University of California at Berkeley
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EECS150, Spring 2010

Quiz 10: April 9^{th}
Solution

Consider the functions F and G:

$$F(a,b,c,d) = \bar{a}\bar{b}\bar{c}\bar{d} + a\bar{b}\bar{c}\bar{d} + \bar{a}\bar{b}c\bar{d} + a\bar{b}c\bar{d} + \bar{a}b\bar{c}d + ab\bar{c}d + \bar{a}bcd + abcd$$

$$G(a,b,c,d,e,f) = F(a,b,c,d)(F(a,b,c,d) + abef)$$

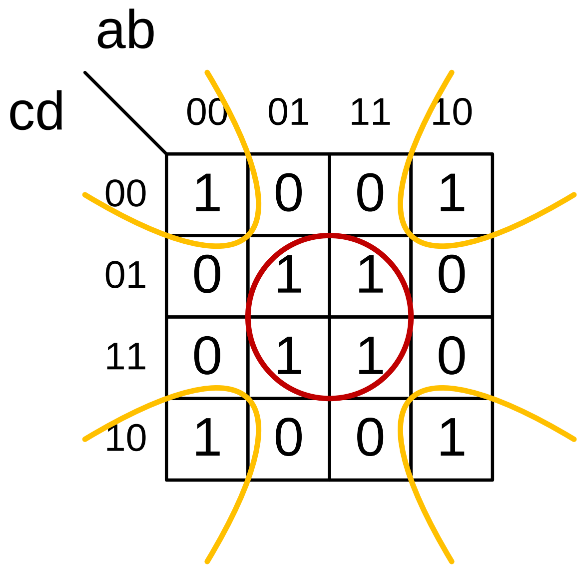
1. Express F in as few literals as possible.

   This problem could have been done using algebraic simplification or KMaps. Notice that the terms are grouped in canonical sum-of-products form. This makes the KMap approach straightforward:

   (a) *Derive a truth-table given the minterms:*

a	b	c	d	F
0	0	0	0	1
0	0	0	1	0
0	0	1	0	1
0	0	1	1	0
0	1	0	0	0
0	1	0	1	1
0	1	1	0	0
0	1	1	1	1
1	0	0	0	1
1	0	0	1	0
1	0	1	0	1
1	0	1	1	0
1	1	0	0	0
1	1	0	1	1
1	1	1	0	0
1	1	1	1	1

(b) *Fill in and cover a 4-variable KMap given the truth table:*



(c) *By inspecting the KMap, we see that*

$$F(a,b,c,d) = \bar{b}\bar{d} + bd = \overline{b \oplus d}$$

2. Express G in as few literals as possible.

This problem might have looked labor-intensive at first glance. The trick is to use the "absorption" property of boolean algebra:

$$F(a,b) = a(a+b) = a$$

Think about why this is true! If $a = 0$*, there is no way that F can evaluate to 1. Likewise, if* $a = 1$*, we are left with this expression:*

$$1(1+b)$$

Which $= 1$ *always. Hence, the answer to this part is:*

$$G(a,b,c,d,e,f) = F(a,b,c,d)(F(a,b,c,d) + abef) = F(a,b,c,d) = \overline{b \oplus d}$$

That is, the same result that we obtained from the first problem.