Recap and Outline

• Pipeline list processor:
  - 31ns \(\rightarrow\) 13 ns

<table>
<thead>
<tr>
<th>component</th>
<th>delay</th>
</tr>
</thead>
<tbody>
<tr>
<td>simple logic gates</td>
<td>0.5ns</td>
</tr>
<tr>
<td>n-bit register</td>
<td>clk-to-Q=0.5ns</td>
</tr>
<tr>
<td>n-bit 2-1 mux</td>
<td>setup=0.5ns</td>
</tr>
<tr>
<td>n-bit adder</td>
<td>1ns</td>
</tr>
<tr>
<td>memory</td>
<td>(2 log(n) + 2)ns</td>
</tr>
<tr>
<td>zero compare</td>
<td>10ns read</td>
</tr>
<tr>
<td>FSM &quot;setup&quot;</td>
<td>0.5 log(n)</td>
</tr>
<tr>
<td>FSM output</td>
<td>1.5 ns</td>
</tr>
</tbody>
</table>

• serial multiplier

• Today: Parallelism, looping, pipelining
**Parallelism**

Parallelism is the act of **doing more than one thing at a time**. Optimization in hardware design often involves using parallelism to trade between cost and performance.

- Example, Student final grade calculation:
  ```
  read mt1, mt2, mt3, project;
  grade = 0.2 \times mt1 + 0.2 \times mt2 \\
  + 0.2 \times mt3 + 0.4 \times project;
  write grade;
  ```

- High performance hardware implementation:

```plaintext
count X X X \\
+ + + grade
```

*As many operations as possible are done in parallel.*

- Is there a lower cost hardware implementation? Different tree organization?
- Can factor out multiply by 0.2:

```plaintext
count + + grade
```

- How about sharing operators (multipliers and adders)?
Time-Multiplexing

- *Time multiplex* single ALU for all adds and multiplies:
- Attempts to minimize cost at the expense of time.
  - Need to add extra register, muxes, control.

- If we adopt above approach, we can then consider the combinational hardware circuit diagram as an abstract computation-graph.

- This time-multiplexing “covers” the computation graph by performing the action of each node one at a time. (Sort of *emulates* it.)

HW versus SW

- This *time-multiplexed ALU* approach is very similar to what a conventional software version would accomplish:
  - add r2,r1,r3
  - add r2,r2,r4
  - mult r2,r4,r5

- CPUs time-multiplex function units (ALUs, etc.)

- This model matches our tendency to express computation sequentially - even though most computations naturally contain parallelism.
- Our programming languages also strengthen a sequential tendency.
- In hardware we have the ability to exploit problem parallelism - gives us a "knob" to tradeoff performance & cost.
- Maybe best to express computations as abstract computations graphs (rather than "programs") - should lead to wider range of implementations.
- *Note*: modern processors spend much of their cost budget attempting to restore execution parallelism: “super-scalar execution".
Exploiting Parallelism in HW

- Example: Video Codec

- Separate algorithm blocks implemented in separate HW blocks, or HW is time-multiplexed.
- Entire operation is pipelined (with possible pipelining within the blocks).
- "Loop unrolling used within blocks" or for entire computation.

Optimizing Iterative Computations

- Hardware implementations of computations almost always involves looping. Why?
- Is this true with software?
- Are there programs without loops?
  - Maybe in "throw away" code.
- We probably would not bother building such a thing into hardware, would we?
  - (FPGA may change this.)
- Fact is, our computations are closely tied to loops. Almost all our HW includes some looping mechanism.
- What do we use looping for?
Optimizing Iterative Computations

Types of loops:
1) Looping over input data (streaming):
   - ex: MP3 player, video compressor, music synthesizer.
2) Looping over memory data
   - ex: vector inner product, matrix multiply, list-processing
     • 1) & 2) are really very similar. 1) is often turned into 2) by buffering up input data, and processing “offline”. Even for “online” processing, buffers are used to smooth out temporary rate mismatches.
3) CPUs are one big loop.
   - Instruction fetch ➔ execute ➔ Instruction fetch ➔ execute ➔ …
   - but change their personality with each iteration.
4) Others?

Loops offer opportunity for parallelism
by executing more than one iteration at once,
using parallel iteration execution &/or pipelining

Pipelining Principle

• With looping usually we are less interested in the latency of one iteration and more in the loop execution rate, or throughput.
• These can be different due to parallel iteration execution &/or pipelining.
• Pipelining review from CS61C:
  Analog to washing clothes:

  
  | step 1: wash (20 minutes) |
  | step 2: dry (20 minutes)  |
  | step 3: fold (20 minutes) |
  |
  60 minutes x 4 loads ➔ 4 hours

  wash | load1 | load2 | load3 | load4 |
  dry  | load1 | load2 | load3 | load4 |
  fold | load1 | load2 | load3 | load4 |

  20 min overlapped ➔ 2 hours
Pipelining

- In the limit, as we increase the number of loads, the average time per load approaches 20 minutes.
- The latency (time from start to end) for one load = 60 min.
- The throughput = 3 loads/hour
- The pipelined throughput ~ # of pipe stages x un-pipelined throughput.

Pipelining

- General principle:
  
  ![Diagram](image)
  
  Assume T=8ns
  \[ T_{FF}(\text{setup} + \text{clk} \rightarrow q) = 1\text{ns} \]
  \[ F = 1/9\text{ns} = 111\text{MHz} \]
  
  Cut the CL block into pieces (stages) and separate with registers:

  ![Diagram](image)

  Assume T1 = T2 = 4ns

  \[ T' = 4\text{ns} + 1\text{ns} + 4\text{ns} + 1\text{ns} = 10\text{ns} \]
  \[ F = 1/(4\text{ns} + 1\text{ns}) = 200\text{MHz} \]

- CL block produces a new result every 5ns instead of every 9ns.
Limits on Pipelining

- Without FF overhead, throughput improvement \( \approx \# \) of stages.
- After many stages are added FF overhead begins to dominate:

\[
\text{throughput} = \frac{1}{T} = \frac{1}{n + \text{FF overhead}}
\]

- Other limiters to effective pipelining:
  - clock skew contributes to clock overhead
  - unequal stages
  - FFs dominate cost
  - clock distribution power consumption
  - feedback (dependencies between loop iterations)

Pipelining Example

- \( F(x) = y_i = a x_i^2 + b x_i + c \)
- \( x \) and \( y \) are assumed to be "streams"
- Divide into 3 (nearly) equal stages.
- Insert pipeline registers at dashed lines.
- Can we pipeline basic operators?
Example: Pipelined Adder

- Possible, but usually not done.
  (arithmetic units can often be made sufficiently fast without internal pipelining)

Pipelining Loops with Feedback

"Loop carry dependency"

- Example 1: $y_i = y_{i-1} + x_i + a$

  Unpipelined version:
  
  \[
  \begin{array}{c|c|c}
  \text{add}_1 & x_i + y_{i-1} & x_{i+1} + y_i \\
  \text{add}_2 & y_i & y_{i+1} \\
  \end{array}
  \]

  Can we “cut” the feedback and overlap iterations?

  Try putting a register after add1:
  
  \[
  \begin{array}{c|c|c}
  \text{add}_1 & x_i + y_{i-1} & x_{i+1} + y_i \\
  \text{add}_2 & y_i & y_{i+1} \\
  \end{array}
  \]

  • Can’t overlap the iterations because of the dependency.
  • The extra register doesn’t help the situation (actually hurts).
  • In general, can’t pipeline feedback loops.
Pipelining Loops with Feedback

“Loop carry dependency”

However, we can overlap the “non-feedback” part of the iterations:

Add is associative and commutative. Therefore we can reorder the computation to shorten the delay of the feedback path:

\[ y_i = (y_{i-1} + x_i) + a = (a + x_i) + y_{i-1} \]

“Shorten” the feedback path.

- Pipelining is limited to 2 stages.

Beyond Pipelining - SIMD Parallelism

- An obvious way to exploit more parallelism from loops is to make multiple instances of the loop execution data-path and run them in parallel, sharing the same controller.
- For P instances, throughput improves by a factor of P.
- example: \( y_i = f(x_i) \)
- Usually called SIMD parallelism. Single Instruction Multiple Data

- Assumes the next 4 x values available at once. The validity of this assumption depends on the ratio of f repeat rate to input rate (or memory bandwidth).
- Cost \( \sim P \). Usually, much higher than for pipelining. However, potentially provides a high speedup. Often applied after pipelining.
- Limited, once again, by loop carry dependencies. Feedback translates to dependencies between parallel data-paths.
- Vector processors use this technique.
SIMD Parallelism with Feedback

- Example, from earlier:
  \[ y_i = a \cdot y_{i-1} + x_i + b \]

- In this example end up with “carry ripple” situation.
- Could employ look-ahead / parallel-prefix optimization techniques to speed up propagation.
- As with pipelining, this technique is most effective in the absence of a loop carry dependence.

Conclusions

- Parallelism
- Pipelining
- Single Instruction Multiple Data (SIMD)