1. Note the pattern:

square #	# grains	... in binary	bits
1	1	1	1
2	2	10	2
3	4	100	3
4	8	1000	4
⋮	⋮	⋮	⋮
k	2^{k-1}	$1\underbrace{00\ldots0}_{k-1 \text{ times}}$	k

a. From the table, $\boxed{k}$

b. From the table, $\boxed{2^{k-1}}$

c. A chess board has 64 squares so we have,

$$\sum_{k=0}^{63} 2^k = 2^{64} - 1 = \boxed{\underbrace{11\ldots1}_{64}{}_2 = 1.845 \times 10^{19} \text{ grains}}$$

d. If one grain is 10 mm^3, the total volume is

$(1.845 \times 10^{19}$ ~~grains~~$)(10\ mm^3/$~~grain~~$) = (1.845 \times 10^{20}\ mm^3)(10^{-9}\ m^3/mm^3)$

$= 1.845 \times 10^{11}\ m^3$

Therefore, the depth of the pile would be

$1.845 \times 10^{11}\ m^3 / (.3\ m)^2 = \boxed{2.05 \times 10^{12}\ m}$

2a.

	50 000	gates
8192×8 =	65 536	"gates" from RAM
65 536/10×8 =	52 428.8	"gates" from ROM
	167 964.8	total "gates" per ASIC

The size of an ASIC is

$(167\,964.8\ \text{gates/ASIC})(100\,000^{-1}\ \text{mm}^2/\text{gate}) = \boxed{1.680\ \text{mm}^2/\text{ASIC}}$

b. A 200 mm wafer has a surface area of

$\pi(100\ \text{mm})^2 = 31\,416\ \text{mm}^2$

on which we can fit

$(31\,416\ \text{mm}^2/\text{wafer})(1.680^{-1}\ \text{ASICs}/\text{mm}^2) = 18\,700\ \text{ASICs/wafer}$

With 90% yield, our usable ASICs/wafer is

$(18\,700\ \text{ASICs/wafer})(.9) = 16\,830\ \text{ASICs/wafer}$

With a 20% testing overhead, the cost of a wafer is \$1200 so the cost per ASIC is

$(\$1200/\text{wafer})(16\,830^{-1}\ \text{wafers/ASIC}) = \boxed{\$.0713/\text{wafer}}$

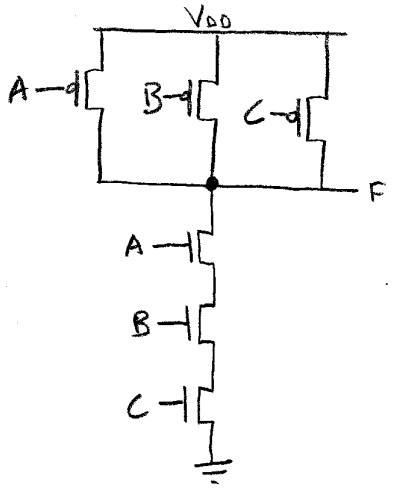
c. We save \$10 − \$.0713 = \$9.9287 per product, thus recouping the mask cost in

\$200 000/\$9.9287 = **20 144 products**

d. Using the same method as above,

\$5 200 000/\$9.9287 = **523 735 products**

3. As in lecture, a 3-input NAND gate scales from the 2-input version:



Since this design requires six transistors as opposed to the standard four (of which we can fit 100 000/mm²), we assume that 66 666 can fit per mm². We can therefore fit

$(66\,666 \text{ gates/mm}^2)(31\,416 \text{ mm}^2\text{/wafer}) = 2.094\times10^9 \text{ gates/wafer}$

of which

$(2.094\times10^9 \text{ gates/wafer})(.9) = 1.858\times10^9 \text{ gates/wafer}$

work. It therefore costs

$(\$1200/\text{wafer})(1.858\times10^{9^{-1}} \text{ wafers/gate}) = \boxed{\$6.459\times10^{-7}/\text{gate}}$

You could therefore get

$(6.459\times10^{-7^{-1}} \text{ gates/\$})(.01\ \$/¢) = \boxed{15\,482 \text{ gates/penny}}$