CS 170 Fall 2006 — Discussion Handout #3

October 12, 2006

1. Dijkstra with negative edges!
Consider a directed graph in which the only negative edges are those that leave s; all other edges are positive. Can Dijkstra's algorithm, started at s, fail on such a graph? Prove your answer.

2. Dijkstra Jr.
Suppose we want to run Dijkstra's algorithm on a graph whose edge weights are integers in the range $0, \cdots, W$ where W is a relatively small number.

- (a) Show how Dijkstra's algorithm can be made to run in time $O(W|V| + |E|)$.
- (b) Show an alternative implementation that takes time just $O(|V| + |E|)logW)$

3. No one like you
Let $G = (V, E)$ be an undirected graph. Prove that if all its edge weights are distinct, then it has a unique minimum spanning tree.

4. To tell you the truth...
The following statements may or may not be correct. In each case, either prove it(if it is correct) or give a counter example (if it isn't correct). Always assume that the graph $G = (V, E)$ is undirected and connected. Do not assume that edge weights are distinct unless this is specifically stated.

1. If graph G has more than $|V| - 1$ edges, and there is a unique heaviest edge, then this edge cannot be part of a minimum spanning tree.
2. Let e be an edge of minimum weight in G. Then e must be part of some MST.
3. Kruskal's algorithm fails to find the minimum spanning tree if the graph has negative edges.
4. If e is the heaviest edge in some cycle of the graph G, then there is a minimum spanning tree *not* containing e.