Due: Fri, 3 May 2019

1. A definition (that is, an assignment) of a simple variable is said to reach a point in the program if it might be the last assignment to that variable executed before execution reaches that point in the program. So for example, definition $A$ below reaches points $B$ and $C$, but not $D$ :
```
x = 3 # A
if a < 2:
    x = 2
    pass # D
else:
    y = 5
    pass # B
pass # C
```

Suppose we want to compute $R(p)$, the set of all definitions that reach point $p$ in a program. Give forward rules (in the style of the lecture) for computing the reaching definitions, $R_{\text {out }}(s)$ for a statement $s$ (the set of definitions that reach the point immediately after the statement) as a function of $R_{\text {in }}(s)$ (the definitions that reach the beginning) for each assignment statement $s$ and give the rules for computing $R_{\mathrm{in}}(s)$ as a function of the $R_{\text {out }}$ values of its predecessors.
2. Consider the loop

```
for i := 0 to n-1 do
    for j := 0 to n-1 do
        for k := 0 to n-1 do
            c[i,j] := c[i,j] + a[i,k] * b[k,j]
```

In this nested loop, a, b, and c are two-dimensional arrays of 4-byte integers. Here is a translation into intermediate code (assume that a, b, and c are addresses of static memory, and that all other variables are in registers):

| Entry: |  | $\mathrm{t} 11:=4 * \mathrm{n}$ | \#17 |
| :---: | :---: | :---: | :---: |
| i : = 0 | \#1 | t12 : $=$ t11 * k | \#18 |
| goto L6 | \#2 | t13 := 4 * j | \#19 |
| L1: |  | $\mathrm{t} 14:=\mathrm{t} 12+\mathrm{t} 13$ | \#20 |
| j : = 0 | \#3 | $\mathrm{t} 15:=*(\mathrm{t} 14+\mathrm{b})$ | \#21 |
| goto L5 | \#4 | $\mathrm{t} 16:=\mathrm{t} 10$ * t15 | \#22 |
| L2: |  | t 17 := t5 + t16 | \#23 |
| $\mathrm{k}:=0$ | \#5 | $\mathrm{t} 18:=4 * \mathrm{n}$ | \#24 |
| goto L4 | \#6 | t19 := t18 * i | \#25 |
| L3: |  | t20 := 4 * j | \#26 |
| $\mathrm{t} 1:=4 * \mathrm{n}$ | \#7 | $\mathrm{t} 21:=\mathrm{t} 19+\mathrm{t} 20$ | \#27 |
| t2 : $=\mathrm{t} 1 * \mathrm{i}$ | \#8 | *(t21+c) $:=\mathrm{t} 17$ | \#28 |
| t3 : $=4 * j$ | \#9 | $\mathrm{k}:=\mathrm{k}+1$ | \#29 |
| $\mathrm{t} 4:=\mathrm{t} 2+\mathrm{t} 3$ | \#10 | L4: |  |
| $\mathrm{t} 5:=*(\mathrm{t} 4+\mathrm{c})$ | \#11 | if $\mathrm{k}<\mathrm{n}$ : goto L3 | \#30 |
| t6 := $4 * \mathrm{n}$ | \#12 | $j \quad:=j+1$ | \#31 |
| t7 := t6 * i | \#13 | L5: |  |
| t8 := 4* k | \#14 | if $\mathrm{j}<\mathrm{n}$ : goto L2 | \#32 |
| $\mathrm{t} 9:=\mathrm{t} 7+\mathrm{t} 8$ | \#15 | i $:=\mathrm{i}+1$ | \#33 |
| $\mathrm{t} 10:=*(\mathrm{t} 9+\mathrm{a})$ | \#16 | L6: |  |
|  |  | if i < n: goto L1 Exit: | \#34 |

To notate accesses to memory, we've used C-like notation:

```
r1 := *(r2+K)
*(r1+K) := r2
*K := r3
r3 := *K
```

K is an integer literal, and L is a static-storage label (a constant address in memory). Unlike C, the additions here are just straight addition: no automatic scaling by word size.
a. According to this code, how are the elements of the three two-dimensional arrays laid out in memory (in what order do the elements of the arrays appear)?
b. Divide the instructions into basic blocks, each headed by a label and with no other labels in the program.
c. The program is almost in SSA form, except for variables i, j, and k. Introduce new variables and $\phi$ functions as needed to put the program into SSA form (try to minimize $\phi$ functions).
d. Now optimize this code as best you can, moving assignments of invariant expressions out of loops, eliminating common subexpressions, removing dead statements, performing copy propagation, etc.

