University of California at Berkeley
College of Engineering
Department of Electrical Engineering and Computer Science

SOLUTIONS

EECS 150 K. Pister
Fall 2006

Problem Set #2

Only some problems will be graded. The rest are for your entertainment your torment for you to study from. Good potential quiz problems are marked, but there are no guarantees that material from other problems won't show up on the quiz (especially if it's basic stuff)

1. Write down two binary numbers A and B whose sum requires the use of every row of the full adder truth table.

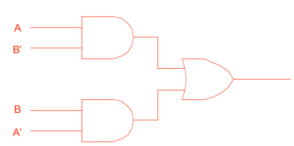
```
1 1 1 1 0 0 0 0    Cin
0 1 0 1 1 1 0 0    B
0 0 1 1 1 0 1 0    A
--------------------------
1 0 0 1 0 1 1 0
```

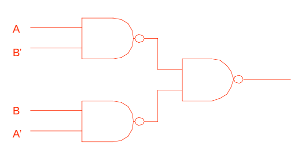
2. [Graded] How many rows are there in a truth table with 6 input variables? How many possible functions are there of 6 variables?

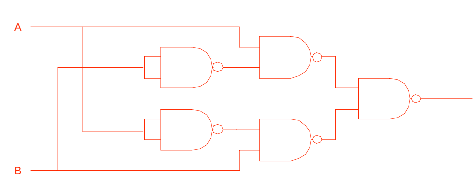
Rows = $2^6 = 64$
Possible Functions = $2^{rows} = 2^{64}$

3. Implement a 2-input XOR gate using only 2-input NAND gates. What is the worst case gate delay of the XOR gate in terms of NAND delays? Same two questions but use only 2-input NOR gates.

In Sum of Product Form:
A XOR B = AB' + BA'

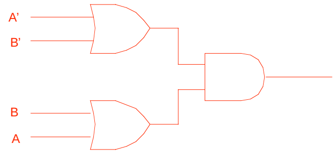


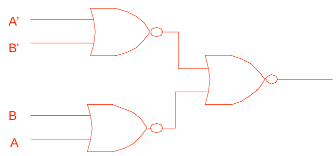


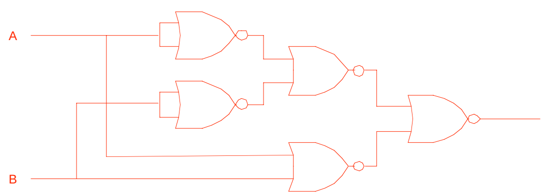


Worst case delay is 3 NAND delays

In Product of Sum Form,
A XOR B = (A'+B')(A+B)







Worst case delay is 3 NOR delays

4. Is it possible to write two different Boolean expressions which have the same truth table? Is it possible to write two different truth tables which have the same Boolean expression?

a. Yes, Take A and A*1
b. No. If a Boolean expression satisfies one truth table, it cannot satisfy the other.

5. [Graded] Using the BCD-to-7-seg functions given in CLD2 figure 2.25,

a. Write C0 in canonical minterm and maxterm form, and draw the logic gates to implement each.

$C_0 = \Sigma m\ (0,2,3,5,6,7,8,9)$
$= \pi M\ (1,4)$

b. Write C4 in canonical minterm and maxterm form, and draw the logic gates to implement each.

$C_4 = \Sigma m\ (0,2,6,8,9)$
$= \pi M\ (1,3,4,5,7)$

c. Is there one form which gives the smallest number of gates?

No, there does not exist one form that is _universally_ smaller than the other. In most cases, one form will require fewer gates but which form this is depends on the function.

d. Looking at the truth table, how would decide whether to use a minterm or maxterm implementation to get the fewest gates?

If there are more 0's then 1's, use max-terms; if there are more 1's than 0's, use min-terms; else the two require the same number of gates.

6. [Graded, quiz material] Given the function F = $\Sigma m(0,1,2,4,5,6,8,10,13,14)$

   a. Draw a Karnaugh map of this function

A

1	1	0	1
1	1	1	0
0	0	0	0
1	1	1	1

D

C

B

      i. Find the prime implicants

         A'C', BC'D, B'C'D', AB'D', A'D', CD'

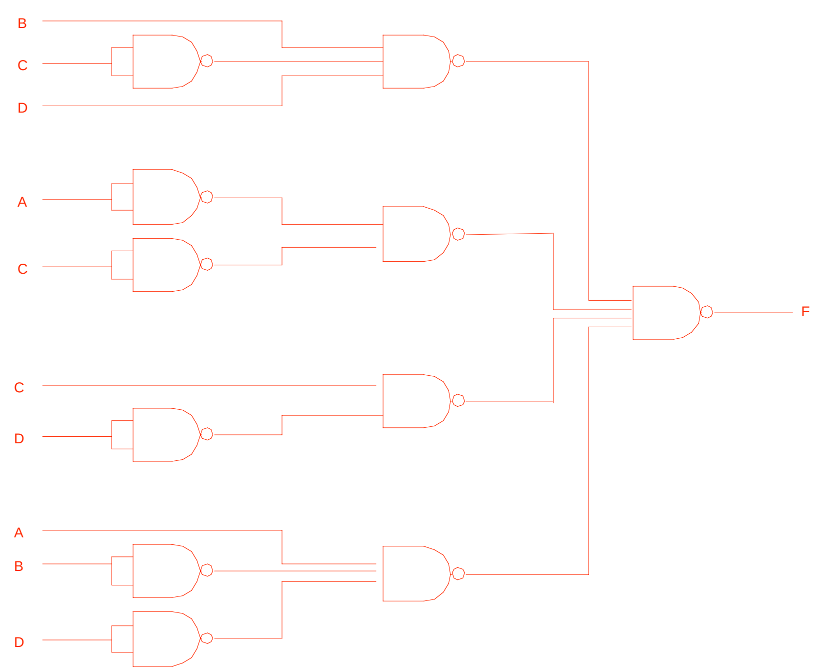
      ii. Find the essential prime implicants

         BC'D, A'C', CD'

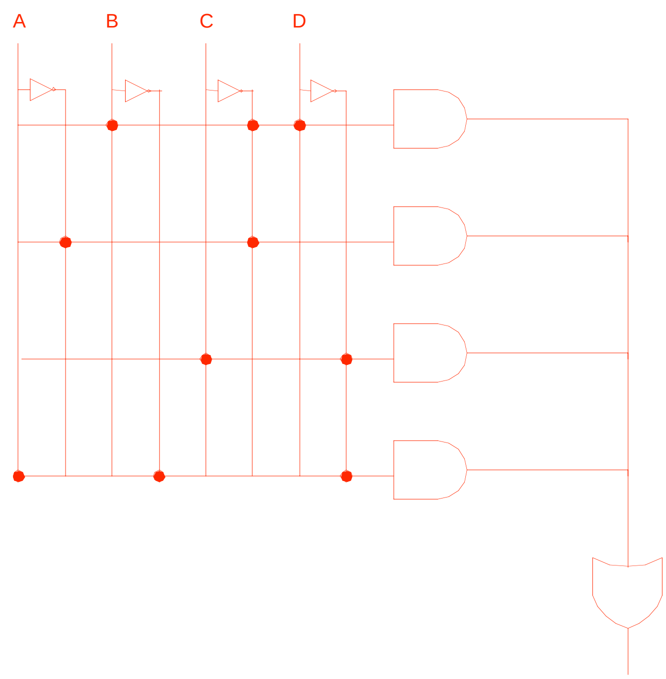
   b. Write down a minimal sum of products formula for the function

      F = BC'D + A'C' + CD' + AB'D'

      i. draw an implementation of the function using only NAND gates



      ii. draw an implementation of the function using a PLA



c. draw a Karnaugh map of the complement of this function, !F

A

0	0	1	0
0	0	0	1
1	1	1	1
0	0	0	0

D

C

B

i. Find the prime implicants
   ABC’D’, CD, AB’D

ii. Find the essential prime implicants
   ABC’D’, CD, AB’D

d. Write down a minimal sum of products implementation of !F.
   F! = ABC’D’ + CD + AB’D

e. Write down a minimal product of sums implementation of F
   F = !F’ = (A’ + B’ + C + D)*(C’ + D’)*(A’ + B + D’)

7. [quiz material] Same questions as in 6, for the following functions
   a. F = Σm(0,2,5,7,8,10,13,15)
   b. F = Σm(3,4,6,7,8,12,13,15)
   c. F = Σm(3,4,5,6,7,11,15)
   d. F = πM(0,3,5,6,9,10,12,15)

e. F = πM(0,1,4,5,7,13,15)

8. [quiz material] Pick a random mapping from the decimal numbers 0 through 9 to the four bit binary numbers 0000 through 1111 (BCD is one example of such a mapping). Repeat parts 6.a.ii and 6.b.ii for these functions, trying to minimize the number of product terms in the PLA.

Let's use:

A	B	C	D	X_3	X_2	X_1	X_0
0	0	0	0	1	0	1	1
0	0	0	1	0	1	0	0
0	0	1	0	1	0	0	1
0	0	1	1	0	0	1	0
0	1	0	0	1	0	0	0
0	1	0	1	0	1	1	1
0	1	1	0	0	1	1	0
0	1	1	1	1	1	1	0
1	0	0	0	0	0	0	0
1	0	0	1	1	0	1	0
1	0	1	0	X	X	X	X
.	.	.	.	.	.	.	.
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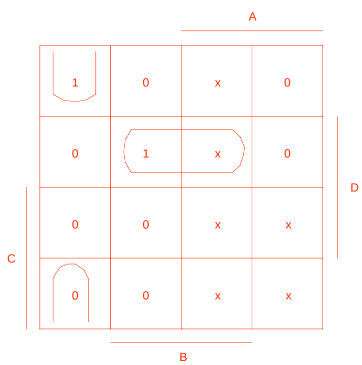
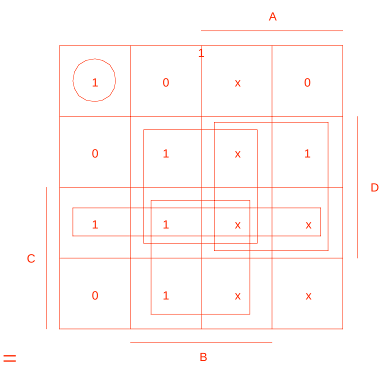
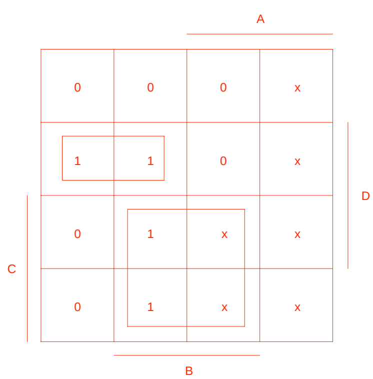


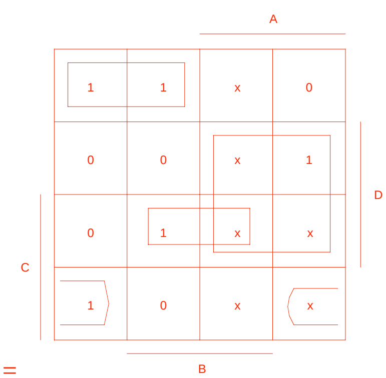
$X_0 =$



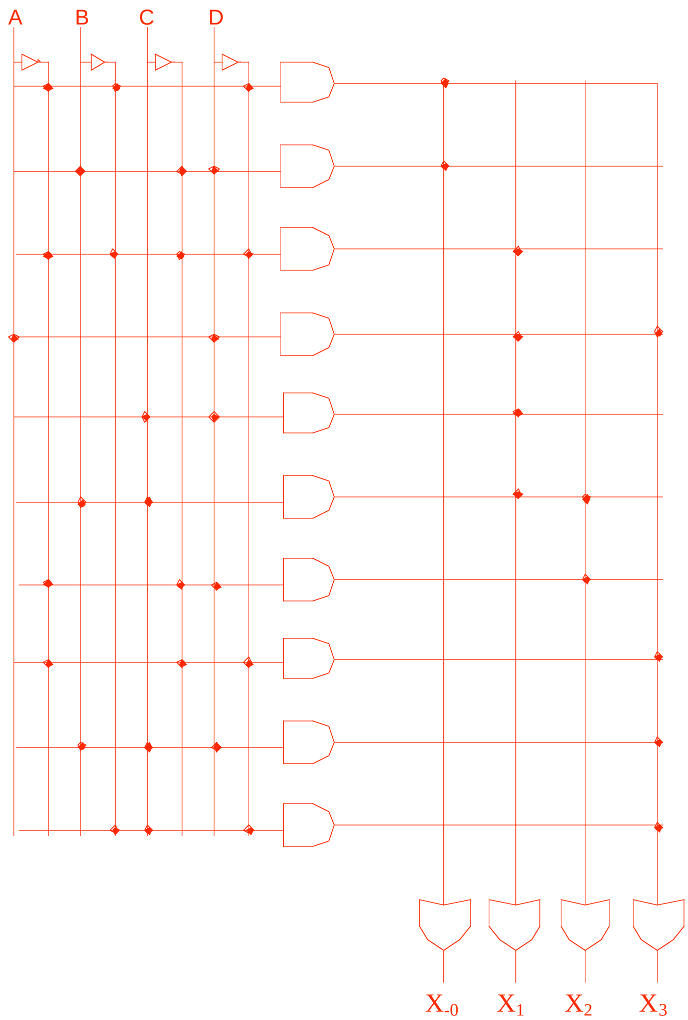
$X_1 =$



$X_2 =$



$X_3 =$



9. How many different mappings are there in problem 8?

$_{16}P_{10} = 16!/6! = 29{,}059{,}430{,}400$