Homework #7 – Solution

The following solutions are reproduced from the Katz textbook’s solutions manual.

2.7

a.) \( (X + \bar{Y}) (X + \bar{Y}) = X \)
   \[= (X + \bar{Y}) (X + \bar{Y}) \]
   \[= XX + X\bar{Y} + X\bar{Y} + Y \bar{Y} \]
   \[= X + X(\bar{Y} + Y) + 0 \]
   \[= X + X (1) \]
   \[= X \]

b.) \( X (X + Y) = X \)
   \[= X (X + Y) \]
   \[= XX + XY \]
   \[= X + XY \]
   \[= X (1 + Y) \]
   \[= X(1) \]
   \[= X \]

c.) \( (X + \bar{Y}) Y = XY \)
   \[= XY + \bar{Y} Y \]
   \[= XY + 0 \]
   \[= XY \]

d.) \( (X + Y)(\bar{X} + Z) = XZ + \bar{X}Y \)
   \[= X\bar{X} + XZ + \bar{X}Y + YZ \]
   \[= 0 + XZ + \bar{X}Y + YZ \]
   \[= XZ + \bar{X}Y + YZ (1) \]
   \[= XZ + \bar{X}Y + YZ (X + \bar{X}) \]
   \[= XZ + \bar{X}Y + XZ + \bar{X}YZ \]
   \[= (XZ + XZ + \bar{X}Y Z) (X + \bar{X}) \]
   \[= XZ + \bar{X}Y + XZ + \bar{X}YZ \]
   \[= XZ(1 + Y) + \bar{X}Y(1 + Z) \]
   \[= XZ(1) + \bar{X}Y(1) \]
   \[= XZ + \bar{X}Y \]
2.9

\[ XY + YZ + \overline{XZ} \quad \overline{XY} + \overline{XZ} \]

<table>
<thead>
<tr>
<th></th>
<th>X</th>
<th>Y</th>
<th>Z</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

2.10

a.)  \( f = A (B + CD) \)
    \[ \overline{f} = \overline{A} + \overline{B} + \overline{C} + \overline{D} \]
    \[ \overline{f} = \overline{A} + \overline{B} + \overline{CD} \]
    \[ \overline{f} = \overline{A} + \overline{B} + (C + D) \]

b.)  \( f = ABC + B(\overline{C} + \overline{D}) \)
    \[ \overline{f} = \overline{ABC} + \overline{(C + D)} \]
    \[ \overline{f} = \overline{A} \overline{B} \overline{C} + \overline{B} + \overline{(C + D)} \]
    \[ \overline{f} = (\overline{A} + \overline{B} + \overline{C}) (\overline{B} + (C + D)) \]
    \[ \overline{f} = (\overline{A} + \overline{B} + \overline{C}) (B + CD) \]

c.)  \( f = \overline{X} + \overline{Y} \)
    \[ \overline{f} = X \cdot Y \]

d.)  \( f = X + Y \overline{Z} \)
    \[ \overline{f} = X + Y \overline{Z} \]
    \[ \overline{f} = \overline{X} (Y \overline{Z}) \]
    \[ \overline{f} = \overline{X} (Y + Z) \]

e.)  \( f = \overline{(X + Y)} \overline{Z} \)
    \[ \overline{f} = \overline{(X + Y)} \overline{Z} \]
    \[ \overline{f} = \overline{(X + Y)} + Z \]
    \[ \overline{f} = X \overline{Y} + Z \]
2.12

\[ F = T_2 \cdot T_3 = (X \cdot T_1) \cdot (\overline{Y} \cdot T_1) = X \cdot T_1 + Y \cdot T_1 \]
\[ = (X + Y) \cdot T_1 = (X + Y) \cdot \overline{X} \cdot \overline{Y} \]
\[ = (X + Y) (\overline{X} + \overline{Y}) = \overline{X} + \overline{X} \cdot \overline{Y} + Y \cdot \overline{X} + Y \cdot \overline{Y} \]
\[ = X \cdot \overline{Y} + Y \cdot \overline{X} \]

2.13

a.)
\[ f(X, Y) = X \cdot Y + X \cdot \overline{Y} \]
\[ = X(\overline{Y} + \overline{Y}) \]
\[ = X \cdot 1 \]
\[ = X \]

One literal

Distributive Law
Theorem of Complementarity
Operations with 0 and 1

b.)
\[ f(X, Y) = (X + Y) (X + \overline{Y}) \]
\[ = X \cdot X + X \cdot \overline{Y} + Y \cdot X + Y \cdot \overline{Y} \]
\[ = X \cdot X + X \cdot \overline{Y} + Y \cdot X + Y \cdot 0 \]
\[ = X \cdot X + X \cdot \overline{Y} + Y \cdot X \]
\[ = X + X \cdot \overline{Y} + Y \cdot X \]
\[ = X + XY + X \cdot \overline{Y} \]
\[ = X \]

One literal

Distributive Law
Theorem of Complementarity
Operations with 0 and 1
Idempotent Theorem
Commutative Law
Simplification Theorem

C.)
\[ f(X, Y, Z) = Y \cdot \overline{Z} + \overline{X} \cdot Y \cdot Z + X \cdot Y \cdot Z \]
\[ = Y \cdot \overline{Z} + (\overline{X} + X) \cdot Y \cdot Z \]
\[ = Y \cdot \overline{Z} + (1) \cdot Y \cdot Z \]
\[ = Y \cdot \overline{Z} + Y \cdot Z \]
\[ = Y (\overline{Z} + Z) \]
\[ = Y (1) \]

One literal

Distributive Law
Theorem of Complementarity
Operations with 0 and 1

D.)
\[ f(X, Y, Z) = (X + Y) (\overline{X} + Y + Z) (\overline{X} + Y + Z) \]
\[ = (X + Y) (\overline{X} + Y) \]
\[ = Y \]

One literal

Simplification Theorem
Simplification Theorem
2.17

a.) Minimum sum of products form and its complement:

\[ F = \overline{B} \overline{D} + \overline{B} \overline{C} + B \overline{C} \overline{D} \]

\[ \overline{F} = B \overline{C} + B \overline{D} + B \overline{C} \overline{D} \]

2.18

a.)

\[ f(X, Y, Z) = \overline{X} \overline{Y} + X \overline{Y} \]

Four literals

b.)

\[ f(W, X, Y, Z) = \overline{Z} + X \overline{Y} \]

Three literals
c.)

\[ f(V, W, X, Y, Z) = \overline{V} Y + \overline{V} Z + W \overline{Z} + W Y + V \overline{W} \overline{Y} \]

Eleven literals

v = 0

v = 1

d.)

\[ f(A, B, C, D) = \overline{A} \overline{D} \]

Two literals

e.)

\[ f(A, B, C, D) = \overline{A} \overline{C} + B \overline{C} \]

Four literals
f(A, B, C, D, E) = A \overline{B} \overline{D} + \overline{A} \overline{B} \overline{D} \overline{E} + A B \overline{D} E \\
Eleven literals

f(A, B, C, D) = \overline{A} B D + \overline{A} B \overline{C} + B \overline{C} D \\
OR \\
f(A, B, C, D) = \overline{A} B D + \overline{A} \overline{C} D + B \overline{C} D

2.15

a.) Canonical minterm form:
\[ \overline{A} B C \overline{D} + A \overline{B} C D + \overline{A} B C \overline{D} + A B C D + \overline{A} \overline{B} \overline{C} D + A \overline{B} \overline{C} D + \overline{A} \overline{B} C D + A \overline{B} C D + \overline{A} B C D + A B C D \]

b.) Canonical maxterm form:
\[ \Pi M(3, 4, 5, 6, 11, 12, 13, 14) \]
\[ = (A + \overline{B} + C + D) \cdot (A + \overline{B} + C + D) \cdot (A + \overline{B} + \overline{C} + D) \cdot (\overline{A} + B + C + D) \]
\[ \cdot (\overline{A} + \overline{B} + C + D) \cdot (A + \overline{B} + C + D) \]

c.) Complement of f in "little m" notation and as a canonical minterm expression:
\[ \overline{f} = \Sigma m(3, 4, 5, 6, 11, 12, 13, 14) \]
\[ = \overline{A} B C D + \overline{A} B C D + \overline{A} B C D + \overline{A} B C D + A \overline{B} \overline{C} D + A B \overline{C} D + A \overline{B} C D + A B C D + A B C D + A B C D \]

d.) Complement of f in "big M" notation as a canonical maxterm expression:
\[ \overline{f} = \Pi M(0, 1, 2, 7, 8, 9, 10, 15) \]
\[ = (A + \overline{B} + C + D) \cdot (A + \overline{B} + C + D) \cdot (A + B + \overline{C} + D) \cdot (\overline{A} + B + C + D) \]
\[ \cdot (\overline{A} + B + C + D) \cdot (A + \overline{B} + C + D) \cdot (A + \overline{B} + C + D) \]
<table>
<thead>
<tr>
<th>Σm</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>7</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>9</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>10</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>11</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>12</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>13</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>14</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>15</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>16</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>17</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>18</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>19</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>20</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>21</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>22</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>23</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>24</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>25</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>26</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>27</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>28</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
b.)

\[ F = \Sigma m(3, 5, 6, 9, 10, 12, 17, 18, 20, 24) \]

c.)

\[ \Pi M(0, 1, 2, 4, 7, 8, 11, 13, 14, 15, 17, 19, 21, 22, 23, 25, 26, 27, 28, 29, 30, 31) \]

d.)

\[
\begin{array}{cccc}
00 & 01 & 11 & 10 \\
00 & 0 & 1 & 0 \\
01 & 0 & 1 & 0 \\
11 & 1 & 0 & 0 \\
10 & 0 & 1 & 0 \\
\end{array}
\]

\[ F = \overline{A}BCDE + \overline{A}BEDE + A\overline{B}CDE + \overline{A}BCDE + \overline{A}BCDE + \overline{A}BCDE + A\overline{B}CDE + A\overline{B}CDE + A\overline{B}CDE + ABCDE \]
<table>
<thead>
<tr>
<th>Σm</th>
<th>D₂</th>
<th>D₁</th>
<th>D₀</th>
<th>C₁</th>
<th>C₀</th>
<th>R₁</th>
<th>R₀</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>7</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>9</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>11</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>12</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>13</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>14</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>15</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>16</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>X</td>
<td>X</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Σm</th>
<th>D₂</th>
<th>D₁</th>
<th>D₀</th>
<th>C₁</th>
<th>C₀</th>
<th>R₁</th>
<th>R₀</th>
</tr>
</thead>
<tbody>
<tr>
<td>17</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>18</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>19</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>20</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>21</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>22</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>23</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>24</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>25</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>26</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>27</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>28</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>29</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>30</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>31</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>
b)

\[
\begin{array}{cccc}
C_1 & C_0 & D_1 & D_0 \\
00 & 01 & 11 & 10 \\
00 & X & X & X & X \\
01 & 0 & 0 & 0 & 0 \\
11 & 0 & 0 & 0 & 1 \\
10 & 0 & 0 & 0 & 0 \\
\end{array}
\]

\[
\begin{array}{cccc}
C_1 & C_0 & D_1 & D_0 \\
00 & 01 & 11 & 10 \\
00 & X & X & X \\
01 & 0 & 0 & 0 \\
11 & 0 & 1 & 0 & 0 \\
10 & 0 & 0 & 0 \\
\end{array}
\]

\[
R_1 = \overline{D_2} D_1 D_0 C_1 C_0 + D_2 \overline{D_1} D_0 C_1 C_0
\]

\[
\begin{array}{cccc}
C_1 & C_0 & D_1 & D_0 \\
00 & 01 & 11 & 10 \\
00 & X & X & X \\
01 & 0 & 0 & 0 \\
11 & 0 & 1 & 0 & 0 \\
10 & 0 & 1 & 1 & 0 \\
\end{array}
\]

\[
\begin{array}{cccc}
C_1 & C_0 & D_1 & D_0 \\
00 & 01 & 11 & 10 \\
00 & X & X & X \\
01 & 0 & 0 & 0 \\
11 & 0 & 1 & 0 & 0 \\
10 & 0 & 0 & 0 \\
\end{array}
\]

\[
R_0 = D_0 \overline{C_0} + \overline{D_2} \overline{D_1} D_0 C_1 + D_2 D_1 D_0 C_1 + \overline{D_2} \overline{D_1} D_0 C_1 \overline{C_0}
\]