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Student Name:

Class Account Username:

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**Instructions: Read them carefully!**

*The exam begins at 5:10pm and ends at 8:00pm. You must turn your exam in when time is announced or risk not having it accepted.*

*Make sure you fill in your name and the above information, and that you sign below. Anonymous tests will not be graded.*

**Write legibly.** *If the person grading the test cannot read something, he/she will simply assume that you meant the illegible portion as a note to yourself and they will ignore it. If you lose points because part of your answer could not be read, you will not be given the opportunity to explain what it says.*

**Be clear and concise.** *The answers to most questions should be short. If you find yourself writing an excessively long response, you may want to think more carefully about the question. Long rambling answers generally get fewer points than short ones do because there are more opportunities to mark something wrong.*

*You may use two pages of notes while taking the exam. You may not ask questions of other students, look at another student's exam, use a textbook, use a phone or calculator, or seek any other form of assistance. In summary: do not cheat. Persons caught cheating will be subject to disciplinary action.*

**Do not ask questions during the exam.** *Most questions are unnecessary and they disturb other students. Figuring out what the exam question is asking is part of the test. If you think you have to make some unusual assumption to answer a problem, note what that assumption is on the test.*

***I have read these instructions, I understand them, and I will follow them.***

Your Signature: \_\_\_\_\_

Date: \_\_\_\_\_

Student ID: \_\_\_\_\_

Total Points: 223 + 5      You Scored: \_\_\_\_\_ + Extra \_\_\_\_\_

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1. Please fill in each of the blanks with an appropriate answer. *2 points each blank, 68 total*

The Euler integration scheme tends to cause simulations to “blow up.” The implicit version of this scheme, known as \_\_\_\_\_, is much more stable but has a tendency to damp motions artificially.

Explicit integration schemes make use of the accelerations at the \_\_\_\_\_ of each simulation timestep.

Inverse kinematics algorithms we discussed in class involve numerical root finding for a set of non-linear equations and can be solved using \_\_\_\_\_.

\_\_\_\_\_ colors consist of light at a single wavelength.

\_\_\_\_\_ motion capture systems record the position and orientation of the sensors on the subject.

The \_\_\_\_\_ of a matrix can be computed using the singular value decomposition algorithm.

The tangent vectors of an parametric surface generally can be used to compute the surface \_\_\_\_\_.

When representing \_\_\_\_\_ in 3D using homogenized coordinates, the fourth coordinate (i.e. “w”) will be one.

Temporal anti-aliasing is often called \_\_\_\_\_.

The \_\_\_\_\_ method operates from the assumption that all surfaces in a scene act like diffuse reflectors.

The final gather often is the most time consuming step of the \_\_\_\_\_ rendering method.

Computing form factors is between patches often is the most time consuming step of the \_\_\_\_\_ rendering method.

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\_\_\_\_\_ approximates global illumination by making diffuse shading proportional to the un-occluded area over a surface.

Radiosity is measured in units of \_\_\_\_\_.

Catmull-Clack subdivision surfaces are a generalization of uniform, cubic, tensor-product \_\_\_\_\_.

A Bezier curve is always \_\_\_\_\_ by the convex hull of its control points.

In Catmull-Clark subdivision, the number of extraordinary points \_\_\_\_\_ after the first round of subdivision.

The \_\_\_\_\_ in the human eye are used in bright-light situations.

Under linear perspective projection, straight lines always appear as \_\_\_\_\_.

Red spectral colors appear at the \_\_\_\_\_ end of the visible spectrum.

The \_\_\_\_\_ is a function that describes how well a material reflects incoming light from one direction out in another direction.

The \_\_\_\_\_ of an orthonormal matrix is equal to its transpose.

The \_\_\_\_\_ parameterization of 3D rotations is plagued by the fearful phenomena known as gimbal lock.

\_\_\_\_\_ encode 3D rotations as 3D point inside a ball of radius  $\pi$ .

Waiting until the last day to start working on your inverse kinematics assignment is a \_\_\_\_\_ idea.

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The "B" in BSP-Tree stands for \_\_\_\_\_.

\_\_\_\_\_ is a special case of a point light source where the viewer is infinitely far away.

In the context of a scan-line renderer, Z-buffers are used for \_\_\_\_\_.

A bump map is used to change the \_\_\_\_\_ vectors when shading an object.

When two curve segments join at a point and both curves approach that point with non-zero parallel tangent vectors from opposite directions, the joining is said to be \_\_\_\_\_ continuous.

When two curve segments join at a point and both curves approach that point with non-parallel tangent vectors, the joining is said to be \_\_\_\_\_ continuous.

NURBS are b-splines that use \_\_\_\_\_ for control points.

Steradians are the dimensionless units used to measure \_\_\_\_\_.

When the view point used to generate a radiosity solution changes, updating the solution for the new viewpoint recomputing \_\_\_\_\_.

**2. Answer the following questions with True (T) or False (F)**

*2 points each, 46 total*

\_\_\_\_\_ Cloth can be modeled reasonably well using a collection of particles attached by springs.

\_\_\_\_\_ The pseudo inverse of a matrix can be computed using the Singular Value Decomposition (SVD) algorithm.

\_\_\_\_\_ The Jacobian of a valid kinematic system will always be invertible.

\_\_\_\_\_ Polished metallic surfaces typically have bright white specularities.

\_\_\_\_\_ Radiance grows with distance along a straight line.

\_\_\_\_\_ The implicit representation of a given geometric entity is unique.

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- \_\_\_\_\_ The rods in the human eye have a flat spectral response function.
  - \_\_\_\_\_ Under linear perspective projection, squares always appear as a square.
  - \_\_\_\_\_ Under linear perspective projection, triangles always appear as a triangle.
  - \_\_\_\_\_ Under orthographic projection, all sets of parallel lines will remain parallel.
  - \_\_\_\_\_ Quaternions represent rotations as points in 4D space on the surface of a hypersphere.
  - \_\_\_\_\_ Any set of non-intersecting polygons can be sorted in front-to-back order.
  - \_\_\_\_\_ Ink-based color printers could be designed to use other colors besides cyan, magenta, and yellow.
  - \_\_\_\_\_ Shining an ultraviolet light on scorpions induces a chemical response that causes them to glow green and become paralyzed, thus making them easy to find and safe to handle.
  - \_\_\_\_\_ The force exerted by a spring with zero rest length is given by a function that is linear in terms of the endpoint locations.
  - \_\_\_\_\_ Cubic Bézier curves will always be  $C^2$  across segment boundaries.
  - \_\_\_\_\_ Light fields are (ideally) records of the light passing through all points in a region of space in all directions.
  - \_\_\_\_\_ In a kinematic skeleton, every body must have exactly one inboard joint.
  - \_\_\_\_\_ Modern LCD displays have a dynamic range approximately twice that of the human eye.
  - \_\_\_\_\_ A rotation matrix always has determinant of  $\pm 1$ .
  - \_\_\_\_\_ Pasteurized coordinates facilitate representing perspective and translation using matrices.
  - \_\_\_\_\_ Ambient occlusion tends to enhance the appearance of surface detail.
  - \_\_\_\_\_ The sky is blue because water vapor scatters red light.
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3. Imagine that you have a RGB monitor where the wires have been swapped so that the red, green, and blue outputs from the computer have been respectively attached to the blue, red, and green inputs on the monitor. When one attempts to display the following colors, what colors will actually appear on the screen? *8 points*

Red \_\_\_\_\_

Green \_\_\_\_\_

Blue \_\_\_\_\_

Cyan \_\_\_\_\_

Magenta \_\_\_\_\_

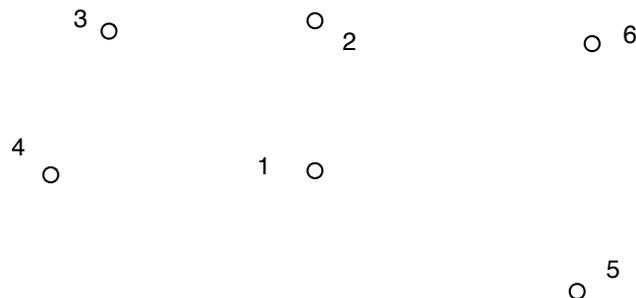
Yellow \_\_\_\_\_

Black \_\_\_\_\_

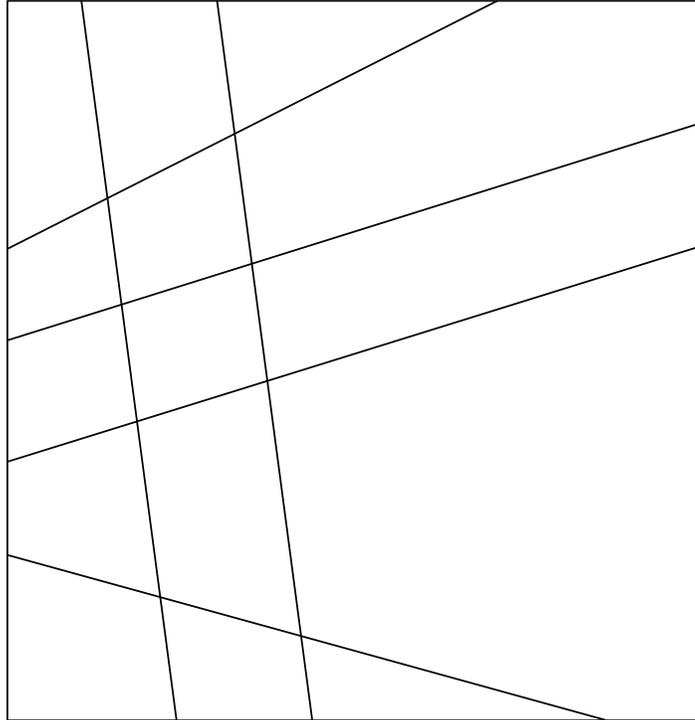
White \_\_\_\_\_

4. If a surface is defined implicitly by the function  $f(x) = 0$ , write out the equation you would use to compute the surface's normal at some point. (Assume that negative values are inside the surface.) *2 points*

5. The diagram below shows control points for a curve made by joining two cubic Bezier segments. However control point #5 has been removed. Indicate location(s) where #5 may be placed to achieve  $G^1$  continuity and where it may be placed to achieve  $C^1$  continuity. Clearly label your diagram. *6 points*



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6. Here is a piece of mesh. Draw the result of applying one iteration of Catmull-Clark subdivision. Then circle all vertices (both original and the new ones you added) that are extraordinary. *Note: I am only interested in the topology of your answer.* 7 points



7. Name a phenomenon that can be modeled easily using radiosity but that cannot be modeled with a basic ray-tracing algorithm. Give an example. 3 points
8. Briefly state why interpolating transformation matrices by linearly interpolating the matrix components is a bad idea. 4 points
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9. Below are two 4x4 homogenized transformation matrices. What does the first one do? How does the effect produced by the second one differ from that produced by the first? 4 points

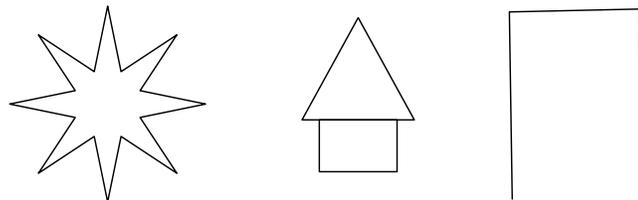
$$\begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 6 \end{bmatrix} \quad \begin{bmatrix} 6 & 0 & 0 & 0 \\ 0 & 6 & 0 & 0 \\ 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

The first one will: \_\_\_\_\_

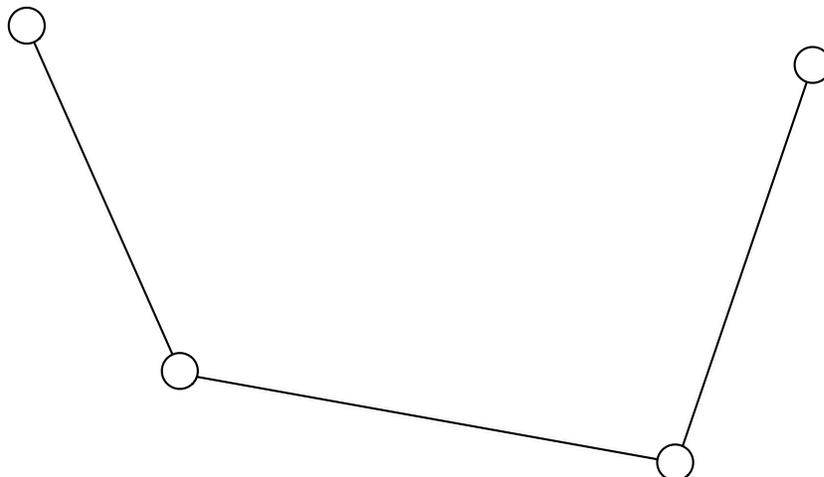
The second one will: \_\_\_\_\_

10. Draw the convex hull of the shapes shown below.

6 points

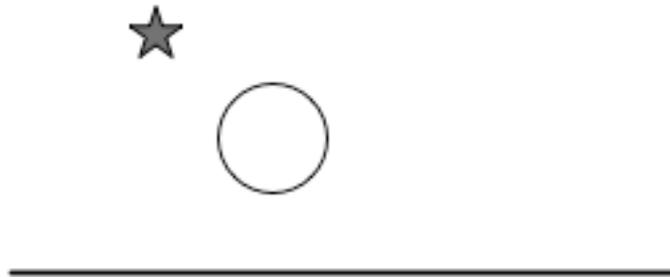


11. The diagram below is the control polygon for a Bezier curve segment. Draw the curve and show how de Casteljau's algorithm can be used to subdivide the curve into two halves. Make sure your drawing is geometrically reasonable and shows correct curve tangents for the beginning, middle, and end of each segment. 5 points



12. Given a rotation matrix, how would you determine the number of degrees that it rotates by? *3 points*

13. In the diagram below of a light source, a clear glass ball, and a diffuse surface, draw lines illustrating the path traveled by light to form a refraction caustic on the surface. *3 points*



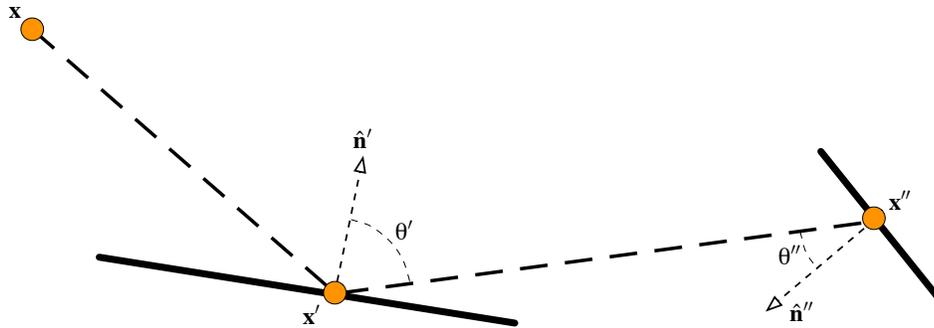
14. Write out an explicit equation for a sphere. Be sure to specify the range of your parametric variables to exactly cover the sphere once. *6 points*

15. Given two points, A and B, write out an implicit equation for the line in 3D that contains both points. You may assume A and B are distinct points. *4 points*

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16. Consider the following equation and diagram:

$$L_s(\mathbf{x}, \mathbf{x}') = \delta(\mathbf{x}, \mathbf{x}') \left[ E(\mathbf{x}, \mathbf{x}') + \int_S \rho_{x'}(\mathbf{x}, \mathbf{x}'') L_s(\mathbf{x}', \mathbf{x}'') \frac{\cos(\theta') \cos(\theta'')}{\|\mathbf{x}' - \mathbf{x}''\|^2} d\mathbf{x}'' \right]$$



Explain what effects each of the following is responsible for.

10 points

$\delta(\mathbf{x}, \mathbf{x}')$

\_\_\_\_\_

$E(\mathbf{x}, \mathbf{x}')$

\_\_\_\_\_

$\|\mathbf{x}' - \mathbf{x}''\|^2$

\_\_\_\_\_

$\cos(\theta')$

\_\_\_\_\_

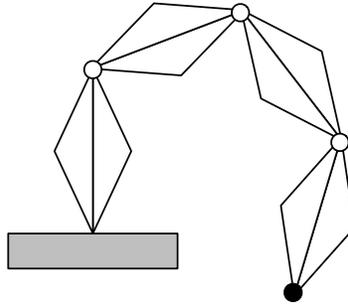
$\rho_{x'}(\mathbf{x}, \mathbf{x}'')$

\_\_\_\_\_

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17. If a surface is defined explicitly by the function  $\mathbf{x} = f(u, v)$ , write out the equation you would use to compute the surface's normal at some point. 4 points

18. Consider this diagram showing a four-joint arm in 2D where each joint is a simple pin joint and the base is fixed in space.



If we are solving an IK problem to place the tip of the arm (the black dot) at a particular location, what is the size of the Jacobian matrix we will be working with? 3 points

Draw any one configuration of the arm where two columns of the Jacobian will be parallel vectors. 5 points

In the drawing you made above, *clearly* show the direction of the parallel vectors. 3 points

In the drawing you made, will the Jacobian have rank less than, greater than, or equal to two? 3 points

When will the this system's Jacobian be fully invertable? 3 points

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19. Name two types of useful image effects that would require shooting more than one ray per pixel in a ray-tracer. 4 points

20. Draw a set of polygons that cannot be sorted in front-to-back order. 3 points

21. Consider the two diagrams below. All four surfaces are identical ideal diffuse reflectors. In each diagram circle the surface that will appear brighter to the observer. 4 points

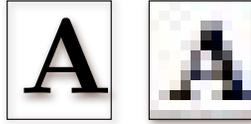
Diagram #1



Diagram #2



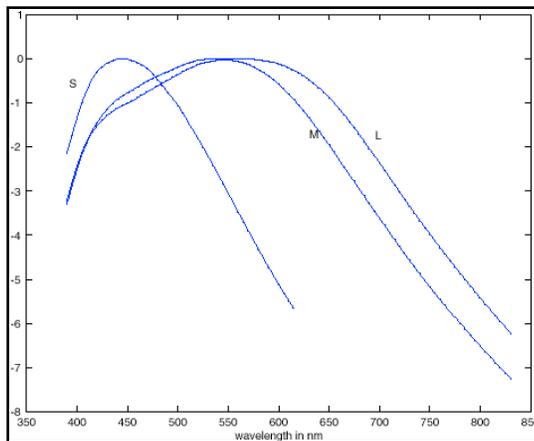
22. The two images below show two 12 point versions of the letter “A” that have been significantly enlarged. Concisely explain the most likely difference between the two fonts used in the images. 4 points



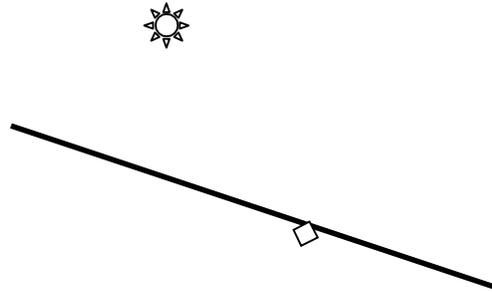
23. The following equation describes how the radiosity at patch  $P_i$  is a function of the radiosity of other patches. Circle the part(s) that are responsible for taking into account the visibility of other patches when viewed from  $P_i$ . 3 points

$$H_i = E_i + \rho_i \sum_j H_j \int_{S_j} \delta_{ij} \frac{\cos(\theta_i) \cos(\theta_j)}{2\pi \|\mathbf{c}_i - \mathbf{x}\|^2} d\mathbf{x}$$

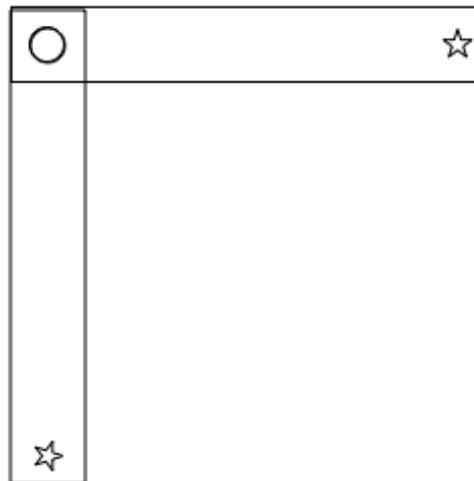
24. The following are the response curves for the cones in the human eye. Which type of cone is most sensitive to red light? 2 point



25. Consider the diagram below. A location has been marked on the surface. Indicate a viewer position such that a viewer looking at the surface from that position would see a specular highlight on the surface at the marked location. *3 points*



26. Below is a diagram showing a bar that has been rotated 90 degrees about the point indicated with a circle. If intermediate positions were generated by linearly interpolating the transformation matrices, how would the point indicated by the star move? Give your answer by drawing the path of the star. *4 points*



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EXTRA CREDIT

5 points

Given a plane and a sphere:

Plane:  $\hat{\mathbf{n}} \cdot \mathbf{x} + f = 0$

Sphere:  $\|\mathbf{c} - \mathbf{x}\| - r = 0$

**Write out an equation that will, for the case where the plane intersects the sphere, implicitly define the circle of their intersection.**

Your answer must be neat and clear. No points will be awarded for imprecise answers. Your answer should be in the form of a simple explicit equation that you have drawn a box around. Do not attempt this question until you have completed the rest of the exam! There will be no partial credit for this question.

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## Final project / Assignment 6 report submission instructions:

- The report for your final project is due tomorrow (Friday the 18th) no later than 5pm.
  - Reports should be posted as a webpage linked from your class home page.
  - You may, and probably should, include images and videos.
  - The names and login IDs for all project members should appear on your report.
  - There is no leeway for accepting late submissions. Once I go home on Friday I will not be back on campus until after grades have been submitted.
  - Submissions by e-mail will not be accepted. They will be deleted.
  - Make sure your report accurately and fully describes your results. Your grade will be based entirely on this report.
  - Use the submission system to turn in your code as if it were assignment 6 (even if you did a project). We will spot check to make sure the code you submit matches the results shown in your report.
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