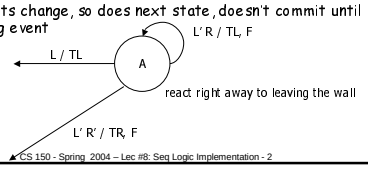
Sequential Logic Implementation

- Models for representing sequential circuits
  - Abstraction of sequential elements
  - Finite state machines and their state diagrams
  - Inputs/outputs
  - Mealy, Moore, and synchronous Mealy machines
- Finite state machine design procedure
  - Deriving state diagram
  - Deriving state transition table
  - Determining next state and output functions
  - Implementing combinational logic

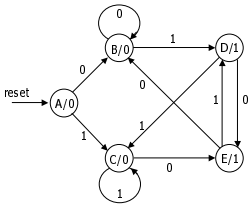
Mealy vs. Moore Machines

- Moore: outputs depend on current state only
- Mealy: outputs depend on current state and inputs
- Ant brain is a Moore Machine
  - Output does not react immediately to input change
- We could have specified a Mealy FSM
  - Outputs have immediate reaction to inputs
  - As inputs change, so does next state, doesn't commit until clocking event



Specifying Outputs for a Moore Machine

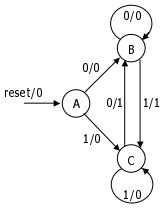
- Output is only function of state
  - Specify in state bubble in state diagram
  - Example: sequence detector for 01 or 10



reset	input	current state	next state	output
1	–	–	A	
0	0	A	B	0
0	1	A	C	0
0	0	B	B	0
0	1	B	D	0
0	0	C	E	0
0	1	C	C	0
0	0	D	E	1
0	1	D	C	1
0	0	E	B	1
0	1	E	D	1

Specifying Outputs for a Mealy Machine

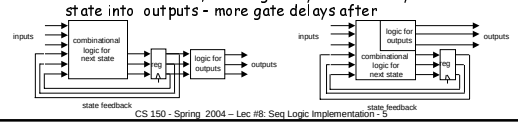
- Output is function of state and inputs
  - Specify output on transition arc between states
  - Example: sequence detector for 01 or 10



reset	input	current state	next state	output
1	–	–	A	0
0	0	A	B	0
0	1	A	C	0
0	0	B	B	0
0	1	B	C	1
0	0	C	B	1
0	1	C	C	0

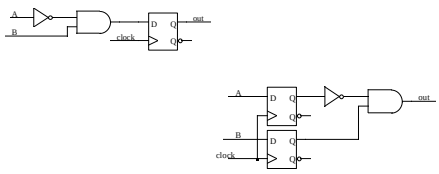
Comparison of Mealy and Moore Machines

- Mealy Machines tend to have less states
  - Different outputs on arcs (n^2) rather than states (n)
- Moore Machines are safer to use
  - Outputs change at clock edge (always one cycle later)
  - In Mealy machines, input change can cause output change as soon as logic is done - a big problem when two machines are interconnected - asynchronous feedback
- Mealy Machines react faster to inputs
  - React in same cycle - don't need to wait for clock
  - In Moore machines, more logic may be necessary to decode state into outputs - more gate delays after



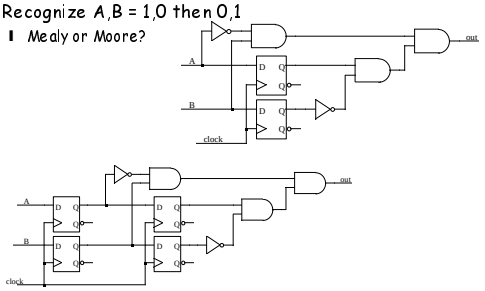
Mealy and Moore Examples

- Recognize A,B = 0,1
  - Mealy or Moore?



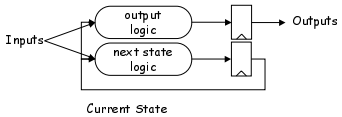
Mealy and Moore Examples (cont'd)

- Recognize A,B = 1,0 then 0,1
  - Mealy or Moore?



Registered Mealy Machine (Really Moore)

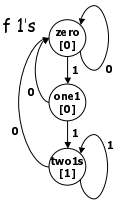
- Synchronous (or registered) Mealy Machine
  - Registered state AND outputs
  - Avoids 'glitchy' outputs
  - Easy to implement in PLDs
- Moore Machine with no output decoding
  - Outputs computed on transition to next state rather than after entering
  - View outputs as expanded state vector



Verilog FSM - Reduce 1s Example

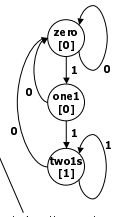
- Change the first 1 to 0 in each string of 1's
  - Example Moore machine implementation

```
// State assignment
parameter zero = 0, one1 = 1, two1s = 2;

module reduce (out, clk, reset, in);
  output out;
  input clk, reset, in;
  reg out;
  reg [1:0] state;       // state register
  reg [1:0] next_state;
```



Moore Verilog FSM (cont'd)

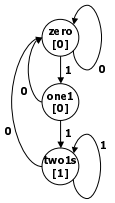
```
always @(in or state)
  case (state)

    zero: begin // last input was a zero
      out = 0;
      if (in) next_state = one1;
      else    next_state = zero;
    end

    one1: begin // we've seen one 1
      out = 0;
      if (in) next_state = two1s;
      else    next_state = zero;
    end

    two1s: begin // we've seen at least 2 ones
      out = 1;
      if (in) next_state = two1s;
      else    next_state = zero;
    end

    default: begin // in case we reach a bad state
      out = 0;
      next_state = zero;
  endcase
```



include all signals that are input to state and output equations

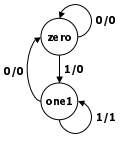
Moore Verilog FSM (cont'd)

```
// Implement the state register
  always @(posedge clk)
    if (reset) state <= zero;
    else       state <= next_state;

endmodule
```



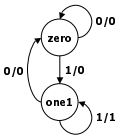
Mealy Verilog FSM for Reduce-1s Example

```
module reduce (clk, reset, in, out);
  input clk, reset, in; output out;
  reg out; reg state;     // state register
  reg next_state;
  parameter zero = 0, one = 1;

  always @(in or state)
    case (state)
      zero: begin       // last input was a zero
              if (in) next_state = one;
              else    next_state = zero;
              out = 0;
            end
      one:            // we've seen one 1
            if (in) begin
                      next_state = one;
                      out        = 1;
                    end else begin
              next_state = zero;
              out        = 0;
            end
    endcase

  always @(posedge clk)
    if (reset) state <= zero;
    else       state <= next_state;

  endmodule
```



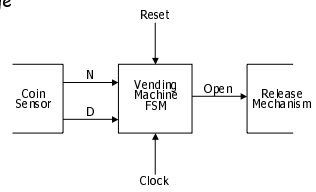
Synchronous Mealy Verilog FSM for Reduce-1s Example

```
module reduce (clk, reset, in, out);
  input clk, reset, in; output out;
  reg out; reg state;    // state register
  reg next_state; reg next_out;
  parameter zero = 0, one = 1;

  always @(in or state)
    case (state)
      zero: begin       // last input was a zero
              if (in) next_state = one;
              else    next_state = zero;
              next_out = 0;
            end
      one:            // we've seen one 1
            if (in) begin
                      next_state = one;
                      next_out   = 1;
                    end else begin
            else begin
                      next_state = zero;
                      next_out   = 0;
                    end
    endcase

  always @(posedge clk)
      if (reset) begin
        state <= zero; out <= 0;
      end
      else begin
        state <= next_state; out <= next_out;
      end

endmodule
```



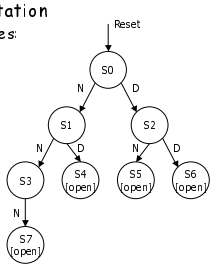
Example: Vending Machine

- Release item after 15 cents are deposited
- Single coin slot for dimes, nickels
- No change



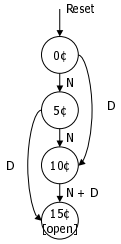
Example: Vending Machine (cont'd)

- Suitable Abstract Representation
  - Tabulate typical input sequences:
    - 3 nickels
    - nickel, dime
    - dime, nickel
    - two dimes
  - Draw state diagram:
    - Inputs: N, D, reset
    - Output: open chute
  - Assumptions:
    - Assume N and D asserted for one cycle
    - Each state has a self loop for N = D = 0 (no coin)



Example: Vending Machine (cont'd)

- Minimize number of states - reuse states whenever possible



present state	inputs D	N	next state	output open
0¢	0	0	0¢	0
	0	1	5¢	0
	1	0	10¢	0
	1	1	–	–
5¢	0	0	5¢	0
	0	1	10¢	0
	1	0	15¢	0
	1	1	–	–
10¢	0	0	10¢	0
	0	1	15¢	0
	1	0	15¢	0
	1	1	–	–
15¢	–	–	15¢	1

symbolic state table

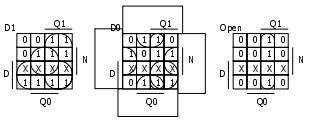
Example: Vending Machine (cont'd)

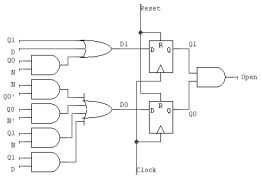
- Uniquely Encode States

present state Q1	Q0	inputs D	N	next state D1	D0	output open
0	0	0	0	0	0	0
		0	1	0	1	0
		1	0	1	0	0
		1	1	–	–	–
0	1	0	0	0	1	0
		0	1	1	0	0
		1	0	1	1	0
		1	1	–	–	–
1	0	0	0	1	0	0
		0	1	1	1	0
		1	0	1	1	0
		1	1	–	–	–
1	1	–	–	1	1	1

Example: Vending Machine (cont'd)

- Mapping to Logic





D1 = Q1 + D + Q0 N

D0 = Q0' N + Q0 N' + Q1 N + Q1 D

OPEN = Q1 Q0

Example: Vending Machine (cont'd)

- One-hot Encoding

present state Q3 Q2 Q1 Q0	inputs D N	next state output D3 D2 D1 D0	open
0 0 0 1	0 0	0 0 0 1	0
	0 1	0 0 1 0	0
	1 0	0 1 0 0	0
	1 1	- - - -	-
0 0 1 0	0 0	0 0 1 0	0
	0 1	0 1 0 0	0
	1 0	1 0 0 0	0
	1 1	- - - -	-
0 1 0 0	0 0	0 1 0 0	0
	0 1	1 0 0 0	0
	1 0	1 0 0 0	0
	1 1	- - - -	-
1 0 0 0	- -	1 0 0 0	1

D0 = Q0 D' N'

D1 = Q0 N + Q1 D' N'

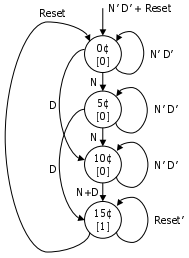
D2 = Q0 D + Q1 N + Q2 D' N'

D3 = Q1 D + Q2 D + Q2 N + Q3

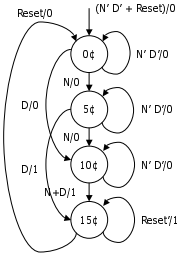
OPEN = Q3

Equivalent Mealy and Moore State Diagrams

- Moore machine
  - outputs associated with state

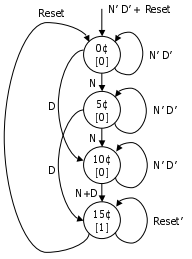


Mealy machine outputs associated with transitions



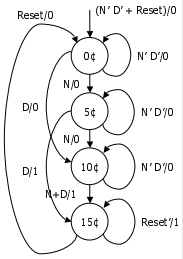
Moore Verilog FSM for Vending Machine

```
module vending (open, Clk, Reset, N, D);
  input Clk, Reset, N, D; output open;
  reg open; reg state;    // state register
  reg next_state;
  parameter zero = 0, five = 1, ten = 2, fifteen = 3;

  always @(N or D or state)
    case (state)
      zero: begin
              if (D)      next_state = five;
              else if (N) next_state = ten;
              else        next_state = zero;
              open = 0;
            end
      …
      fifteen: begin
              if (!Reset) next_state = fifteen;
              else        next_state = zero;
              open = 1;
            end
 endcase

  always @(posedge clk)
    if (Reset || (!N && !D)) state <= zero;
    else                     state <= next_state;

  endmodule
```



Mealy Verilog FSM for Vending Machine

```
module vending (open, Clk, Reset, N, D);
  input Clk, Reset, N, D; output open;
  reg open; reg state;    // state register
  reg next_state; reg next_open;
  parameter zero = 0, five = 1, ten = 2, fifteen = 3;

  always @(N or D or state)
    case (state)
      zero: begin
              if (D) begin
                next_state = ten; next_open = 0;
              end
              else if (N) begin
                next_state = five; next_open = 0;
              end
              else begin
                next_state = zero; next_open = 0;
              end
            end
    …
  endcase

always @(posedge clk)
  if (Reset || (!N && !D)) begin state <= zero; open <= 0; end
  else begin state <= next_state; open <= next_open; end
endmodule
```

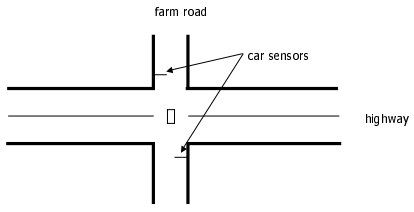


Example: Traffic Light Controller

- A busy highway is intersected by a little used farmroad
- Detectors C sense the presence of cars waiting on the farmroad
  - with no car on farmroad, light remain green in highway direction
  - if vehicle on farmroad, highway lights go from Green to Yellow to Red, allowing the farmroad lights to become green
  - these stay green only as long as a farmroad car is detected but never longer than a set interval
  - when these are met, farm lights transition from Green to Yellow to Red, allowing highway to return to green
  - even if farmroad vehicles are waiting, highway gets at least a set interval as green
- Assume you have an interval timer that generates:
  - a short time pulse (TS) and
  - a long time pulse (TL),
  - in response to a set (ST) signal.
  - TS is to be used for timing yellow lights and TL for green lights

Example: Traffic Light Controller (cont')

- Highway/farm road intersection



Example: Traffic Light Controller (cont')

- Tabulation of Inputs and Outputs

inputs	description
reset	place FSM in initial state
C	detect vehicle on the farm road
TS	short time interval expired
TL	long time interval expired

outputs	description
HG, HY, HR	assert green/yellow/red highway lights
FG, FY, FR	assert green/yellow/red highway lights
ST	start timing a short or long interval

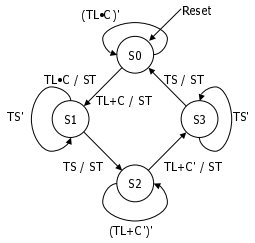
- Tabulation of unique states – some light configurations imply others

state	description
S0	highway green (farm road red)
S1	highway yellow (farm road red)
S2	farm road green (highway red)
S3	farm road yellow (highway red)

Example: Traffic Light Controller (cont')

- State Diagram

S0: HG
S1: HY
S2: FG
S3: FY



Example: Traffic Light Controller (cont')

- Generate state table with symbolic states
- Consider state assignments

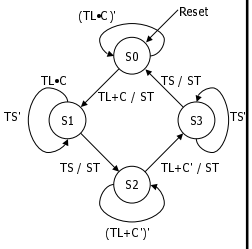
output encoding – similar problem to state assignment (Green = 00, Yellow = 01, Red = 10)

Inputs C	TL	TS	Present State	Next State	Outputs ST	H	F
0	–	–	HG	HG	0	Green	Red
–	0	–	HG	HG	0	Green	Red
1	1	–	HG	HY	1	Green	Red
–	–	0	HY	HY	0	Yellow	Red
–	–	1	HY	FG	1	Yellow	Red
1	0	–	FG	FG	0	Red	Green
0	–	–	FG	FY	1	Red	Green
–	1	–	FG	FY	1	Red	Green
–	–	0	FY	FY	0	Red	Yellow
–	–	1	FY	HG	1	Red	Yellow

SA1:	HG = 00	HY = 01	FG = 11	FY = 10	
SA2:	HG = 00	HY = 10	FG = 01	FY = 11	
SA3:	HG = 0001	HY = 0010	FG = 0100	FY = 1000	(one-hot)

Traffic Light Controller Verilog

```
module traffic (ST, Clk, Reset, C, TL, TS);
  input Clk, Reset, C, TL, TS; output ST;
  reg ST; reg state;
  reg next_state; reg next_ST;
  parameter S0 = 0, S1 = 1, S2 = 2, S3 = 3;

  always @(C or TL or TS or state)
    case (state)
      S0: if (!(TL && C)) begin
            next_state = S0; next_ST = 0;
          else if (TL || C) begin
            next_state = S1; next_ST = 1;
          end
    …
  endcase


  always @(posedge Clk)
    if (Reset) begin state <= S0; ST <= 0; end
    else begin state <= next_state; ST <= next_ST; end
     endmodule
```



Logic for Different State Assignments

- SA1

NS1 = C·TL'·PS1·PS0 + TS·PS1'·PS0 + TS·PS1·PS0' + C'·PS1·PS0 + TL·PS1·PS0
NS0 = C·TL·PS1'·PS0' + C·TL'·PS1·PS0 + PS1'·PS0

ST = C·TL·PS1'·PS0' + TS·PS1'·PS0 + TS·PS1·PS0' + C'·PS1·PS0 + TL·PS1·PS0
H1 = PS1 H0 = PS1'·PS0
F1 = PS1' F0 = PS1·PS0'

- SA2

NS1 = C·TL·PS1' + TS'·PS1 + C'·PS1'·PS0
NS0 = TS·PS1·PS0' + PS1'·PS0 + TS'·PS1·PS0

ST = C·TL·PS1' + C'·PS1'·PS0 + TS·PS1
H1 = PS0 H0 = PS1·PS0'
F1 = PS0' F0 = PS1·PS0

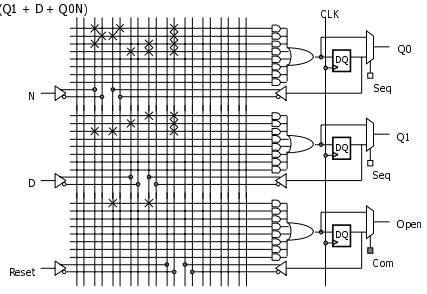
- SA3

NS3 = C'·PS2 + TL·PS2 + TS'·PS3 NS2 = TS·PS1 + C·TL'·PS2
NS1 = C·TL·PS0 + TS'·PS1 NS0 = C'·PS0 + TL'·PS0 + TS·PS3

ST = C·TL·PS0 + TS·PS1 + C'·PS2 + TL·PS2 + TS·PS3
H1 = PS3 + PS2 H0 = PS1
F1 = PS1 + PS0 F0 = PS3

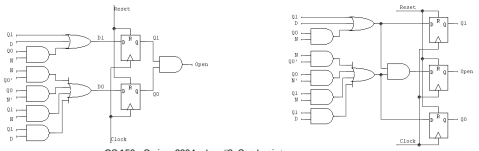
Vending Machine Example Revisted (PLD mapping)

D0 = reset'(Q0'N + Q0N' + Q1N + Q1D)
D1 = reset'(Q1 + D + Q0N)
OPEN = Q1Q0



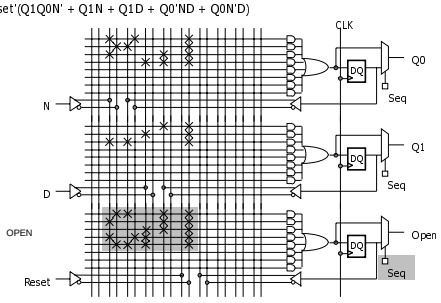
Vending Machine (cont'd)

- OPEN = Q1Q0 creates a combinational delay after Q1 and Q0 change
- This can be corrected by retiming, i.e., move flip-flops and logic through each other to improve delay
- OPEN = reset'(Q1 + D + Q0N)(Q0'N + Q0N' + Q1N + Q1D)
  = reset'(Q1Q0N' + Q1N + Q1D + Q0'ND + Q0N'D)
- Implementation now looks like a synchronous Mealy machine
  - Common for programmable devices to have FF at end of logic



Vending Machine (Retimed PLD Mapping)

OPEN = reset'(Q1Q0N' + Q1N + Q1D + Q0'ND + Q0N'D)



Sequential Logic Implementation Summary

- Models for representing sequential circuits
  - Abstraction of sequential elements
  - Finite state machines and their state diagrams
  - Inputs/outputs
  - Mealy, Moore, and synchronous Mealy machines
- Finite state machine design procedure
  - Deriving state diagram
  - Deriving state transition table
  - Determining next state and output functions
  - Implementing combinational logic