LR Parsing, LALR Parser Generators

Lecture 10

Outline

- Review of bottom-up parsing
- Computing the parsing DFA
- Using parser generators

Bottom-up Parsing (Review)

- A bottom-up parser rewrites the input string to the start symbol
- The state of the parser is described as

$$\alpha \blacktriangleright \gamma$$

  - α is a stack of terminals and non-terminals
  - γ is the string of terminals not yet examined

- Initially: $\blacktriangleright x_1 x_2 \ldots x_n$

The Shift and Reduce Actions (Review)

- Recall the CFG: $E \rightarrow \text{int} \mid E + (E)$
- A bottom-up parser uses two kinds of actions:

- Shift pushes a terminal from input on the stack

$$E + (\blacktriangleright\ \text{int}) \Rightarrow E + (\text{int} \blacktriangleright)$$

- Reduce pops 0 or more symbols off the stack (the rule's rhs) and pushes a non-terminal on the stack (the rule's lhs)

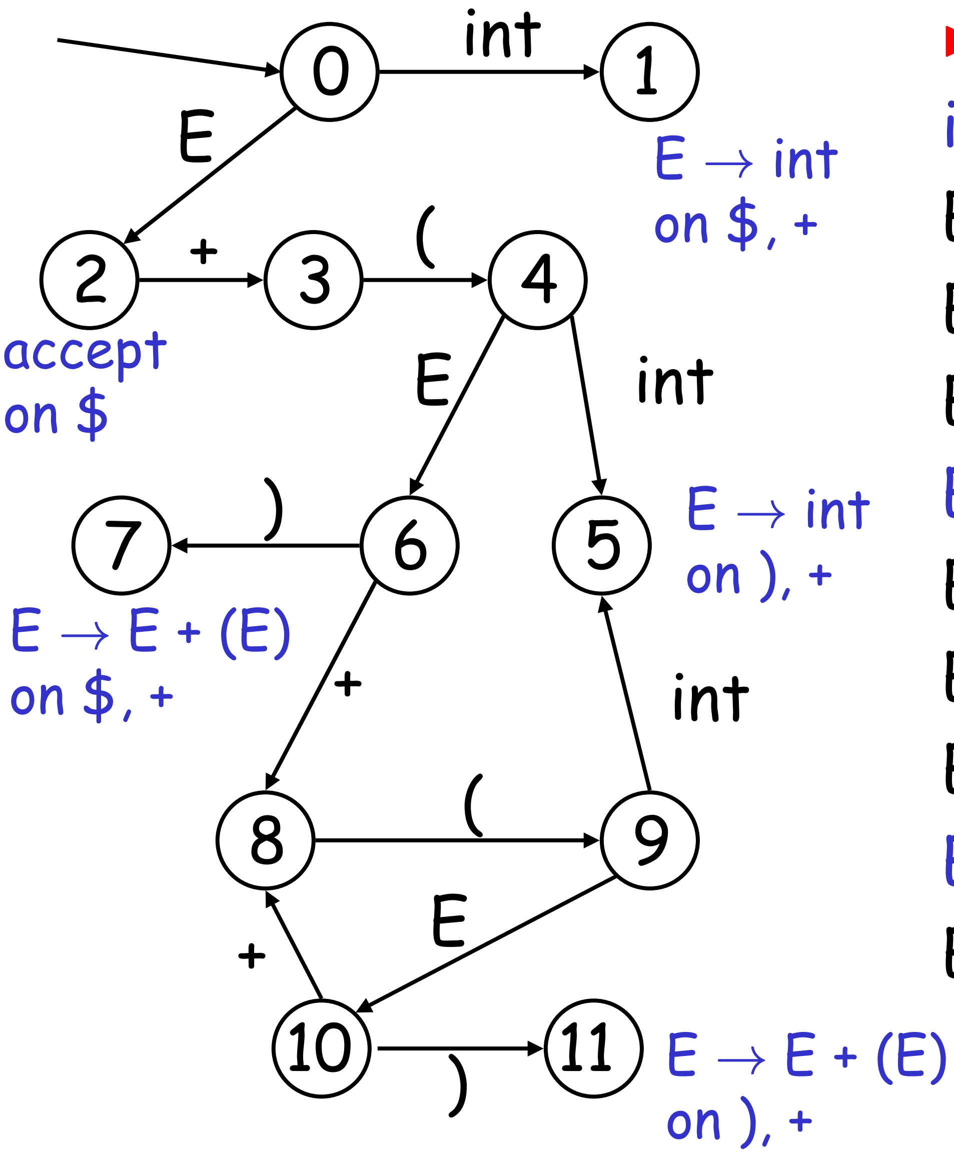
$$E + (\underline{E + (E)} \blacktriangleright) \Rightarrow E + (\underline{E} \blacktriangleright)$$

Key Issue: When to Shift or Reduce?

- Idea: use a finite automaton (DFA) to decide when to shift or reduce
  - The input is the stack
  - The language consists of terminals and non-terminals

- We run the DFA on the stack and we examine the resulting state X and the token tok after ▶
  - If X has a transition labeled tok then shift
  - If X is labeled with “$A \to \beta$ on tok” then reduce

LR(1) Parsing. An Example



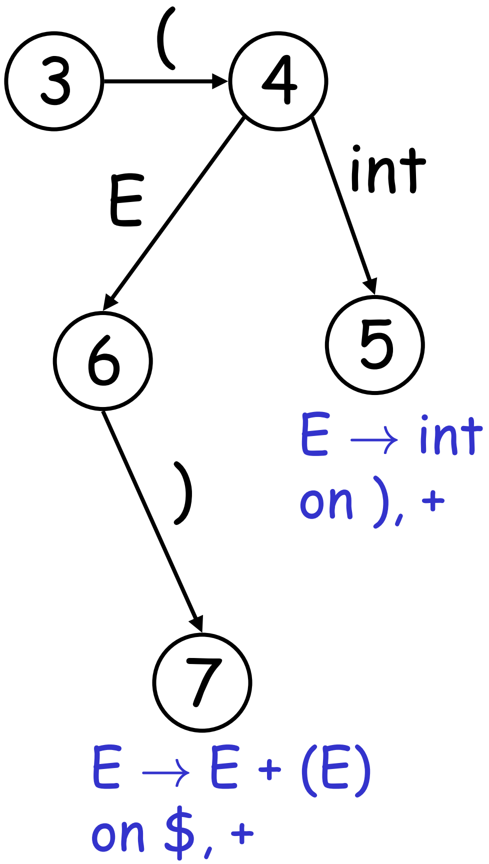
▶ int + (int) + (int)$	shift
int ▶ + (int) + (int)$	E → int
E ▶ + (int) + (int)$	shift(x3)
E + (int ▶) + (int)$	E → int
E + (E ▶) + (int)$	shift
E + (E) ▶ + (int)$	E → E+(E)
E ▶ + (int)$	shift (x3)
E + (int ▶)$	E → int
E + (E ▶)$	shift
E + (E) ▶ $	E → E+(E)
E ▶ $	accept

Representing the DFA

- Parsers represent the DFA as a 2D table
  - Recall table-driven lexical analysis
- Lines correspond to DFA states
- Columns correspond to terminals and non-terminals
- Typically columns are split into:
  - Those for terminals: the action table
  - Those for non-terminals: the goto table

Representing the DFA. Example

- The table for a fragment of our DFA:



	int	+	(	)	$	E
…						
3			s4			
4	s5					g6
5		$r_{E\to \text{int}}$		$r_{E\to \text{int}}$		
6	s8		s7			
7		$r_{E\to E+(E)}$			$r_{E\to E+(E)}$	
…						

sk is shift and goto state k
$r_{X\to\alpha}$ is reduce
gk is goto state k,

The LR Parsing Algorithm

- After a shift or reduce action we rerun the DFA on the entire stack
  - This is wasteful, since most of the work is repeated

- Remember for each stack element on which state it brings the DFA; use extra memory.

- LR parser maintains a stack

$$\langle \text{sym}_1, \text{state}_1 \rangle \ldots \langle \text{sym}_n, \text{state}_n \rangle$$

state_k is the final state of the DFA on $\text{sym}_1 \ldots \text{sym}_k$

The LR Parsing Algorithm

```
Let I = w$ be initial input
Let j = 0
Let DFA state 0 be the start state
Let stack = ⟨ dummy, 0 ⟩
    repeat
        case action[top_state(stack), I[j]] of
            shift k:  push ⟨ I[j++], k ⟩
            reduce X → A:
                pop |A| pairs,
                push ⟨X, Goto[top_state(stack), X]⟩
            accept: halt normally
            error: halt and report error
```

Key Issue: How is the DFA Constructed?

- The stack describes the context of the parse
  - What non-terminal we are looking for
  - What production rhs we are looking for
  - What we have seen so far from the rhs

- Each DFA state describes several such contexts
  - E.g., when we are looking for non-terminal E, we might be looking either for an int or a E + (E) rhs

LR(1) Items

- An LR(1) item is a pair:

$$X \to \alpha \bullet \beta,\ a$$

  - $X \to \alpha\beta$ is a production
  - a is a terminal (the lookahead terminal)
  - LR(1) means 1 lookahead terminal

- $[X \to \alpha \bullet \beta,\ a]$ describes a context of the parser
  - We are trying to find an X followed by an a, and
  - We have (at least) α already on top of the stack
  - Thus we need to see next a prefix derived from βa

Note

- The symbol ▶ was used before to separate the stack from the rest of input
  - α ▶ γ, where α is the stack and γ is the remaining string of terminals
- In items • is used to mark a prefix of a production rhs:

$$X \rightarrow \alpha \bullet \beta, a$$

  - Here β might contain terminals as well
- In both case the stack is on the left

Convention

- We add to our grammar a fresh new start symbol S and a production S → E
  - Where E is the old start symbol

- The initial parsing context contains:

$$S \rightarrow \bullet E,\ \$$$

  - Trying to find an S as a string derived from E$
  - The stack is empty

LR(1) Items (Cont.)

- In context containing

$$E \to E + \bullet (E), +$$

  - If (follows then we can perform a shift to context containing

$$E \to E + (\bullet E), +$$

- In context containing

$$E \to E + (E) \bullet, +$$

  - We can perform a reduction with $E \to E + (E)$
  - But only if a + follows

LR(1) Items (Cont.)

- Consider the item

  $E \rightarrow E + (\bullet E), +$

- We expect a string derived from E) +
- There are two productions for E

  $E \rightarrow \text{int}$ and $E \rightarrow E + (E)$

- We describe this by extending the context with two more items:

  $E \rightarrow \bullet\ \text{int},)$

  $E \rightarrow \bullet\ E + (E),)$

The Closure Operation

- Extending the context with items is called the <u>closure</u> operation.

```
Closure(Items) =
  repeat
    for each [X → α•Yβ, a] in Items
        for each production Y → γ
            for each b ∈ First(βa)
                add [Y → •γ, b] to Items
  until Items is unchanged
```

Constructing the Parsing DFA (1)

- Construct the start context: Closure({S → •E, $})

S → •E, $
E → •E+(E), $
E → •int, $
E → •E+(E), +
E → •int, +

- We abbreviate as:

S → •E, $
E → •E+(E), $/+
E → •int, $/+

Constructing the Parsing DFA (2)

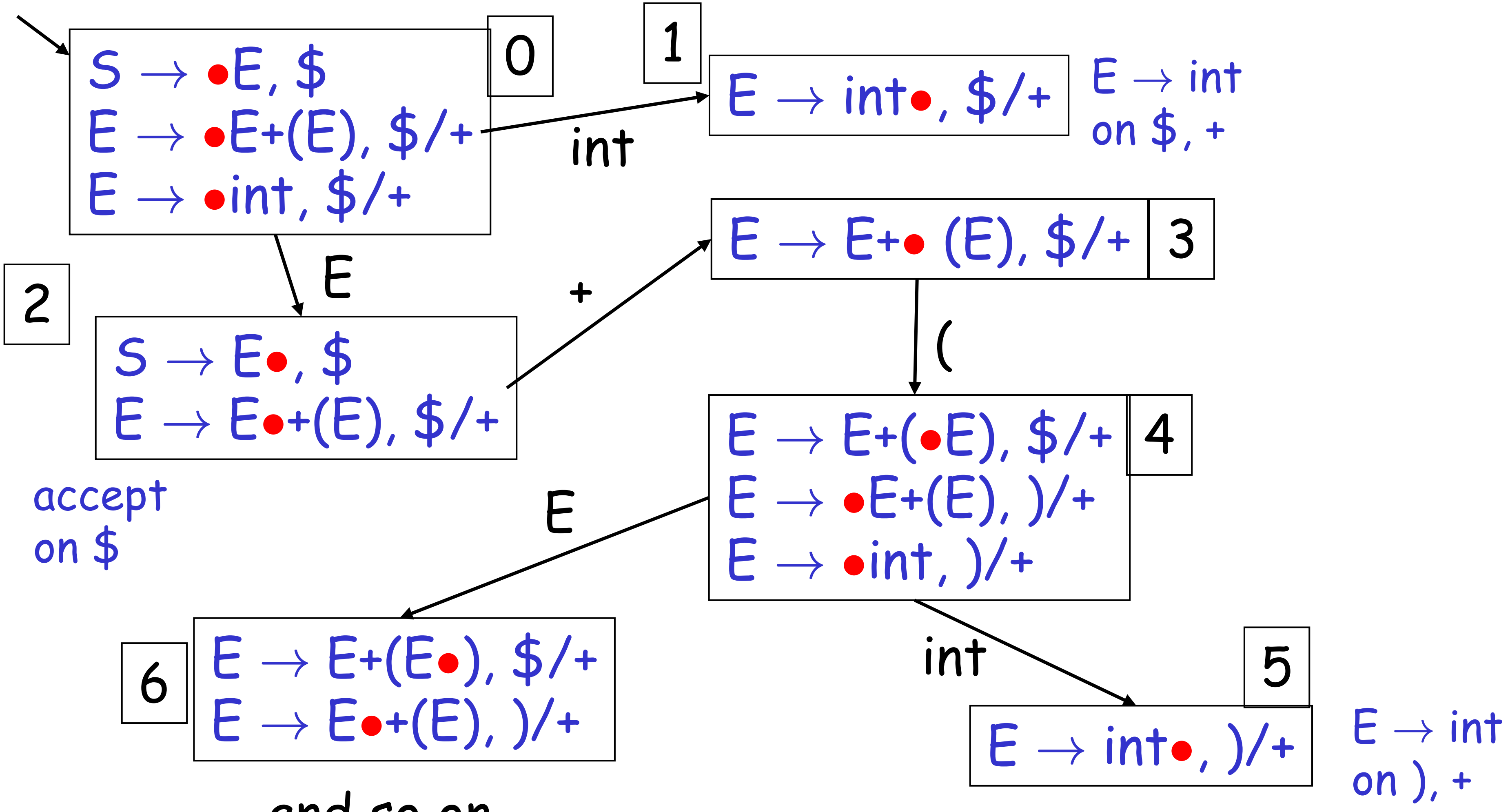
- A DFA state is a closed set of LR(1) items
- The start state contains [S $\rightarrow$ •E, \$]
- A state that contains [X $\rightarrow$ α•, b] is labeled with "reduce with X $\rightarrow$ α on b"
- And now the transitions …

The DFA Transitions

- A state "State" that contains $[X \to \alpha \bullet y\beta, b]$ has a transition labeled y to a state that the items "Transition(State, y)"
  - y can be a terminal or a non-terminal

```
Transition(State, y)
  Items $\leftarrow \emptyset$
  for each $[X \to \alpha \bullet y\beta, b] \in$ State
      add $[X \to \alpha y \bullet \beta, b]$ to Items
  return Closure(Items)
```

Constructing the Parsing DFA. Example.



0

S → •E, $

E → •E+(E), $/+

E → •int, $/+

int

1

E → int•, $/+

E → int

on $, +

E

2

S → E•, $

E → E•+(E), $/+

accept

on $

+

E → E+• (E), $/+

3

(

E → E+(•E), $/+

E → •E+(E),)/+

E → •int,)/+

4

E

6

E → E+(E•), $/+

E → E•+(E),)/+

int

5

E → int•,)/+

E → int

on), +

and so on…

LR Parsing Tables. Notes

- Parsing tables (i.e. the DFA) can be constructed automatically for a CFG

- Why study this at all in CS164? We still need to understand the construction to work with parser generators
  - E.g., they report errors in terms of sets of items

- What kind of errors can we expect?

Shift/Reduce Conflicts

- If a DFA state contains both

  $[X \to \alpha \bullet a\beta, b]$ and $[Y \to \gamma \bullet, a]$

- Then on input "a" we could either
  - Shift into state $[X \to \alpha a \bullet \beta, b]$, or
  - Reduce with $Y \to \gamma$

- This is called a <u>shift-reduce conflict</u>

Shift/Reduce Conflicts

- They are a typical symptom if there is an ambiguity in the grammar
- Classic example: the *dangling else*

  S → if E then S | if E then S else S | OTHER
- Will have DFA state containing

  [S → if E then S•, else]

  [S → if E then S• else S, x]
- If else follows then we can shift or reduce
- Default (bison, CUP, JLALR, etc.) is to shift
  - Default behavior is right in this case

More Shift/Reduce Conflicts

- Consider the ambiguous grammar

$$E \rightarrow E + E \mid E * E \mid \text{int}$$

- We will have the states containing

$$\begin{array}{lcl} {[E \rightarrow E * \bullet E,\ +]} & & {[E \rightarrow E * E \bullet,\ +]} \\ {[E \rightarrow \bullet E + E,\ +]} & \Rightarrow^{E} & {[E \rightarrow E \bullet + E,\ +]} \\ \dots & & \dots \end{array}$$

- Again we have a shift/reduce on input +
  - We need to reduce (* binds more tightly than +)
  - Solution: somehow impose the precedence of * and +

More Shift/Reduce Conflicts

Some parser generators (YACC, BISON) provide precedence declarations.

  - `Precedence left PLUS,`
  - `Precedence left TIMES`
  - `Precedence right EXP`

- Bison, YACC
  - Declare precedence and associativity:

```
%left +
%left *
```

More Shift/Reduce Conflicts

Our LALR generator doesn't do this. Instead, we "Stratify" the grammar. (Less explanation!)

E → E + E | E * E | int ;; original

New

E → E + E1 | E1 ;; E1+E would be right associative

E1 → E1 * int | int

(Many "layers" may be necessary for elaborate languages. (13 in C++, and some operators appear at several levels, e.g. "(". Some operators are right-associative like =, +=; most are left associative.)

Reduce/Reduce Conflicts

- If a DFA state contains both

  $[X \to \alpha\bullet, a]$ and $[Y \to \beta\bullet, a]$

  - Then on input "a" we don't know which production to reduce

- This is called a reduce/reduce conflict

Reduce/Reduce Conflicts

- Usually due to gross ambiguity in the grammar
- Example: a sequence of identifiers

$$S \rightarrow \varepsilon \mid id \mid id\ S$$

- There are two parse trees for the string id

$$S \rightarrow id$$

$$S \rightarrow id\ S \rightarrow id$$

- How does this confuse the parser?

More on Reduce/Reduce Conflicts

- Consider the states

$$\begin{array}{lcl}
 & & {[}S \to \mathrm{id} \bullet, \quad \$] \\
{[}S' \to \bullet\, S, \quad \$] & & {[}S \to \mathrm{id} \bullet S, \quad \$] \\
{[}S \to \bullet, \quad \$] & \Rightarrow^{\mathrm{id}} & {[}S \to \bullet, \quad \$] \\
{[}S \to \bullet\, \mathrm{id}, \quad \$] & & {[}S \to \bullet\, \mathrm{id}, \quad \$] \\
{[}S \to \bullet\, \mathrm{id}\ S, \quad \$] & & {[}S \to \bullet\, \mathrm{id}\ S, \quad \$]
\end{array}$$

- Reduce/reduce conflict on input $\$$

$$S' \to S \to \mathrm{id}$$

$$S' \to S \to \mathrm{id}\ S \to \mathrm{id}$$

- Better rewrite the grammar: $S \to \varepsilon \mid \mathrm{id}\ S$

Using Parser Generators

- Parser generators construct the parsing DFA given a CFG
  - Use precedence declarations and default conventions to resolve conflicts
  - The parser algorithm is the same for all grammars (and is provided as a library function)
- But most parser generators do not construct the DFA as described before
  - Because the LR(1) parsing DFA has 1000s of states even for a simple language

LR(1) Parsing Tables are Big

- But many states are similar, e.g.

1

$E \rightarrow$ int•, \$/+

$E \rightarrow$ int on \$, +

and

5

$E \rightarrow$ int•,)/+

$E \rightarrow$ int on), +

- Idea: merge the DFA states whose items differ only in the lookahead tokens
  - We say that such states have the same core
- We obtain

1′

$E \rightarrow$ int•, \$/+/)

$E \rightarrow$ int on \$, +,)

The Core of a Set of LR Items

- Definition: The core of a set of LR items is the set of first components
  - Without the lookahead terminals

- Example: the core of

$$\{ [X \rightarrow \alpha \bullet \beta, b], [Y \rightarrow \gamma \bullet \delta, d]\}$$

is

$$\{X \rightarrow \alpha \bullet \beta,\ Y \rightarrow \gamma \bullet \delta\}$$

LALR States

- Consider for example the LR(1) states

  $\{[X \to \alpha\bullet, a], [Y \to \beta\bullet, c]\}$

  $\{[X \to \alpha\bullet, b], [Y \to \beta\bullet, d]\}$

- They have the same core and can be merged and the merged state contains:

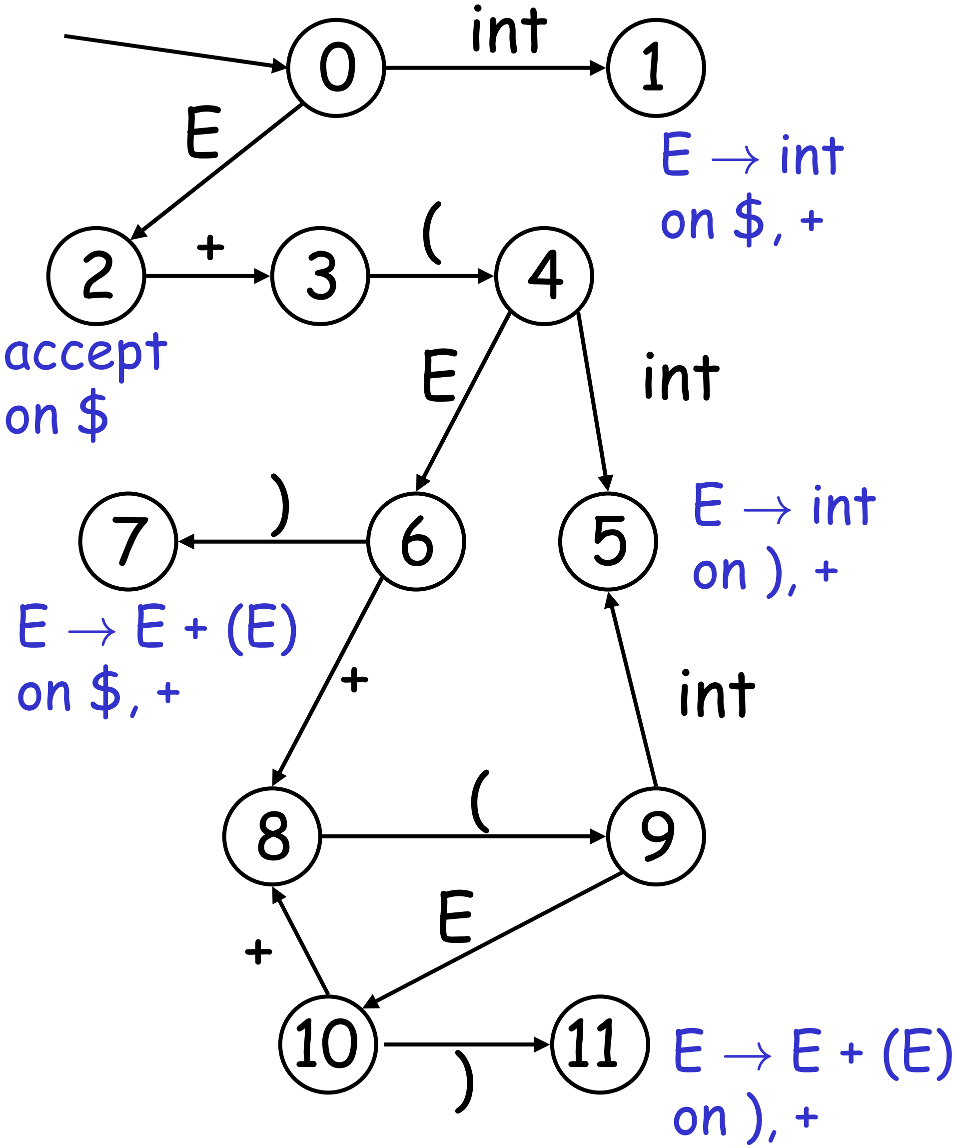
  $\{[X \to \alpha\bullet, a/b], [Y \to \beta\bullet, c/d]\}$

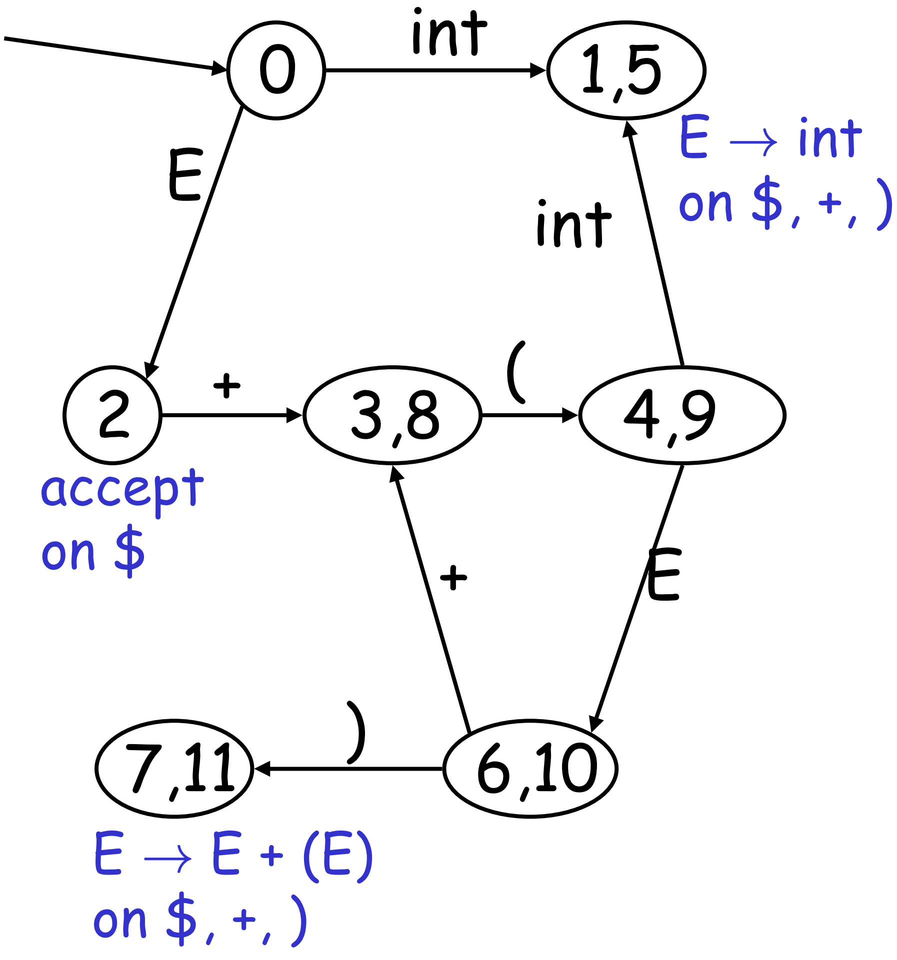
- These are called LALR(1) states
  - Stands for LookAhead LR
  - Typically 10 times fewer LALR(1) states than LR(1)

A LALR(1) DFA

- Repeat until all states have distinct core
  - Choose two distinct states with same core
  - Merge the states by creating a new one with the union of all the items
  - Point edges from predecessors to new state
  - New state points to all the previous successors

Conversion LR(1) to LALR(1). Example.





The LALR Parser Can Have Conflicts

- Consider for example the LR(1) states

  $\{[X \to \alpha\bullet, a], [Y \to \beta\bullet, b]\}$

  $\{[X \to \alpha\bullet, b], [Y \to \beta\bullet, a]\}$

- And the merged LALR(1) state

  $\{[X \to \alpha\bullet, a/b], [Y \to \beta\bullet, a/b]\}$

- Has a new reduce-reduce conflict

- In practice such cases are rare

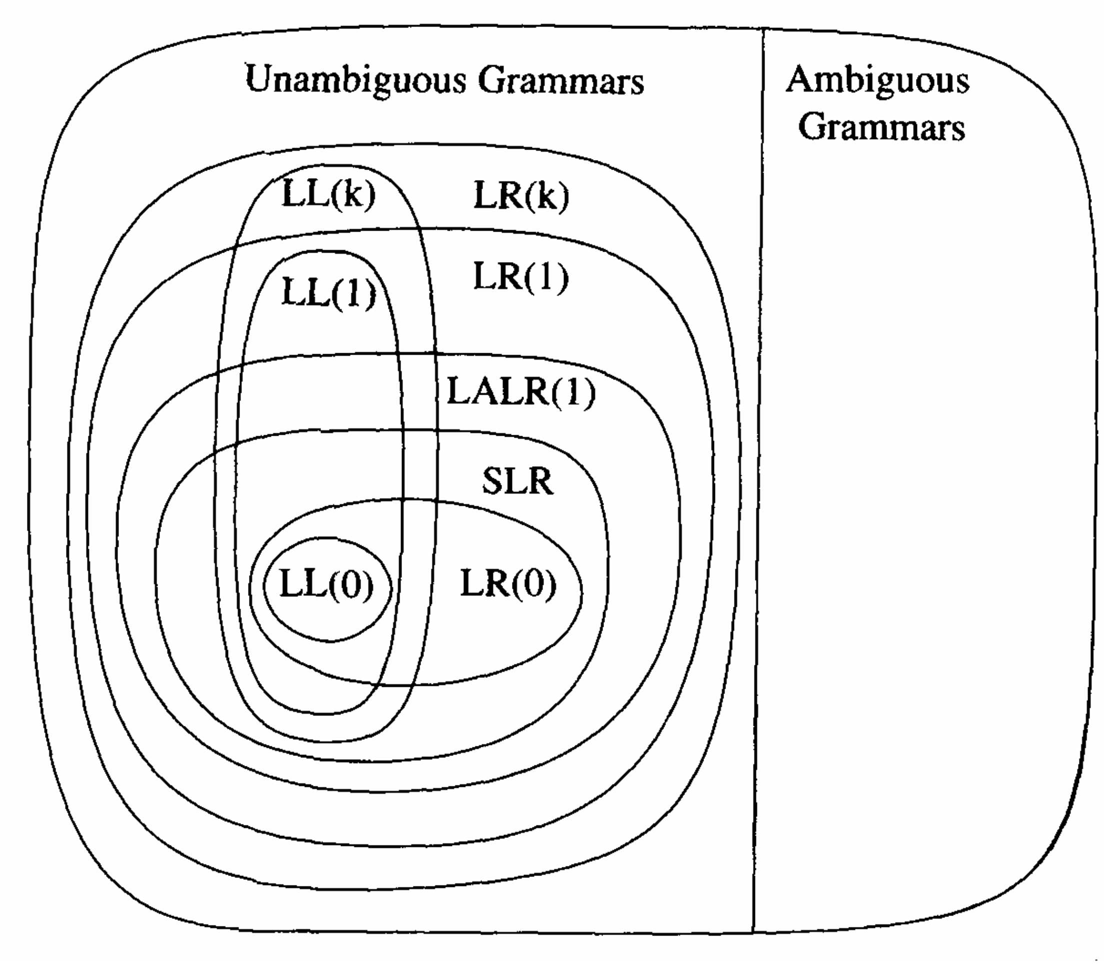
LALR vs. LR Parsing

- LALR languages are not “natural”
  - They are an efficiency hack on LR languages

- You may see claims that any reasonable programming language has a LALR(1) grammar, {Arguably this is done by defining languages without an LALR(1) grammar as unreasonable ☺ }.

- In any case, LALR(1) has become a standard for programming languages and for parser generators, in spite of its apparent complexity.

A Hierarchy of Grammar Classes



From Andrew Appel, “Modern Compiler Implementation in Java”

Notes on Parsing

- Parsing
  - A solid foundation: context-free grammars
  - A simple parser: LL(1)
  - A more powerful parser: LR(1)
  - An efficiency hack: LALR(1)
  - LALR(1) parser generators

- Next time we move on to semantic analysis

Notes on Lisp LALR generator

- lalr.cl is source code; lalr.doc additional documentation.
- A complete parse table can be viewed by
- (setf p (makeparser G lexforms nil))
- (Print-Table stateList)
- (eval p) ;; create the parser named LALR-parser

Sample input for lalr generator

```
(defparameter G2 '(
    (exp --> exp + term      #'(lambda(exp n term)(list '+ exp term)))
    (exp --> exp - term      #'(lambda(exp n term)(list '- exp term)))
    (exp --> term            #'(lambda(term) term))
    (term --> term * factor  #'(lambda(term n fac)(list '* term fac)))
    (term --> factor         #'(lambda(factor) factor))
    (factor --> id           #'(lambda(id) (const-value id)))
    (factor --> |(| exp |)|  #'(lambda(p1 exp p2) exp))
    (factor --> iconst       #'(lambda(iconst) (const-value iconst)))
    (factor --> bconst       #'(lambda(bconst) (const-value bconst)))
    (factor --> fconst       #'(lambda(fconst) (const-value fconst)))))

(defparameter lexforms ‘( + - * |(| |)| id iconst bconst fconst))

(make-parser G2 lexforms nil)
```

Sample table-output for lisp lalr generator

```
STATE-0:
  $Start --> . exp, nil
    On fconst shift STATE-14
    On bconst shift STATE-13
    On iconst shift STATE-12
    On ( shift STATE-7
    On id shift STATE-6
    On factor shift STATE-11
    On term shift STATE-16
    On exp shift STATE-1

STATE-1:
  $Start --> exp ., nil
  exp --> exp . + term, + - nil
  exp --> exp . - term, + - nil
    On + shift STATE-9
    On - shift STATE-2
    On nil reduce exp --> $Start   … up to state 16
```

Each state is embodied in a subroutine

Defined locally in one main program via “labels”
Using local subroutines that shift, reduce, peek at next input
Main parser is called by (lalr-parser #’next-input #’error)

Any number of parsers can be set up in the same environment, though usually only one is tested… I just try out some input

```
(parse-fl ‘( (id a) + (id b)))
```

```
;; if there is a problem, edit the grammar, say G2, then
```

```
(remake G2)
```

```
(remakec G2) ;; COMPILES lalr-parser. Parser runs 20X faster or so.
```

Sample output program for lalr generator

```
(defun lalr-parser (next-input parse-error)
  (let ((cat-la 'nil) (val-la 'nil) (val-stack 'nil) (state-stack 'nil))
    (labels ((input-peek nil …;;these 3 subprograms are standard
             (shift-from (name) …
             (reduce-cat (name cat ndaughters action)…
             (STATE-0 nil  ;; generated specifically from grammar
               (case (input-peek)
                 (fconst (shift-from #'STATE-0) (STATE-14))
                 …
                 (exp (shift-from #'STATE-0) (STATE-1))
                 (otherwise (funcall parse-error))))
             (STATE-1 nil
               (case (input-peek)
                 (+ (shift-from #'STATE-1) (STATE-9))
                 (- (shift-from #'STATE-1) (STATE-2))
                 ((nil) (reduce-cat #'STATE-1 '$Start 1 nil))
                 (otherwise (funcall parse-error))))
         …;etc etc
```

Supplement to LR Parsing

Strange Reduce/Reduce Conflicts
Due to LALR Conversion

Strange Reduce/Reduce Conflicts

- Consider the grammar

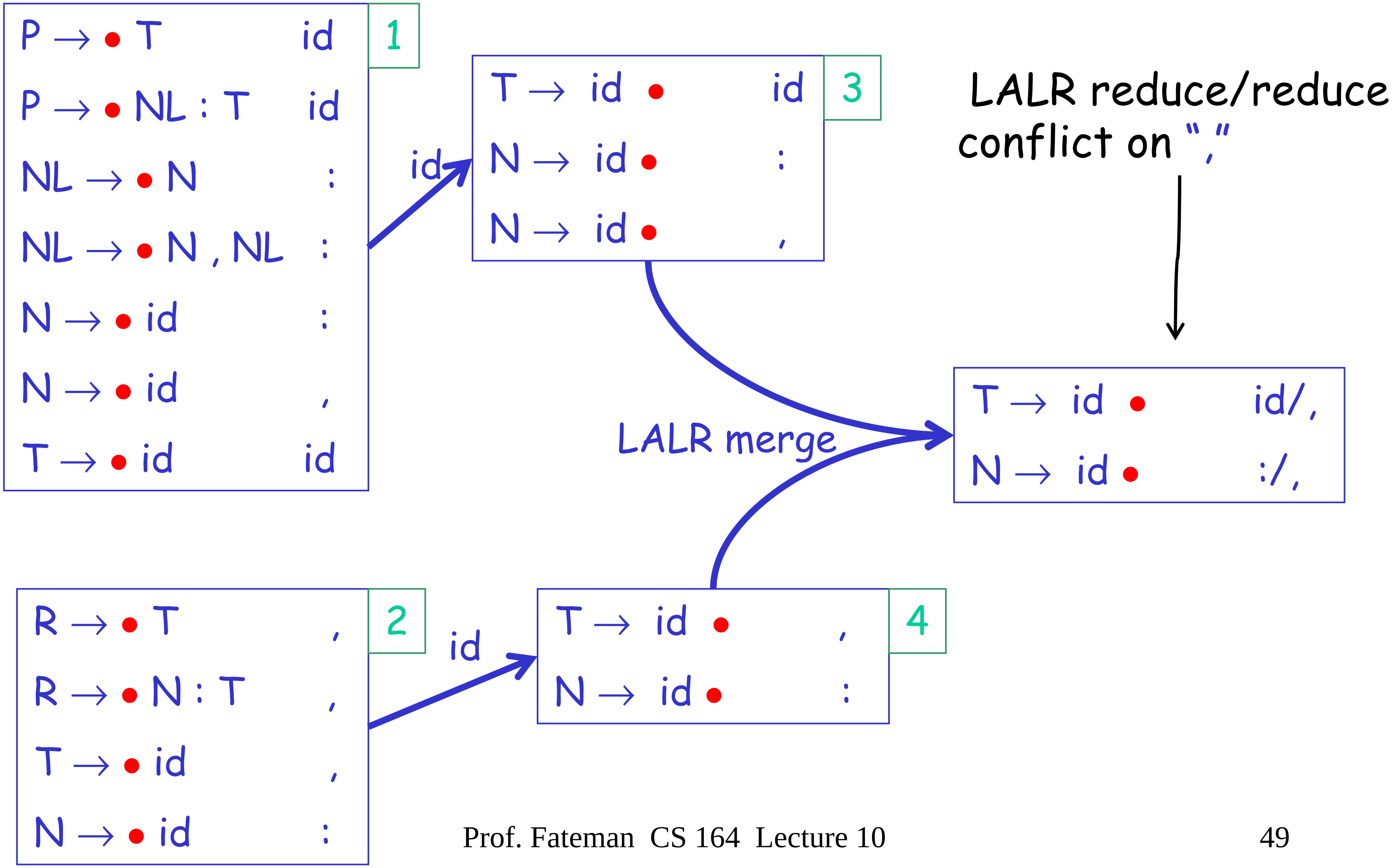
S → P R ,
NL → N | N , NL
P → T | NL : T
R → T | N : T
N → id
T → id

- **P** - parameters specification
- **R** - result specification
- **N** - a parameter or result name
- **T** - a type name
- **NL** - a list of names

Strange Reduce/Reduce Conflicts

- In P an id is a
  - N when followed by , or :
  - T when followed by id
- In R an id is a
  - N when followed by :
  - T when followed by ,
- This is an LR(1) grammar.
- But it is not LALR(1). Why?
  - For obscure reasons

A Few LR(1) States



1

P → • T id
P → • NL : T id
NL → • N :
NL → • N , NL :
N → • id :
N → • id ,
T → • id id

id

3

T → id • id
N → id • :
N → id • ,

LALR reduce/reduce conflict on ","

T → id • id/,
N → id • :/,

LALR merge

2

R → • T ,
R → • N : T ,
T → • id ,
N → • id :

id

4

T → id • ,
N → id • :

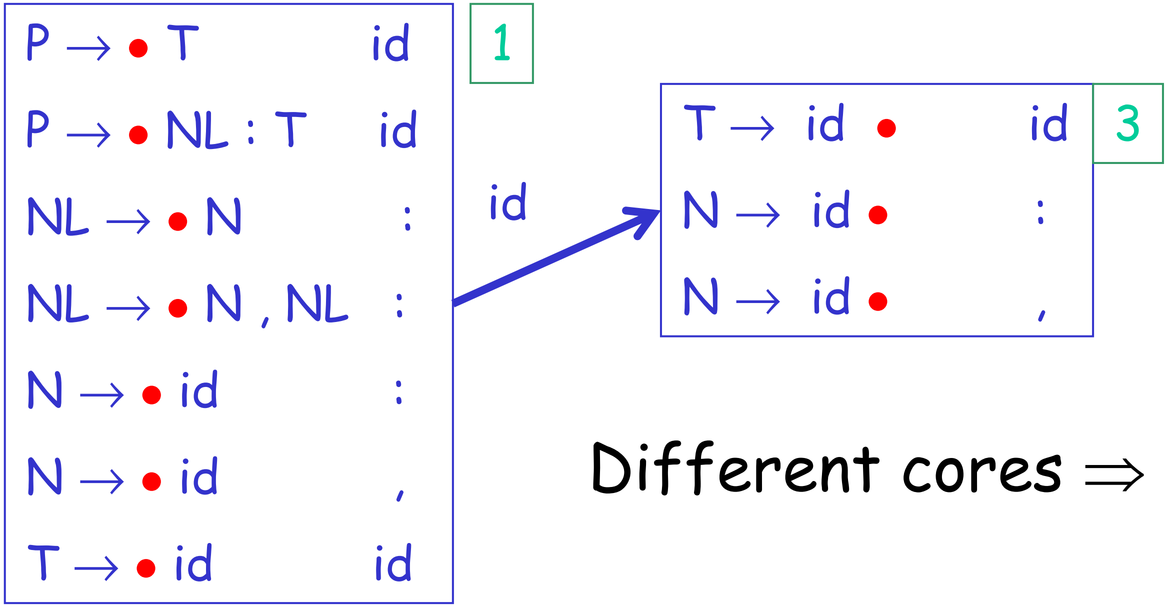
What Happened?

- Two distinct states were confused because they have the same core
- Fix: add dummy productions to distinguish the two confused states
- E.g., add

  $R \rightarrow$ id bogus

  - bogus is a terminal not used by the lexer
  - This production will never be used during parsing
  - But it distinguishes R from P

A Few LR(1) States After Fix

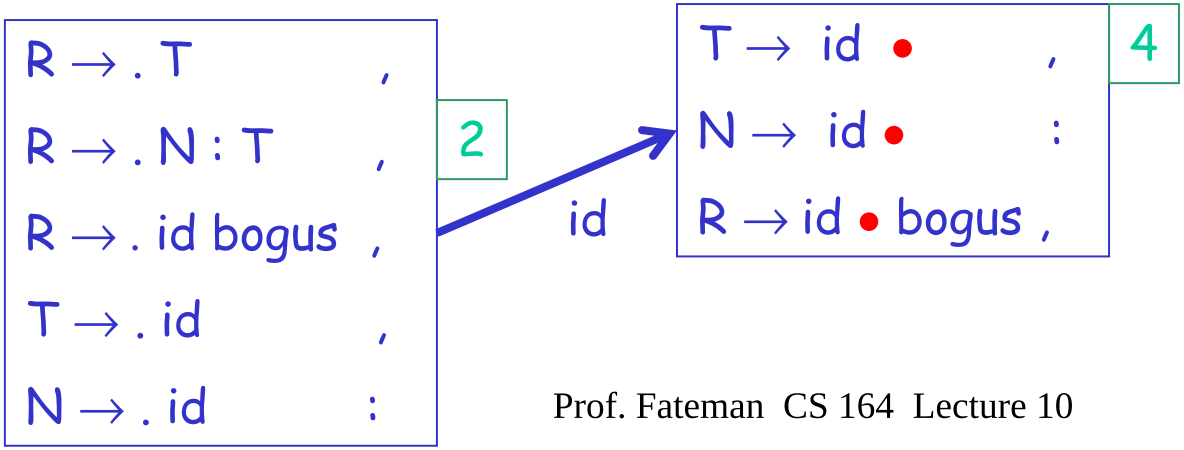


P → • T	id	1
P → • NL : T	id	
NL → • N	:	
NL → • N , NL	:	
N → • id	:	
N → • id	,	
T → • id	id	

id

T → id •	id	3
N → id •	:	
N → id •	,	

Different cores ⇒ no LALR merging



R → . T	,	2
R → . N : T	,	
R → . id bogus	,	
T → . id	,	
N → . id	:	

id

T → id •	,	4
N → id •	:	
R → id • bogus	,	