Block ciphers, stream ciphers

Asymmetric cryptography

CS 161: Computer Security
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Announcements

• Project 1 is out, due Feb 14 midnight
Recall: Block cipher

A function $E : \{0, 1\}^k \times \{0, 1\}^n \rightarrow \{0, 1\}^n$. Once we fix the key $K$, we get

$E_K : \{0,1\}^n \rightarrow \{0,1\}^n$ defined by $E_K(M) = E(K,M)$.

Three properties:

- **Correctness:**
  - $E_K(M)$ is a permutation (bijective/ one-to-one function)
- **Efficiency**
- **Security**
Security

For an unknown key $K$, $E_K$ “behaves” like a random permutation

For all polynomial-time attackers, for a randomly chosen key $K$, the attacker cannot distinguish $E_K$ from a random permutation
Block cipher: security game

- Attacker is given two boxes, one for $E_K$ and one for a random permutation (also called "oracles")
- Attacker does not know which is which (they were shuffled randomly)
- Attacker can give inputs to each box, look at the output, as many times as he/she desires
- Attacker must guess which is $E_K$

??? Which is $E_K$???
Security game

For all polynomial-time attackers,

\[ \Pr[\text{attacker wins game}] \leq \frac{1}{2} + \text{negl} \]
Use block ciphers to construct symmetric-key encryption

• Want two properties:
  – IND-CPA security even when reusing the same key to encrypt many messages
  – Can encrypt messages of any length
Desired security: Indistinguishability under chosen plaintext attack (IND-CPA)

- Strong security definition
- Nothing leaks about the encrypted value other than its length
IND-CPA (Indistinguishability under chosen plaintext attack)

Challenger

K

random bit b

M₀, M₁ (must be same length)

Encₖ(Mₖ)

Encₖ(Mₖ)

Here is my guess: b’

Difference from IND-KPA: no encryption tries
An encryption scheme is IND-CPA if for all polynomial-time adversaries

\[ \Pr[\text{Adv wins game}] \leq \frac{1}{2} + \text{negligible} \]

Note that IND-CPA requires that the encryption scheme is randomized

(An encryption scheme is deterministic if it outputs the same ciphertext when encrypting the same plaintext; a randomized scheme does not have this property)
Difference from known-plaintext attack from last time

• The extra queries to $\text{Enc}_K$
• Q: Why is IND-CPA a stronger security?
  – A: The attacker is given more capabilities so the IND-CPA scheme resists a more powerful attacker
Are block ciphers IND-CPA?

Recall: $E_K : \{0,1\}^n \rightarrow \{0,1\}^n$ is a permutation (bijective)
Are block ciphers IND-CPA?

• No, because they are deterministic
• Here is an attacker that wins the IND-CPA game:
  – Adv asks for encryptions of “bread”, receives $C_{br}$
  – Then, Adv provides ($M_0 = \text{bread}$, $M_1 = \text{honey}$)
  – Adv receives $C$
  – If $C = C_{br}$, Adv says bit was 0 (for “bread”), else Adv says bit was 1 (for “honey”)
  – Chance of winning is 1
Each block encrypted with a block cipher
Later (identical) message again encrypted
Modes of operation

Chain block ciphers in certain modes of operation

- Certain output from one block feeds into next block

Need some initial randomness IV (initialization vector)

Why? To prevent the encryption scheme from being deterministic
Counter mode (CTR)

Last time: ECB, CBC
CTR: Encryption

Enc(K, plaintext):

- If n is the block size of the block cipher, split the plaintext in blocks of size n: P_1, P_2, P_3,...
- Choose a random nonce (Nonce = Same as IV)
- Now compute:

The final ciphertext is (nonce, C_1, C_2, C_3)

Counter (CTR) mode encryption
Dec(K, ciphertext=[nonce, C_1, C_2, C_3, ..]):
- Take nonce out of the ciphertext
- If \( n \) is the block size of the block cipher, split the ciphertext in blocks of size \( n \): \( C_1, C_2, C_3, .. \)
- Now compute this:

Output the plaintext as the concatenation of \( P_1, P_2, P_3, ... \)

Note, CTR decryption uses block cipher’s encryption, not decryption
Want to see CTR explained slowly on “whiteboard”? 
Encrypted with CBC
CBC vs CTR

Security: If no reuse of nonce, both are IND-CPA.

Speed: Both modes require the same amount of computation, but CTR is parallelizable for encryption as well (CBC was parallelizable for decryption but not for encryption)
Pseudorandom generator (PRG)
Pseudorandom Generator (PRG)

- Given a seed, it outputs a sequence of random bits
  \[
  \text{PRG(seed)} \rightarrow \text{random bits}
  \]
- It can output arbitrarily many random bits
PRG security

• Can PRG(K) be truly random?

No. Consider key length $|K|=k$. Have $2^k$ possible initial states of PRG. Deterministic from then on.

• A secure PRG suffices to “look” random (“pseudo”) to an attacker (no attacker can distinguish it from a random sequence)
Example of PRG: using block cipher in CTR mode

If you want $m$ random bits, and a block cipher with $E_k$ has $n$ bits, apply the block cipher $m/n$ times and concatenate the result:

$$PRG(K, IV) = E_k(IV|1), E_k(IV| 2), E_k(IV|3) \ldots E_k(IV| \text{ceil}(m/n)),$$
where $|$ is concatenation.
Application of PRG: Stream ciphers

• Another way to construct encryption schemes
• Similar in spirit to one-time pad: it XORs the plaintext with some random bits
• But random bits are not the key (as in one-time pad) but are output of a pseudorandom generator PRG
Application of PRG: Stream cipher

Enc(K, M):
- Choose a random value IV
- C = PRG(K, IV) XOR M
- Output (IV, C)

Q: How decrypt?
A: Compute PRG(K, IV) and XOR with ciphertext C

Q: What is advantage of OTP?
A: Can encrypt any message length because PRG can produce any number of random bits
Block ciphers summary

• Desirable security: IND-CPA
• Block ciphers have weaker security than IND-CPA
• Block ciphers can be used to build IND-CPA secure encryption schemes by chaining in careful ways
• Stream ciphers provide another way to encrypt, inspired from one-time pads
Start asymmetric cryptography on board