Implementation of Regular Expression Recognizers

CS164

Lecture 6

Outline

- Testing for membership in a “regular” language.
- Specifying lexical structure using regular expressions. A FORMAL high-level approach.
- Could be automatically programmed from spec.

- Finite automata: a “machine” description
  - Deterministic Finite Automata (DFAs)
  - Non-deterministic Finite Automata (NFAs)
  - Implemented in software (but could be in hardware!)
- Implementation of regular expressions as programs

  RegExp => NFA => DFA => Tables or programs

Common Notational Extensions

- There are various extensions used in regular expression notation; this uses up more meta characters but we can generally manage it by escape/quotes when we need them...
- Union: A | B ≡ A + B
- Optional: A + ε ≡ A?
- Sequence: A B ≡ A B
- Kleene Star: A* ≡ A*
- Parens used for grouping: (A+B)C ≡ AC+BC
- Range: 'a'+'b'+…+'z' ≡ [a-z]
- Excluded range:

  complement of [a-z] ≡ [^a-z]

Examples of REs

- R := (0+1)*ab*a
- S := [a-z]([a-z]+[0-9])*

- Described in English:
- an element of R starts optionally with a string of any combination of the digits 0 or 1 of any length, followed by exactly one a then optionally some number of b characters and then an a.
- What is S?

Let's get real

- Do we want yet another language to parse, the language of regular expressions, where A|BC has to be disambiguated? {Is this (A|B)C or A|(BC) ? Is ab* the same as (ab)* or a(b*)? }
- What a mathematician can complicate with notation, we can make more easily constructive by using computer notation.
- What notation is that??

Notation extensions

- We can use lisp…
- Union: A | B ≡ (union A B)
- Option: A + ε ≡ (union A eps)
- Range: 'a'+'b'+…+'z' ≡ alphachar
- Sequence: A B ≡ (seq A B)
- Kleene Star: A* ≡ (star A)
- Excluded range:

  complement of A ≡ (not A)

Notation extensions

Examples in lisp

- (0+1)*(ab*a).
  - (seq (star(union 0 1))(seq a (star b) a))
  - (seq (star(union 0 1)) a (star b) a)
- [a-z]([a-z]+[0-9])*
  - (seq alphachar (star (union alphachar digitchar)))

Regular Expressions in Lexical Specification

- Last lecture: a specification for the predicate

$$s \in L(R)$$

- But a yes/no answer is not enough !
- Instead: we want to partition the input into tokens.

- Tradition is to write an algorithm based on partitioning by regular expressions.

Regular Expressions => Lexical Spec. (1)

1. Select a set of tokens
   - Number, Keyword, Identifier, ...

2. Write a rexp for the lexemes of each token
   - Number = digit^{+}
   - Keyword = 'if' + 'else' + ...
   - Identifier = letter (letter + digit)*
   - OpenPar = '('
   - ...

Regular Expressions => Lexical Spec. (2)

3. Construct R, matching all lexemes for all tokens (and a pattern for everything else..)

$$R = \text{Keyword} + \text{Identifier} + \text{Number} + \ldots$$
$$= R_1 + R_2 + \ldots + R_n = \text{rathole}$$

Facts: If $s \in L(R)$ then s is a lexeme

- Furthermore $s \in L(R_i)$ for some "i"
- This "i" determines the token that is reported

Regular Expressions => Lexical Spec. (3)

4. Let input be $x_1 \ldots x_n$, a SEQUENCE of CHARS
   - (x_1 ... x_n are individual characters)
   - For $1 \leq k \leq n$ check

$$x_1 \ldots x_k \in L(R) \,?$$

5. It must be that

   $x_1 \ldots x_k \in L(R_j)$ for some j , so it is a type-j token

   Remove $x_1 \ldots x_k$ from input and go to (4)

How to Handle Spaces and Comments?

1. We could create a token Whitespace

   Whitespace = (' ' + '\n' + '\t')$^+$

   – We could also add comments in there
   – An input "   \t\n   5555   " is transformed into Whitespace Integer Whitespace

2. Alternatively, Lexer skips spaces (preferred)

   • Modify step 5 from before as follows:
   It must be that $x_k \ldots x_i$ – $L(R_j)$ for some j such that $x_1 \ldots x_{k-1}$ – $L(\text{Whitespace})$
   • Parser is not bothered with (extra) spaces

Ambiguities (1)

- There are ambiguities in the algorithm

- How much input is used? What if
    - $x_1 \ldots x_i \in L(R)$ and also
    - $x_1 \ldots x_K \in L(R)$ for k>i
  - One possible Rule: Pick the longest possible substring
  - The "maximal munch"

Ambiguities (2)

- Which token is used? What if
    - $x_1 \ldots x_i \in L(R_j)$ and also
    - $x_1 \ldots x_i \in L(R_k)$
  - Another possible rule: use rule listed first (j if j < k)

- Example:
  - R_1 = Keyword and R_2 = Identifier
  - "if" matches both.
  - Treats "if" as a keyword not an identifier (many languages just tell user: don't use keyword as identifier.)

Error Handling

- What if

  No rule matches a prefix of input ?
- Problem: Can't just get stuck …
- Solution:
  - Write a rule matching all "bad" strings
  - Put it last (remember, R_n = rathole…)
- Lexer tools allow the writing of:

  $R = R_1 + \ldots + \text{Error}$
  - Token Error matches if nothing else matches

Summary

- Regular expressions provide a concise notation for string patterns
- Use in lexical analysis requires small extensions
  - To resolve ambiguities
  - To handle errors
- Good algorithms known (e.g. r.e. →lexer)
  - Require only single pass over the input
  - Few operations per character (table lookup)

Finite Automata

- Regular expressions = specification
- Finite automata = closer to implementation
- *---(Singular: automaton. Plural: automata.)*
- A finite automaton or (D)FA is an abstraction consisting of
  - An input alphabet Σ
  - A set of states S
  - A start state n
  - A set of accepting states $F \subseteq S$
  - A set of transitions $\text{state}_1 \rightarrow^{\text{input}} \text{state}_2$

Finite Automata

- Transition

$$s_1 \rightarrow^a s_2$$

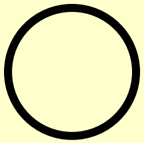
- Is read

  In state s_1 on input a go to state s_2

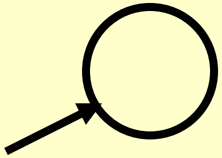
- If end of input (or no transition possible)
  - If in accepting state => accept
  - Otherwise => reject

Finite Automata State Graphs

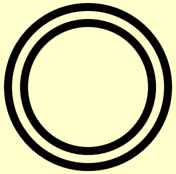
- A state



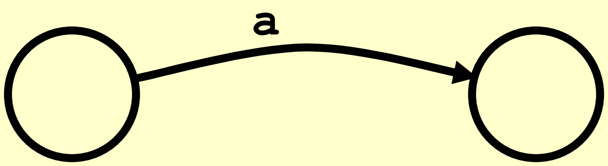
- The start state



- An accepting state

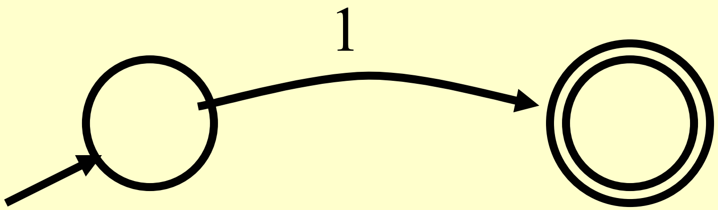


- A transition



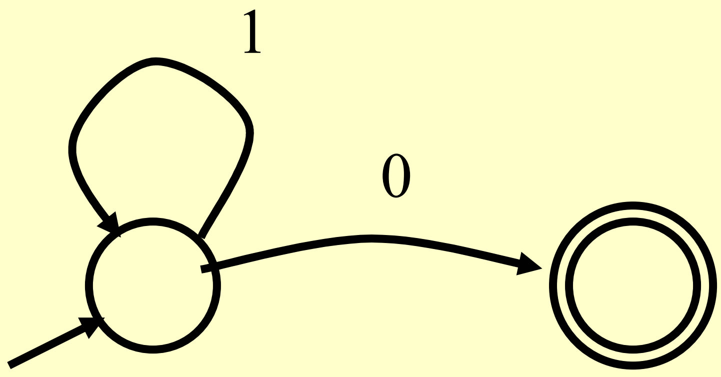
A Simple Example

- A finite automaton that accepts only "1"



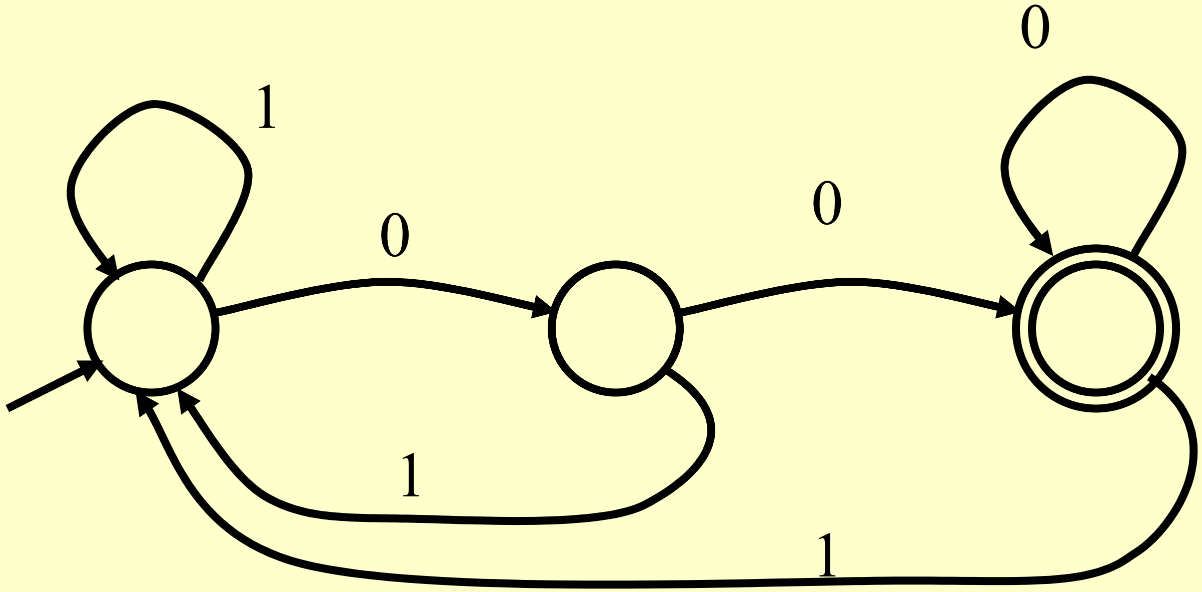
Another Simple Example

- A finite automaton accepting any number of 1's followed by a single 0
- Alphabet: {0,1}; as a RegExp: 1*0



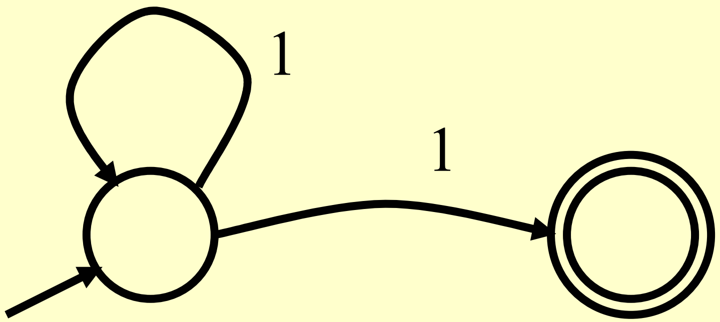
And Another Example

- Alphabet {0,1}
- What language does this recognize?



And Another Example

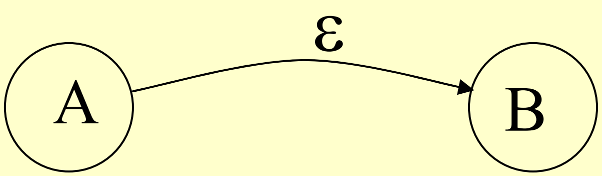
- Alphabet still { 0, 1 }



- The operation of the automaton is not completely defined by the input
  - On input "11" the automaton could be in either state

Epsilon Moves

- Another kind of transition: ε-moves



- Machine can move from state A to state B without reading input. Which state is it really in?

Deterministic and Nondeterministic Automata

- Deterministic Finite Automata (DFA)
  - One transition per input per state
  - No ε-moves
- Nondeterministic Finite Automata (NFA)
  - Can have multiple transitions for one input in a given state
  - Can have ε-moves
- Either kind of finite automaton has finite memory
  - Need only to encode the current state(s)

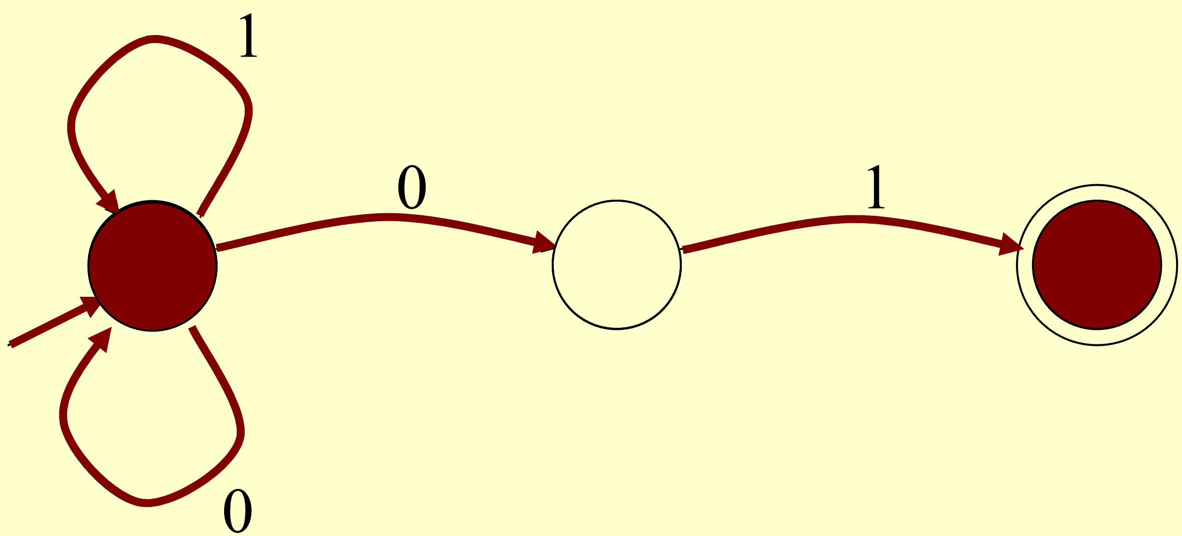
Execution of Finite Automata

- A DFA can take only one path through the state graph
  - Completely determined by input
- One could think that NFAs can "choose"
  - Whether to make ε-moves
  - Which of multiple transitions for a single input to take

Actually, NFAs do not have free will. It would be more accurate to say an execution of an NFA marks "all" choices from a set of states to a new set of states..

Acceptance of NFAs

- An NFA can be “in multiple states”



- Input: 1 0 1
- Rule: NFA accepts if <u>at least one</u> of its current states is a final state

NFA vs. DFA (1)

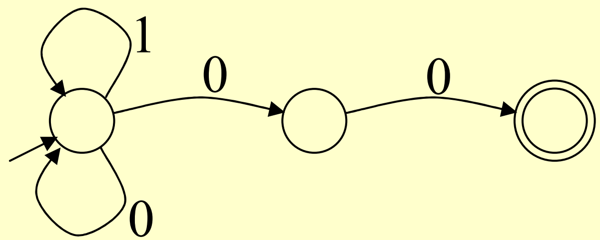
- NFAs and DFAs have the same abstract power to recognize languages. Namely the same set of regular languages.

- DFAs are easier to implement naively as a program
- NFAs can always be converted to DFAs

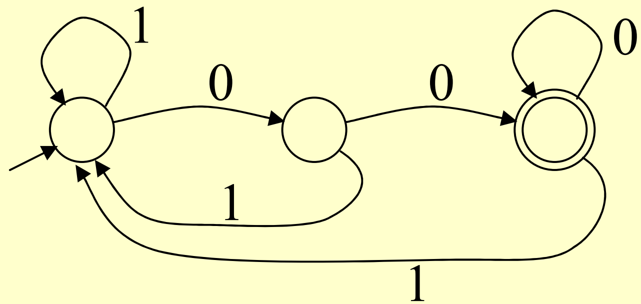
NFA vs. DFA (2)

- For a given language the NFA can be simpler than the DFA

NFA



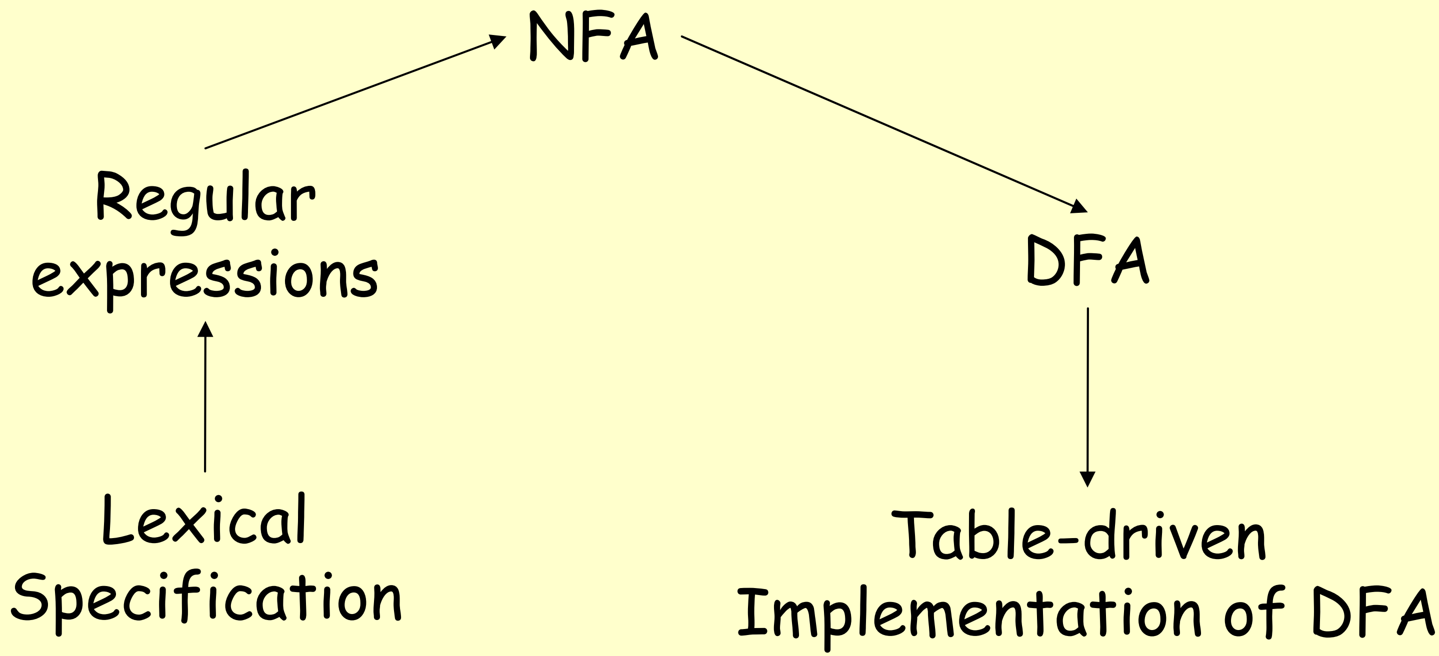
DFA



- DFA can be exponentially larger than NFA (n states in a NFA could require as many as 2^n states in a DFA)

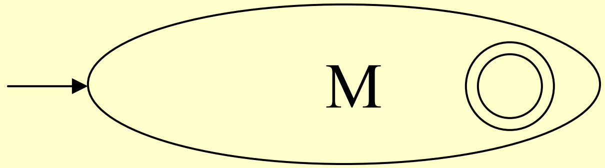
Regular Expressions to Finite Automata

- High-level sketch

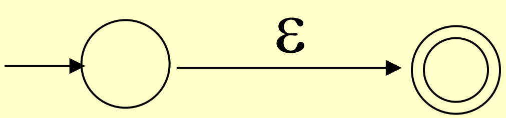


Regular Expressions to NFA (1)

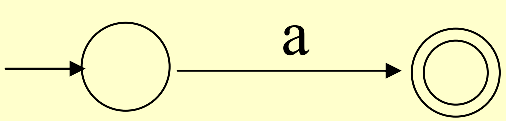
- For each kind of rexp, define an NFA
  - Notation: NFA for rexp M



- For ε

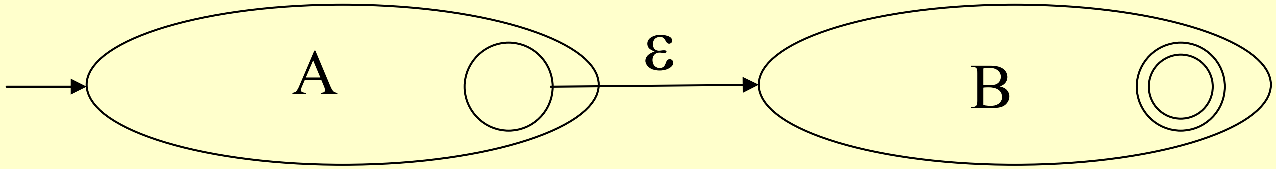


- For input a

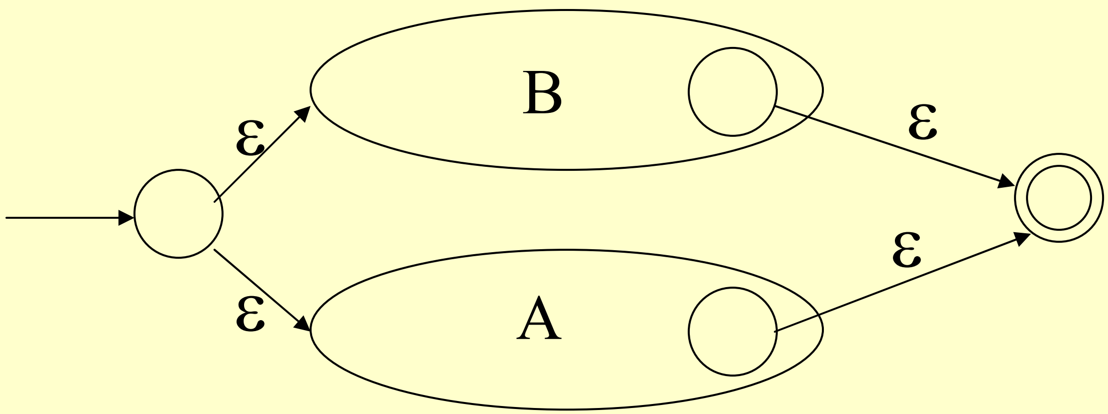


Regular Expressions to NFA (2)

- For AB

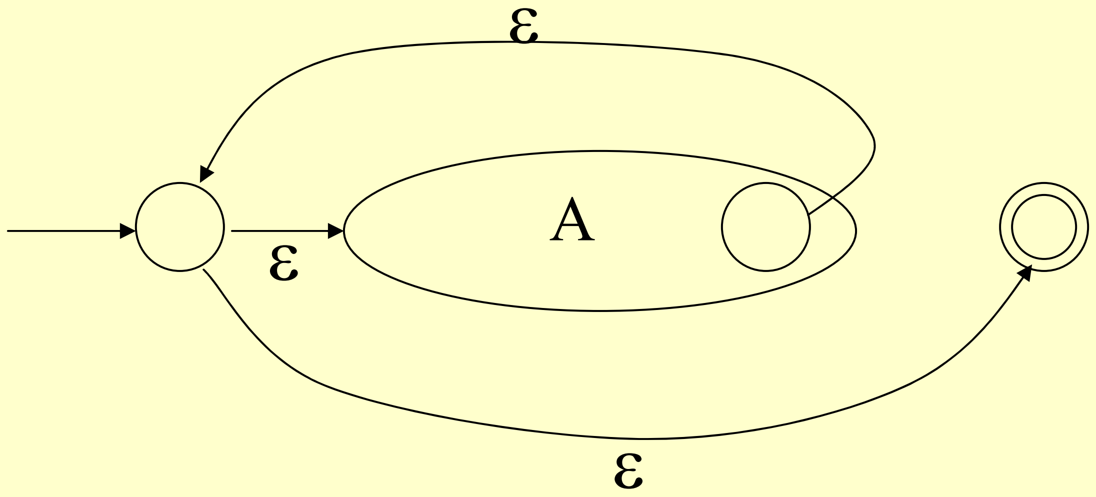


- For A + B



Regular Expressions to NFA (3)

- For *A**

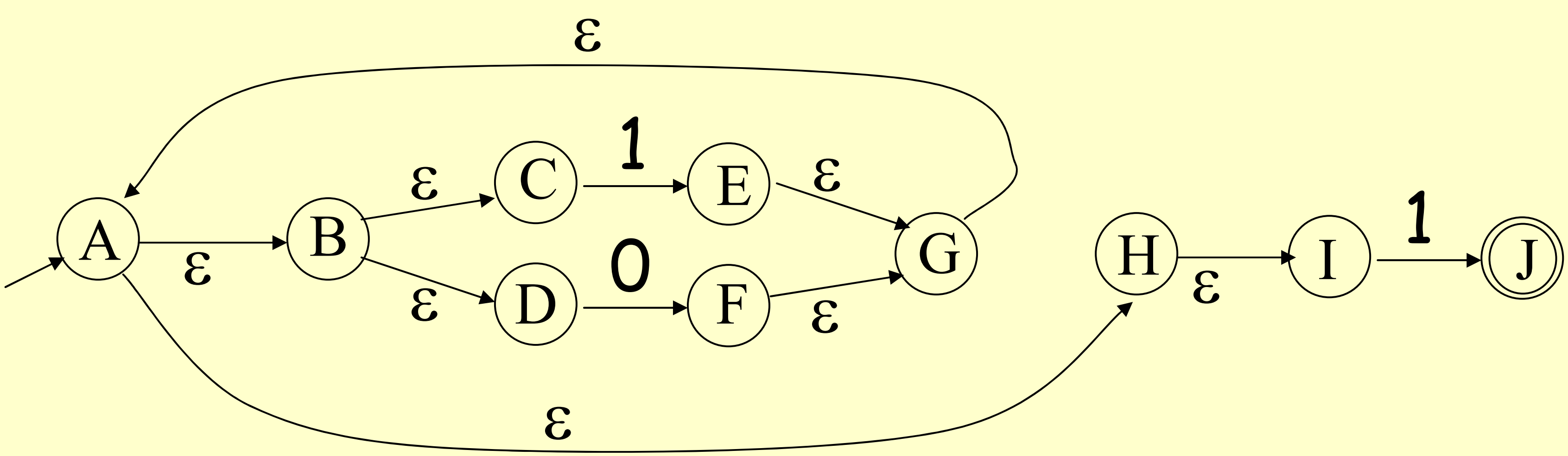


Example of RegExp -> NFA conversion

- Consider the regular expression

  (1+0)*1

- The NFA is



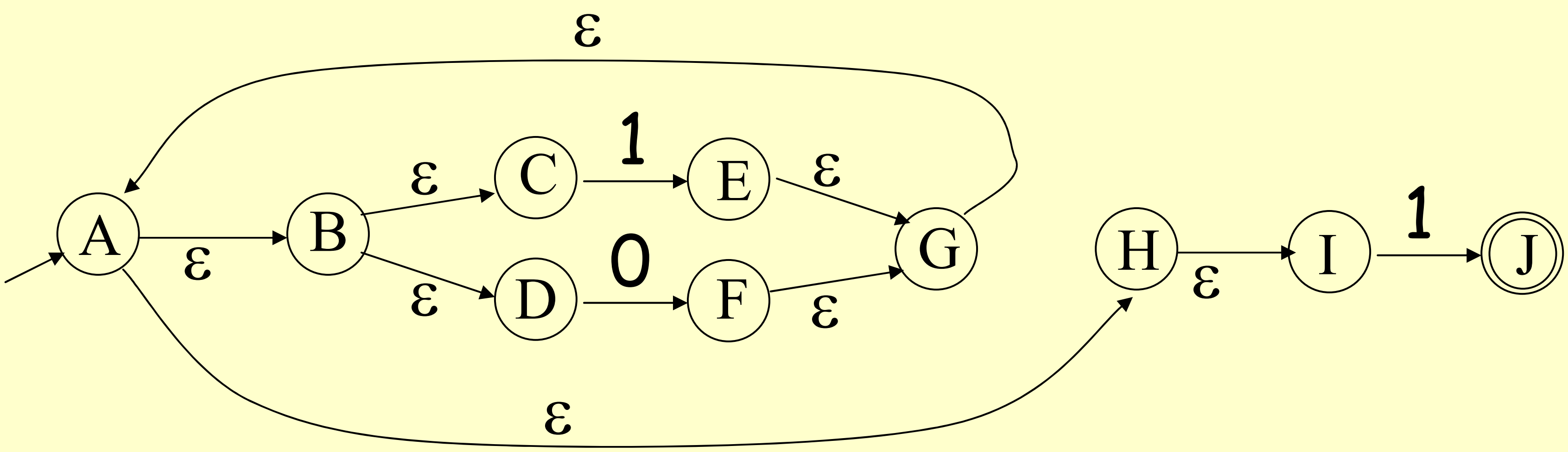
NFA to DFA. The Trick

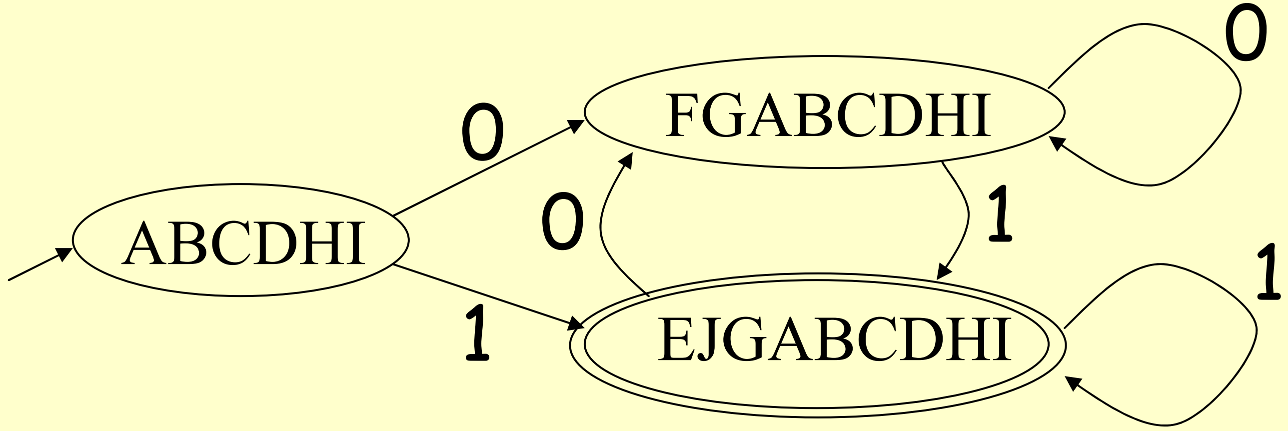
- Simulate the NFA
- Each state of DFA
  = a non-empty subset of states of the NFA
- Start state
  = the set of NFA states reachable through ε-moves from NFA start state
- Add a transition $S \to^{a} S'$ to DFA iff
  - S' is the set of NFA states reachable from <u>any</u> state in S after seeing the input a
    - considering ε-moves as well

NFA to DFA. Remark

- An NFA may be "in many states" at one time
- How many different states ?
- If there are N states, the NFA must be in some subset of those N states
- How many subsets are there (at most)?
  - $2^N - 1$ = finitely many, but usually much more than N

NFA -> DFA Example

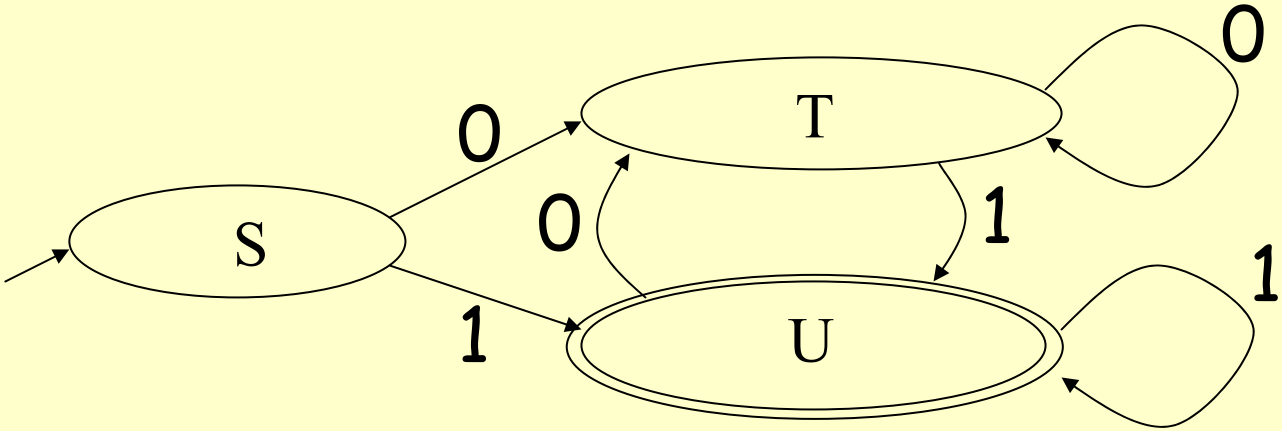




Implementation

- A DFA can be implemented by a 2D table T
  - One dimension is "states"
  - Other dimension is "input symbols"
  - For every transition $S_i \to^a S_k$ define T[i,a] = k
- DFA "execution"
  - If in state S_i and input a, read T[i,a] = k and skip to state S_k
  - Very efficient

Table Implementation of a DFA



state inputs

	0	1
S	T	U
T	T	U
U	T	U

Implementation (Cont.)

- NFA -> DFA conversion is at the heart of tools such as flex.
- But, DFAs can be huge.
- In practice, flex-like tools trade off speed for space in the choice of NFA and DFA representations.
- Oh, there can be many extra states, and usually are, in an auto-generated DFA. Can be mechanically reduced to a minimum number of states, but still may be huge.

Writing a DFA in Lisp

•

```
;;; -*- Mode: Lisp; Syntax: Common-Lisp -*-

;;; A simple finite state machine (fsm) simulator
;;; Note FSM is the same as a DFA (deterministic finite automaton).

;;; Reference to MCIJ is "Modern Compiler Implementation in Java"
;;; by Andrew Appel.

;;; First we show a deterministic finite state machine fsm, then a
;;; non-deterministic fsm: nfsm then a version of nfsm allowing
;;; "epsilon" transitions.

;;;First with no data abstractions. We decide on the representation
;;; and program away.  The correspondence of (state,input) --> next
;;; state is recorded in an association list, as illustrated below.

(defstruct (state (:type list)) transitions final)
;;first use of defstruct
```

Set up Mach1 with 3 states

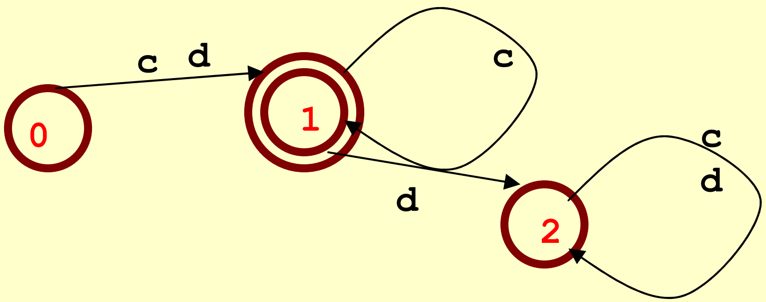
```
(setf Mach1 (make-array 3))

;;The first machine, with 3 states we will denote 0,1,2 will be stored
;; in an array called Mach1.  This machine accepts (c+d)c* and that's all

(setf (aref Mach1 0)                          ; initial state
  (make-state :transitions
            '((#\c 1)  ;; if you read a c go to state 1
              (#\d 1)) ;; if you read a d go to state 1
            ;; if you read anything else it is a error
            :final nil))

(setf (aref Mach1 1)
  (make-state :transitions
            '((#\c 1)
              (#\d 2))
            :final t))

(setf (aref Mach1 2)     ;; dead end state. no way out
  (make-state :transitions
            '( (#\c 2)  ;
               (#\d 2))
            :final nil))
```



FSM program in lisp

```
;; fsm simulates a deterministic finite state machine.
;; given a state number 0,1,2,... returns t for accept, nil for reject.

(defun fsm (state state-table input)
  (cond ((string= input "")
         (state-final (aref state-table state)))
        (t(let ((trans
                 (assoc
                  (elt input 0)
                  (state-transitions
                   (aref state-table state)))))
            (and trans (fsm (cadr trans) state-table (subseq input
1)))))))

;; that's all.  See file fsm.cl for many fluffed-up abstractions,
;; comments, and extensions to NFA
```

Actually, we can write lexers rather simply

- Although RegExps / DFAs/ NFAs are neat, and we teach them in CS164, we are writing lexers on digital computers with <u>memory</u>.
- These are more powerful than DFAs.
- An entirely reasonable lexer can be written using (what amounts to) recursive descent parsing, (later in course!) but in such a simple form that it hardly needs explanation.
- If we insist on automated tools, we can compile patterns into programs simply, too.

Writing stuff in Lisp

- I’d feel bad if too much of this course is specifically about details of Lisp (or for that matter about any particular language)
- But there are features and design issues raised by how Lisp works.
- Some details are inevitably needed… how to read, print, stop loops.
- File: readprintrex (mostly text); iterate.cl

RegExps in Lisp. A recipe for matchers

- Say we want to write a clear metalanguage for RegExps so we can automatically build specific recognizer programs. Like flex. But we will write it in 2 pages of Lisp you can read.
- Step one: Come up with a formal "grammar" for regexps that can be "parsed".
- Step two: Write a parser than produces as output a Lisp program that implements the recognizer.

A data language for constructing REs

- "abc" is the language {"abc"}
- stwildcard matches any string. { [a-z,A-Z]*}
- If r1, r2, … rn are REs then so are
  - (union r1 r2)
  - (star r1)
  - (star+ r1)
  - (sequence r1 r2 …)
  - (assign r1 name) same as r1 with side effect
  - (eval r1 expression) same as r1 with eval side effect

Important: So far we are talking about data not operations

- We are not computing union etc etc. We are merely constructing Lisp lists.
- For example, type '(union "a" "b")
- Or (list 'union "a" "b")

The only interesting operations we need are matching RegExps.

- To match a literal, look for it literally
- To match a sequence, do (and (match r1) (match r2) …) -- (every #'match '(r1 r2 ….))
- To match a union, do (or (match r1) (match r2) …) continues until one succeeds. – (any #'match '(r1 r2 …))
- To match (star r1), in lisp:
- (not (do () ((not (match r1))))) ;;;… restated more conventionally,
- (loop indefinitely until you find a failure to match r1) then return true, for all those forms (maybe none) which matched. *Problem with matching (0+1)*01 which requires backup..*

Here's the matching program (most of it)

```
(defun mymatch (x)
  (declare (special string index end))
  (typecase x
    (list ;; either a list or something else
     (ecase (car x) ;;test the car for something we know
       (sequence (every #'mymatch (cdr x)))
       (union (some #'mymatch (cdr x)))
       (star (not (do ()((not (mymatch (cadr x))) ))))))
    ;; it is not a list
    (t (matchitem x)))
```

Here's the matching program (more of it)

```
(defun mymatch0 (pat string)
  (declare (special string))
  (let ((index 0)
   (end (length string)))
    (declare (special index end))
    ;; this is not very nice lisp: it uses
    ;; global "special" variables instead of
    ;; lexical variables.

    (if (and (mymatch pat)(= end index))
         'success
         `(failed after ,index chars))));first use of ` backquote
          ;;(list 'failed 'after index 'chars) ..
```

Here's the matching program (rest of it)

```
(defun matchitem (x)
  (declare (special index end string))
    (cond ((>= index end) nil)
   ((characterp x) ;match a character
    (if (char= x(elt string index)) (incf index) nil))
   ((stringp x)
    (and (string= x (subseq string index (+ index (length x))))
         (incf index (length x))))
   ((eq x '?) (incf index)) ;single character wildcard
   ((eq x 'alphanumeric) (and
                        (alphanumericp (elt string index))
                        (incf index)))

   ;; generalize this to any predicate
   ((and (symbolp x)(get x 'chartype))
    (and (funcall (get x 'chartype) (elt string index))
         ))
   (t nil)))
```

Here's the matching program (extending it)

```
(setf (get 'digit 'chartype)
 #'(lambda(x)
         (and
        (member x '(#\0 #\1 #\2 #\3 #\4 #\5 #\6 #\7 #\8 #\9))
                        (incf index))))


;;see matchprog.cl
```

What if you don't like (union r1 r2), (seq r1 r2)? / the META system.. (H. Baker)

- [r1 r2] for sequence
- {r1 r2} for union
- R1$ for Kleene star
- ! For evaluation
- @ for indirect "anything of this type"

```
defun parse-int (&aux (s +1) d (n 0))
 (and
  (matchit
   [{#\+ [#\- !(setq s -1)] []}
    @(digit d) !(setq n (ctoi d))
    $[@(digit d) !(setq n (+ (* n 10) (ctoi d)))]])
  (* s n)))
```

Pragmatic parsing (Prag-Parse.html)

- Mostly this is a tour-de-force of Lisp programming to show you can do lex/yacc Unix utilities in a few pages of Lisp. But it also suggests that with appropriate choice of data structure and a versatile language, you can scan/parse a fairly complicated language.
- Rather sophisticated Lisp programming style.

Simpler program (pitman.cl)

- Taken off comp.lang.lisp newsgroup
- Kent Pitman's answer to How does one do lexical analysis in lisp?
- Rather straightforward Lisp programming style.

Conclusion: Regular Expression Programs

- Easy to specify lexical structure of typical language by Regular Expressions.
- Good correspondence between intuition and implementation
- Automated tools can use the RE specs.
- Next time: more on just seat-of-pants systematic programming.