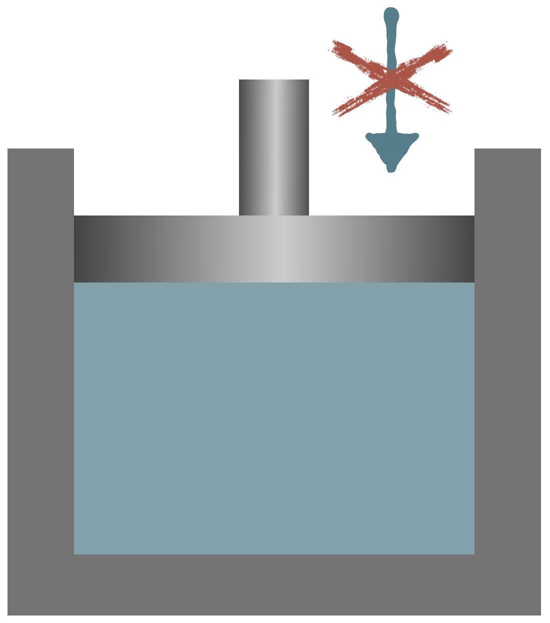
Pressure

- Resists compression and volume change
- Force $\mathbf{f}^{\text{pressure}} = -\nabla p$
- In SPH, we'll assume pressure proportional to density

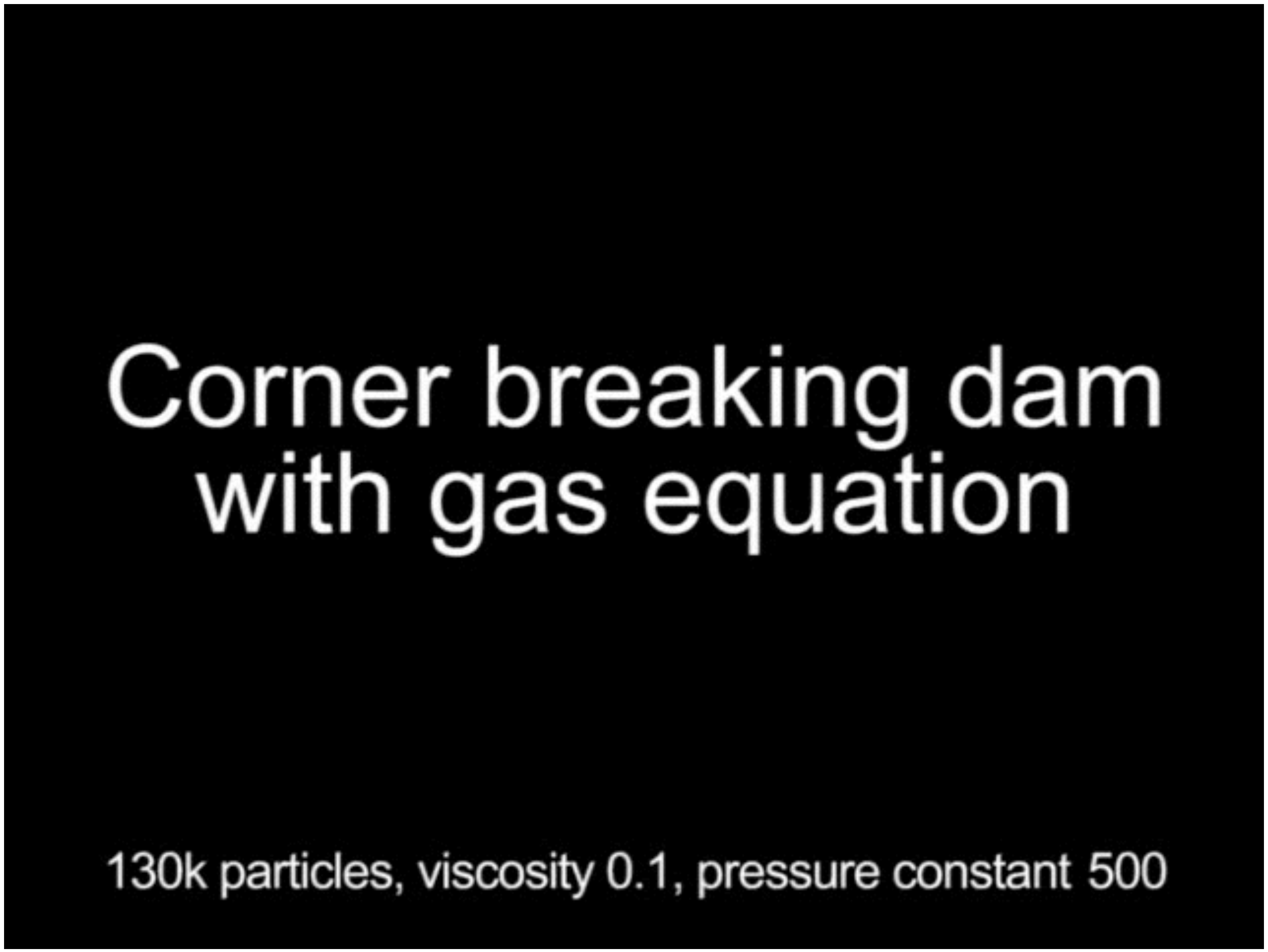
$$p = k\left(\left(\frac{\rho}{\rho_0}\right)^7 - 1\right)$$

Gas constant (k)

Rest density (ρ_0)



Pressure

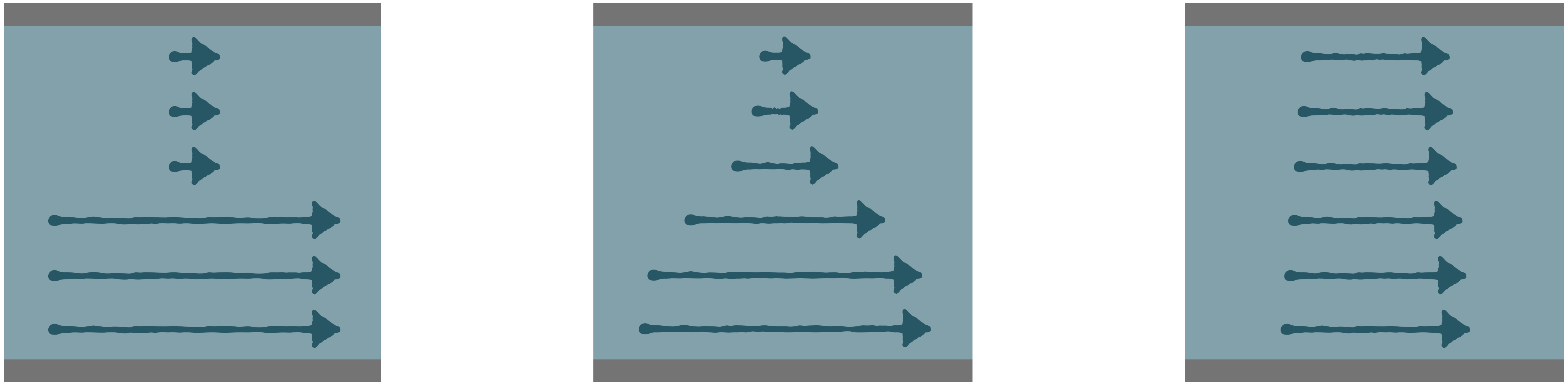

Corner breaking dam
with gas equation
130k particles, viscosity 0.1, pressure constant 500

Viscosity

- Resists relative motion within the fluid

$$\mathbf{f}^{\text{viscosity}} = \mu \nabla^2 \mathbf{v}$$

Coefficient of viscosity



Surface tension

- Tries to minimize surface area
- Only relevant at small scales
- Hard to do correctly

Forces in a fluid

- Forces (per unit volume)

Gravity Viscosity

$$\mathbf{f} = \rho \mathbf{g} - \nabla p + \mu \nabla^2 \mathbf{v}$$

Pressure

- How to evaluate **gradients** of quantities?

Evaluating gradients with SPH

$$A(\mathbf{x}) = \sum_i A_i \frac{m_i}{\rho_i} W(\mathbf{x} - \mathbf{x}_i)$$

- So

$$\nabla A(\mathbf{x}) = \sum_i A_i \frac{m_i}{\rho_i} \nabla W(\mathbf{x} - \mathbf{x}_i)$$

- We just have to differentiate the kernel!
- Same thing works for higher derivatives (for viscosity).

Newton's third law

- Forces between two particles should be equal & opposite

$$\mathbf{f}_i^{\text{pressure}} = -\sum_j p_j \frac{m_j}{\rho_j} \nabla W(\mathbf{x}_i - \mathbf{x}_j)$$

$$\mathbf{f}_i^{\text{viscosity}} = \mu \sum_j \mathbf{v}_j \frac{m_j}{\rho_j} \nabla^2 W(\mathbf{x}_i - \mathbf{x}_j)$$

Newton's third law

- Forces between two particles should be equal & opposite

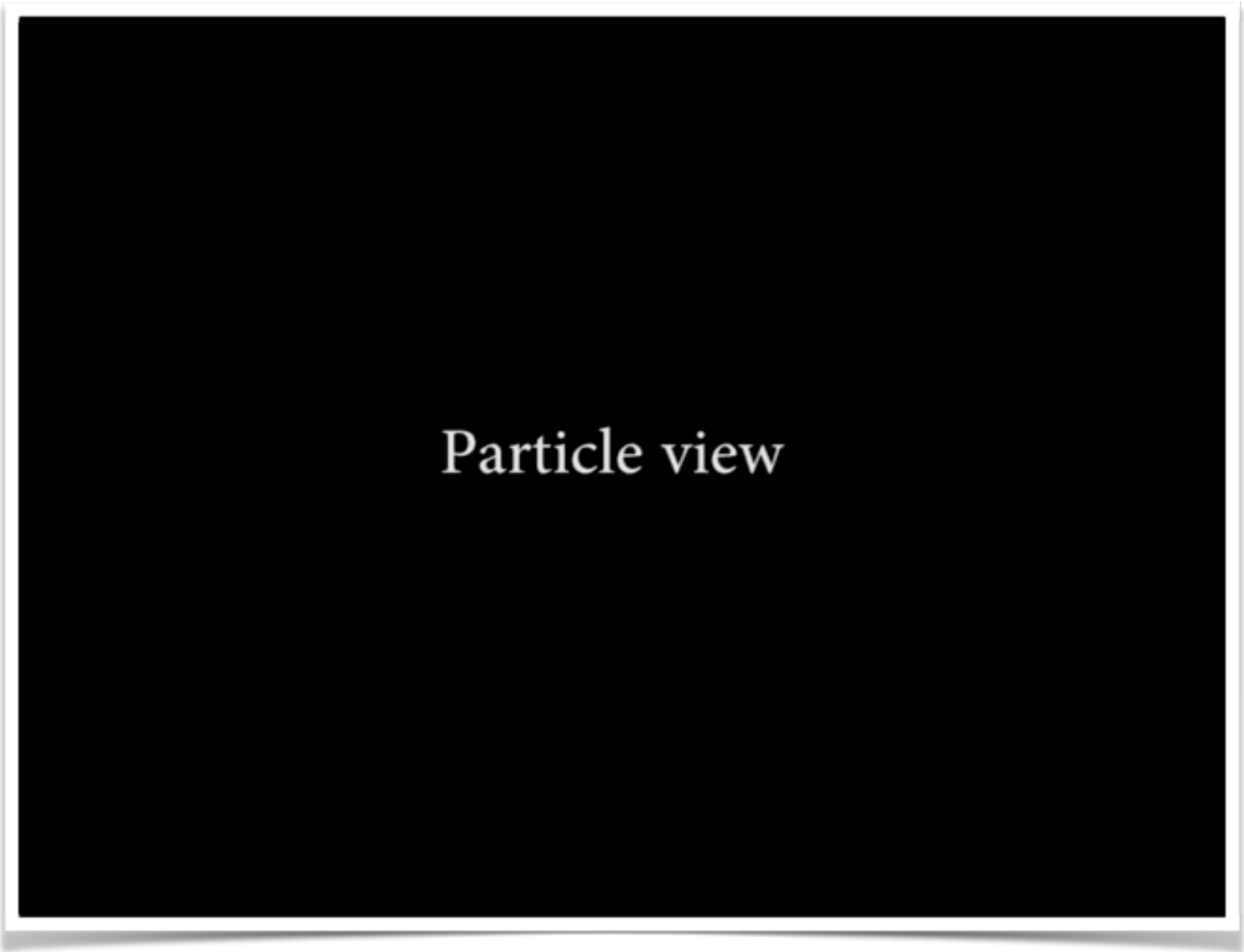
$$\mathbf{f}_i^{\text{pressure}} = -\sum_j \left(\frac{p_i + p_j}{2}\right) \frac{m_j}{\rho_j} \nabla W(\mathbf{x}_i - \mathbf{x}_j)$$

$$\mathbf{f}_i^{\text{viscosity}} = \mu \sum_j (\mathbf{v}_j - \mathbf{v}_i) \frac{m_j}{\rho_j} \nabla^2 W(\mathbf{x}_i - \mathbf{x}_j)$$

Putting it all together

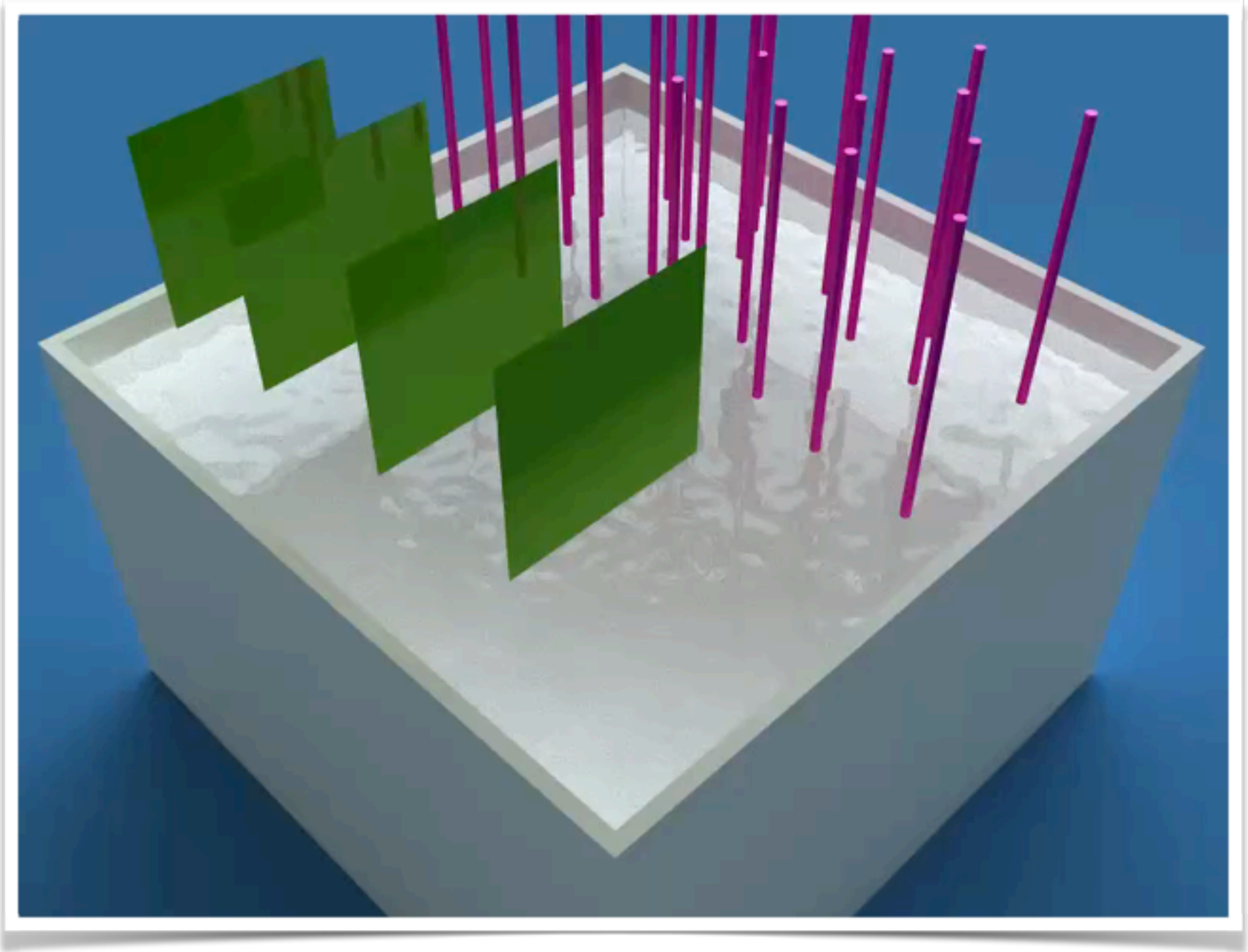
- For each particle:
  - Compute ρ_i for each particle
- For each particle:
  - Evaluate net force $\mathbf{f}_i$
  - Compute acceleration $\mathbf{a}_i = \mathbf{f}_i / \rho_i$
  - Perform leapfrog integration

Particles



Akinci, Ihmsen, Akinci, Solenthaler, and Teschner, 2010

Particles



Akinci, Ihmsen, Akinci, Solenthaler, and Teschner, 2010

Surface reconstruction

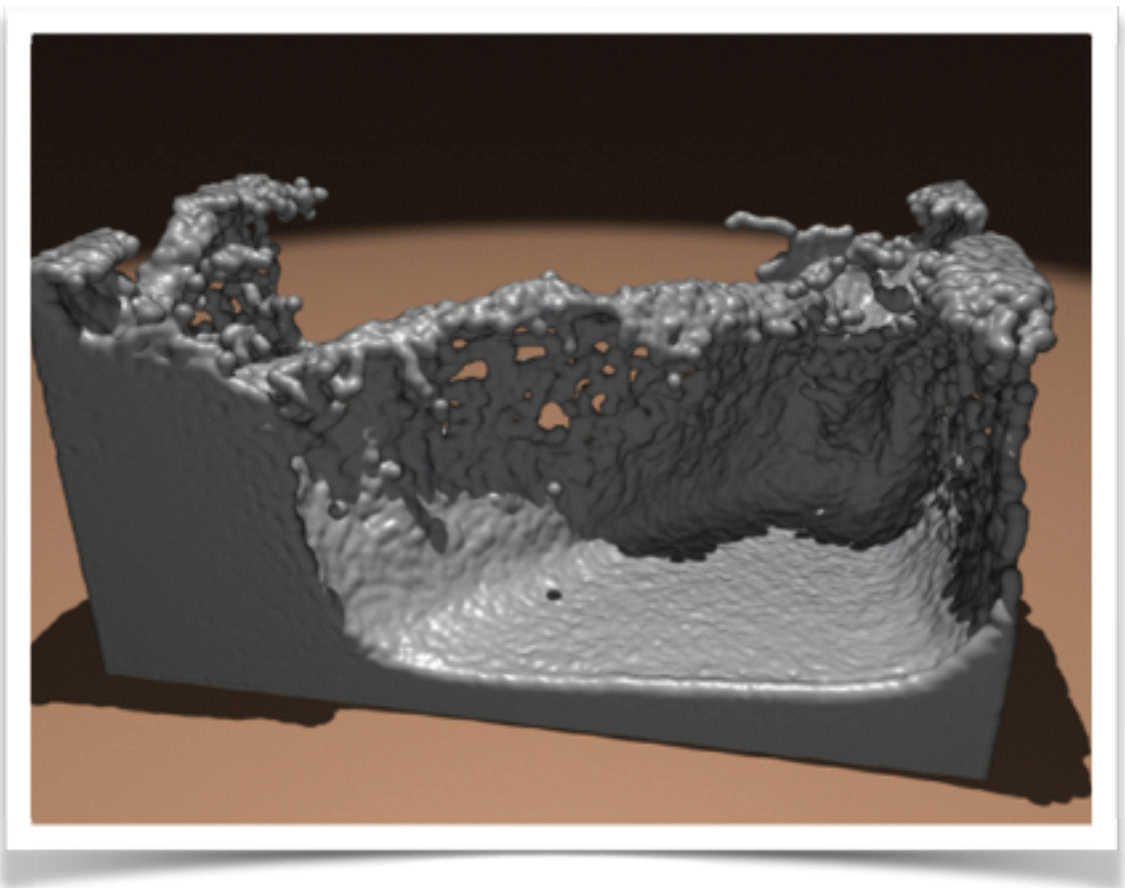
- Define a "color function" $\boldsymbol{c}(\mathbf{x})$ that is 1 inside the fluid and 0 outside
  - *e.g.* do SPH interpolation as usual with $c_i = 1$ always

$$c(\mathbf{x}) = \sum_i c_i \frac{m_j}{\rho_j} W(\mathbf{x} - \mathbf{x}_j)$$

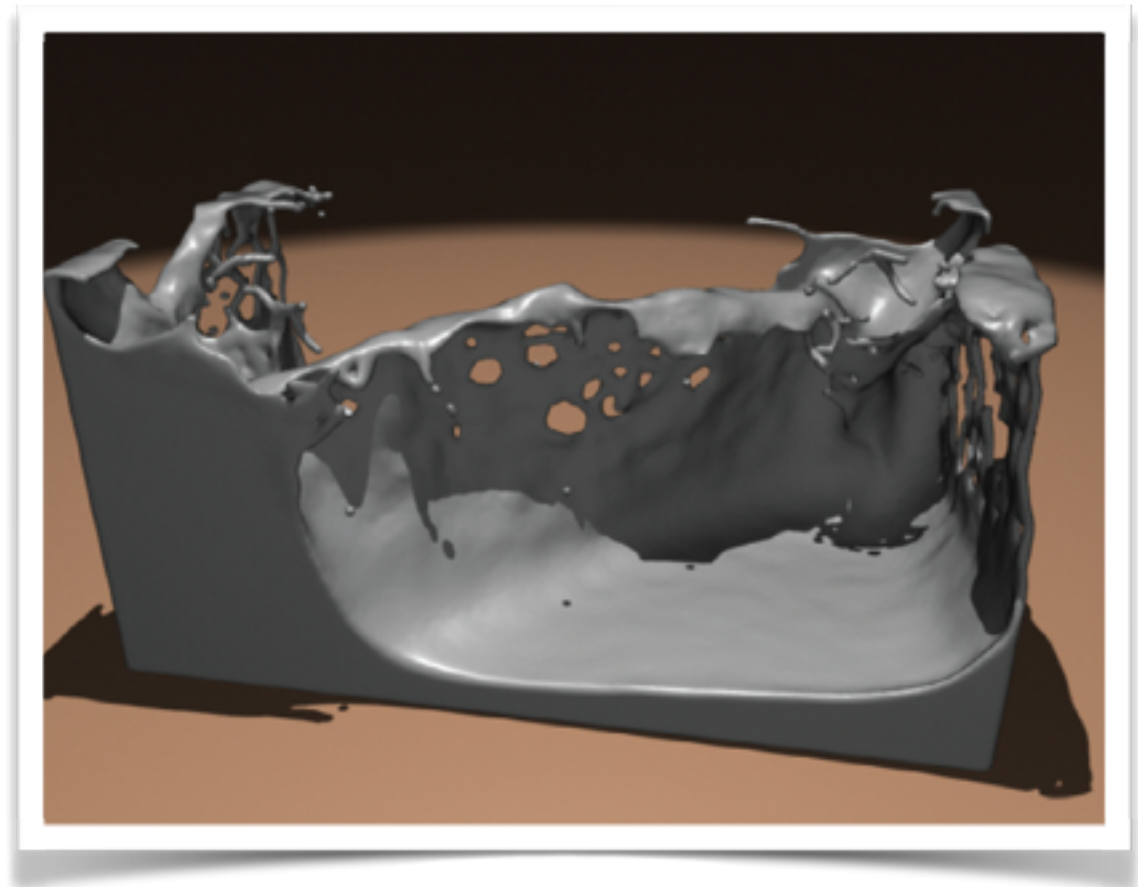
- Extract isosurface at $c = ½$

Surface reconstruction

- Surface can look “lumpy” due to particle distribution
- Solution: use anisotropic kernels along surface



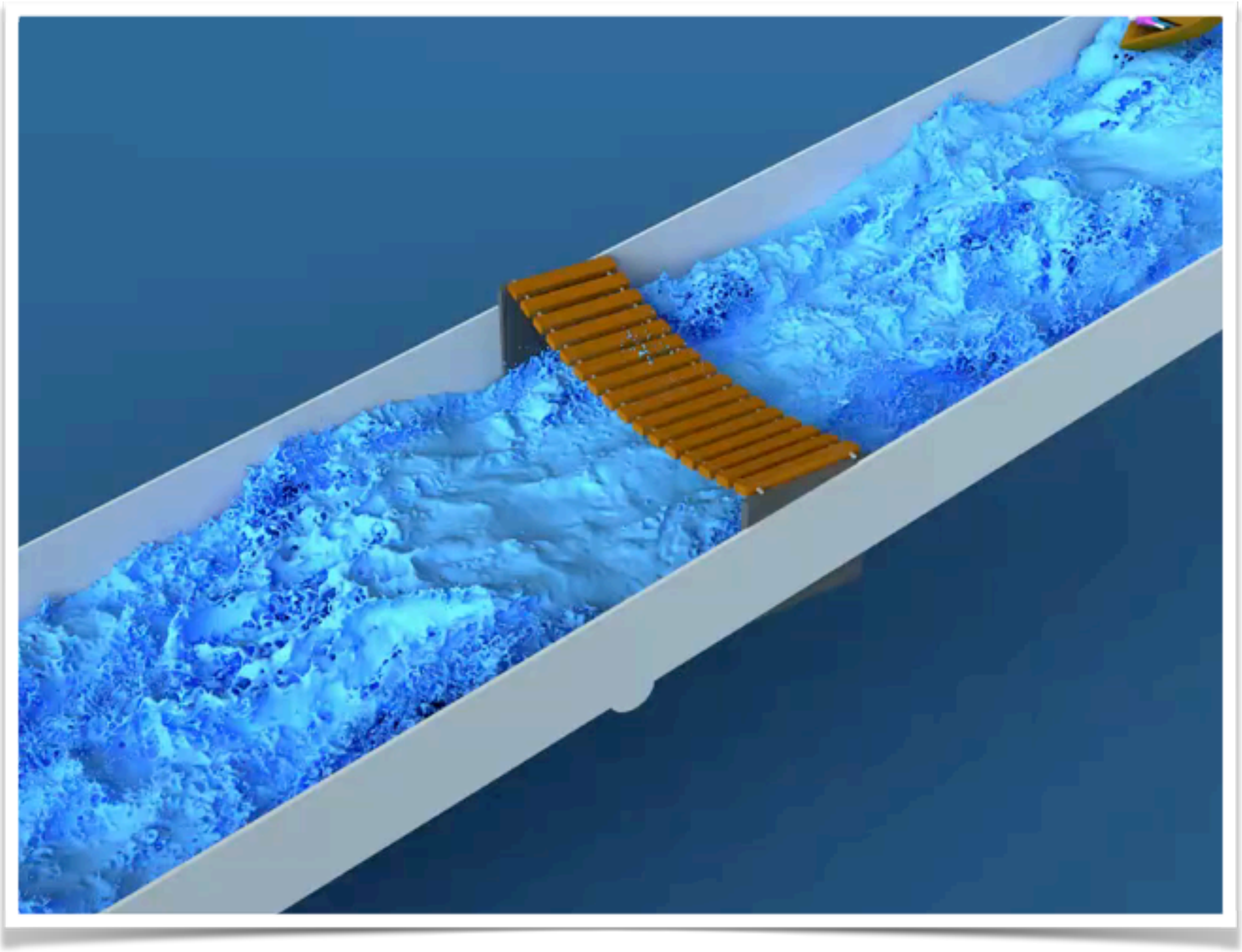
Isotropic kernels



Anisotropic kernels

Yu and Turk, 2010

Smoothed particle hydrodynamics!



Akinci, Ihmsen, Akinci, Solenthaler, and Teschner, 2010

References

- Müller, Charypar, and Gross, "Particle-Based Fluid Simulation for Interactive Applications", 2003
- Becker and Teschner, "Weakly compressible SPH for free surface flows", 2007
- Yu and Turk, "Reconstructing Surfaces of Particle-Based Fluids Using Anisotropic Kernels", 2010