University of California at Berkeley
College of Engineering
Department of Electrical Engineering and Computer Science
SOLUTIONS

EECS 150 R. H. Katz
Spring 2007

Problem Set #1: Combinational Logic forms
Assigned 16 January 2007, Due 26 January at 2 PM

1. Use Boolean algebra to simplify into two-level sum of products form the following Boolean expressions. Show your intermediate steps of applying Boolean algebra axioms and theorems!
   a. F(A,B,C) = A' B' C' + A B C' + A B' C'
      = A'B'C' + AB'C' + AB'C' + ABC'
      = B'C'(A' + A) + AC'(B' + B)
      = B'C' + AC'
   b. F(A, B, C, D) = A' B' C' D' + A' B' C' D + A' B' C D + A' B C D + A B' C' D' + A B' C' D + A B C' D + A B C D
      = A'B'C'(D' + D) + A'CD (B' + B) + AB'C'(D' + D) + ABD (C' + C)
      = A'B'C' + A'CD + AB'C' + ABD
      = B'C'(A' + A) + A'CD + ABD
      = B'C' + A'CD + ABD

2. Given the following minimized two-level sum of products expressions, describe the functions in "little m" notation:
   a. F(A,B,C) = A' B' + A B

A	B	C	Minterms	F(A,B,C)	
0	0	0	A'B'C'	1	m0
0	0	1	A'B'C	1	m1
0	1	0	A'BC'	0	m2
0	1	1	A'BC	0	m3
1	0	0	AB'C'	0	m4
1	0	1	AB'C	0	m5
1	1	0	ABC'	1	m6
1	1	1	ABC	1	m7

= Σm(0,1,6,7)

b. F(A,B,C,D) = C + A B + A' B C

A	B	C	D	Minterms	F(A,B,C,D)	
0	0	0	0	A'B'C'D'	0	m0
0	0	0	1	A'B'C'D	0	m1
0	0	1	0	A'B'C D'	1	m2
0	0	1	1	A'B'C D	1	m3
0	1	0	0	A'B C'D'	0	m4
0	1	0	1	A'B C'D	0	m5
0	1	1	0	A'B C D'	1	m6
0	1	1	1	A'B C D	1	m7
1	0	0	0	A B'C'D'	0	m8
1	0	0	1	A B'C'D	0	m9
1	0	1	0	A B'C D'	1	m10
1	0	1	1	A B'C D	1	m11
1	1	0	0	A B C'D'	1	m12
1	1	0	1	A B C'D	1	m13
1	1	1	0	A B C D'	1	m14
1	1	1	1	A B C D	1	m15

= Σm(2,3,6,7,10,11,12,13,14,15)

3. Given the following minimized two-level product of sums expressions, describe the functions in "big M" notation:
   a. F(A, B, C) = (A' + C) (B + C')

A	B	C	Maxterms	F(A,B,C)	
0	0	0	A+B+C	1	M0
0	0	1	A+B+C'	0	M1
0	1	0	A+B'+C	1	M2
0	1	1	A+B+C'	1	M3
1	0	0	A'+B+C	0	M4
1	0	1	A'+B+C'	0	M5
1	1	0	A'+B'+C	0	M6
1	1	1	A'+B'+C'	1	M7

=Π M(1,4,5,6)

b. F(A, B, C, D) = (A’ + B’) (A’ + C’) (B + D’)

A	B	C	D	Maxterms	F(A,B,C,D)	
0	0	0	0	A+B+C+D	1	M0
0	0	0	1	A+B+C+D’	0	M1
0	0	1	0	A+B+C’+D	1	M2
0	0	1	1	A+B+C’+D’	0	M3
0	1	0	0	A+B’+C+D	1	M4
0	1	0	1	A+B’+C+D’	1	M5
0	1	1	0	A+B’+C’+D	1	M6
0	1	1	1	A+B’+C’+D’	1	M7
1	0	0	0	A’+B+C+D	1	M8
1	0	0	1	A’+B+C+D’	0	M9
1	0	1	0	A’+B+C’+D	0	M10
1	0	1	1	A’+B+C’+D’	0	M11
1	1	0	0	A’+B’+C+D	0	M12
1	1	0	1	A’+B’+C+D’	0	M13
1	1	1	0	A’+B’+C’+D	0	M14
1	1	1	1	A’+B’+C’+D’	0	M15

=Π M(1,3,9,10,11,12,13,14,15)

4. Use K-maps to simplify the following logical expressions into minimized two-level forms. Show how you have used K-maps to obtain your answers!
   a. F(A, B, C) = A C + A B C’ + A B’ C’ + A’ B’ C’ (answer in sum of products form)

	A’B’	A’B	AB	AB’
C’	1	0	1	1
C	0	0	1	1

= A + B’C’

   b. F(A, B, C) = ΠM(2, 4, 6) (answer in product of sums form)

A	B	C	Maxterms	F(A,B,C)	
0	0	0	A+B+C	1	M0
0	0	1	A+B+C’	1	M1
0	1	0	A+B’+C	0	M2
0	1	1	A+B’+C’	1	M3
1	0	0	A’+B+C	0	M4
1	0	1	A’+B+C’	1	M5
1	1	0	A’+B’+C	0	M6
1	1	1	A’+B’+C’	1	M7

	A'B'	A'B	AB	AB'
C'	1	0	0	0
C	1	1	1	1

= (B' + C) (A' + C)

c. F(A, B, C, D) = Σm(0, 4, 5, 8, 11, 12, 13) (answer in sum of products form)

A	B	C	D	Minterms	F(A,B,C,D)	
0	0	0	0	A'B'C'D'	1	m0
0	0	0	1	A'B'C'D	0	m1
0	0	1	0	A'B'C D'	0	m2
0	0	1	1	A'B'C D	0	m3
0	1	0	0	A'B C'D'	1	m4
0	1	0	1	A'B C'D	1	m5
0	1	1	0	A'B C D'	0	m6
0	1	1	1	A'B C D	0	m7
1	0	0	0	A B'C'D'	1	m8
1	0	0	1	A B'C'D	0	m9
1	0	1	0	A B'C D'	0	m10
1	0	1	1	A B'C D	1	m11
1	1	0	0	A B C'D'	1	m12
1	1	0	1	A B C'D	1	m13
1	1	1	0	A B C D'	0	m14
1	1	1	1	A B C D	0	m15

	A'B'	A'B	AB	AB'
C'D'	1	1	1	1
C'D	0	1	1	0
CD	0	0	0	1
CD'	0	0	0	0

= C'D' + BC' + AB'CD

d. F(A, B, C, D) = (A + C + D') (A + B + D) (A' + C + D') (B' + C + D') (answer in product of sums form)

	A'B'	A'B	AB	AB'
C'D'	0	1	1	1
C'D	0	0	0	0
CD	1	1	1	1
CD'	0	1	1	1

= (C + D') (A + B + D)

5. Given that the Boolean function F(A, B, C) = A C' + A' B, answer the following questions. Show the intermediate work to arrive at the answer!
   a. What is the corresponding expression in minimized product of sums form?
      F(A,B,C) = (A + B) (A' + C')
   b. What is the corresponding expression for the complement of F in minimized sum of products form?
      F'(A,B,C) = ((A + B) (A' + C'))' = A'B' + AC
   c. What is the corresponding expression for the complement of F in minimized product of sums form?
      F'(A,B,C) = (AC' + A'B)' = (A' + C) (A + B')

6. F(A,B,C,D) = B(C' + D') in minimized product of sums form. Answer the following questions, showing your intermediate steps:
   a. Show a K-map for F.

	A'B'	A'B	AB	AB'
C'D'	0	1	1	0
C'D	0	1	1	0
CD	0	0	0	0
CD'	0	1	1	0

b. What is F in Big M notation?

A	B	C	D	Maxterms	F(A,B,C,D)	
0	0	0	0	A+B+C+D	0	M0
0	0	0	1	A+B+C+D’	0	M1
0	0	1	0	A+B+C’+D	0	M2
0	0	1	1	A+B+C’+D’	0	M3
0	1	0	0	A+B’+C+D	1	M4
0	1	0	1	A+B’+C+D’	1	M5
0	1	1	0	A+B’+C’+D	1	M6
0	1	1	1	A+B’+C’+D’	0	M7
1	0	0	0	A’+B+C+D	0	M8
1	0	0	1	A’+B+C+D’	0	M9
1	0	1	0	A’+B+C’+D	0	M10
1	0	1	1	A’+B+C’+D’	0	M11
1	1	0	0	A’+B’+C+D	1	M12
1	1	0	1	A’+B’+C+D’	1	M13
1	1	1	0	A’+B’+C’+D	1	M14
1	1	1	1	A’+B’+C’+D’	0	M15

$=\Pi\ M(0,1,2,3,7,8,9,10,11,15)$

c. What is F in Little M notation?

A	B	C	D	Minterms	F(A,B,C,D)	
0	0	0	0	A’B’C’D’	0	m0
0	0	0	1	A’B’C’D	0	m1
0	0	1	0	A’B’C D’	0	m2
0	0	1	1	A’B’C D	0	m3
0	1	0	0	A’B C’D’	1	m4
0	1	0	1	A’B C’D	1	m5
0	1	1	0	A’B C D’	1	m6
0	1	1	1	A’B C D	0	m7
1	0	0	0	A B’C’D’	0	m8
1	0	0	1	A B’C’D	0	m9
1	0	1	0	A B’C D’	0	m10
1	0	1	1	A B’C D	0	m11
1	1	0	0	A B C’D’	1	m12
1	1	0	1	A B C’D	1	m13
1	1	1	0	A B C D’	1	m14
1	1	1	1	A B C D	0	m15

$= \Sigma m(4,5,6,12,13,14)$

d. What is F in minimized two-level sum of products form?

	A'B'	A'B	AB	AB'
C'D'	0	1	1	0
C'D	0	1	1	0
CD	0	0	0	0
CD'	0	1	1	0

= BC' + BD'

e. Apply DeMorgan's Law to derive the complement of F, starting with the original product of sums expression.
F'(A,B,C,D) = (B(C' + D'))' = B' + CD

f. Apply DeMorgan's Law to derive the complement of F given your answer to part d.
F'(A,B,C,D) = (BC' + BD')' = (B' + C) (B' + D)

g. Using the K-map you filled in for part a, show how to derive the minimized complement of F in product of sums form. Is your answer the same as in part f? If not, why not?

	A'B'	A'B	AB	AB'
C'D'	1	0	0	1
C'D	1	0	0	1
CD	1	1	1	1
CD'	1	0	0	1

F' = (B' + C) (B' + D)
Yes, it is the same as part f.

h. Using the K-map you filled in for part a, show how to derive the minimized complement of F in sum of products form. Is your answer the same as in part e? If not, why not?

	A'B'	A'B	AB	AB'
C'D'	1	0	0	1
C'D	1	0	0	1
CD	1	1	1	1
CD'	1	0	0	1

F' = B' + CD
Yes, it is the same as part e.