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EECS 150 R. H. Katz
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Solutions Set # 2

1. Simplify the following expressions using the laws and theorems of Boolean Algebra:
   (a) S(A,B,C) = A’ B’ C + A’ B C’ + A B’ C’ + A B C
   = A’ (B’C + BC’) + A (B’C’ + BC)
   = A’ (B XOR C) + A ((B XOR C)’)
   = A XOR B XOR C

   (b) F(A,B,C) = A’ B’ C’ + A’ B’ C + A B’ C’ + A B’ C + A B C’ + A B C
   = A’B’ (C’ + C) + AB’ (C’ + C) + AB (C’ + C)
   = A’B’ + AB’ + AB
   = A’B’ + AB’ + AB’ +AB
   = B’ (A’ + A) + A (B’ + B)
   = B’ + A

   (c) G(A,B,C,D) = A’ B’ C’ D’ + A’ B’ C D’ + A B’ C’ D’ + A B’ C D + A B C’ D’ + A B C D
   = A’B’D’ (C’ + C) + AC’D’ (B’ +B) + ACD (B’ +B)
   = A’B’D’ + AC’D’ +ACD
   = A’B’D’ + A ((C XOR D)’)

2. Use K-maps on the expressions of Problem 1. Show your work in K-map form. For 1(a)-(c):
   (a) Find the minimized sum of products form.

1a)

C \AB	00	01	11	10
0	0	1	0	1
1	1	0	1	0

S = A’B’C + A’BC’ ABC + AB’C’

1b)

C \AB	00	01	11	10
0	1	0	1	1
1	1	0	1	1

F = B’ + A

1c)

CD\AB	00	01	11	10
00	1	0	1	1
01	0	0	0	0
11	0	0	1	1
10	1	0	0	0

G = AC’D’ + ACD + A’B’D’

(b) Find the minimized product of sums form.

1a)

C \AB	00	01	11	10
0	0	1	0	1
1	1	0	1	0

S’ = A’B’C’ + A’BC + ABC’ + AB’C
= (A’B’C’)’ (A’BC)’ (ABC’)’ (AB’C)’
= (A + B + C) (A + B’ + C’) (A’ + B’ + C) (A’ + B + C’)

1b)

C \AB	00	01	11	10
0	1	0	1	1
1	1	0	1	1

F’ = BA’
= A + B’

1c)

CD\AB	00	01	11	10
00	1	0	1	1
01	0	0	0	0
11	0	0	1	1
10	1	0	0	0

G’ = C’D + A’B + A’D + ACD’
= (C’D)’ (A’B)’ (A’D)’ (ACD’)’
= (C + D’) (A + B’) (A + D’) (A’ + C’ + D)

(c) Find the minimized sum of products form of the function’s complement.

k-maps are the same from part b

1a) S’ = A’B’C’ + A’BC + ABC’ + AB’C

1b) F’ = A’B

1c) $G' = A'D + A'B + C'D + ACD'$

(d) Find the minimized product of sums form of the function's complement.

K-maps are the same from part a

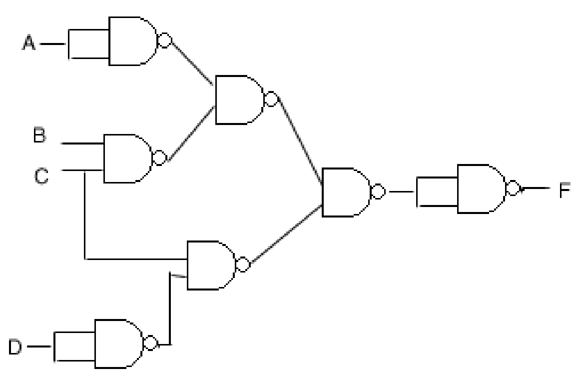
1a) $F = A'B'C + A'BC' + ABC + AB'C'$
$F' = (A + B + C')(A + B' + C)(A' + B' + C')(A' + B + C)$

1b) $F = A + B'$
$F' = A'B$

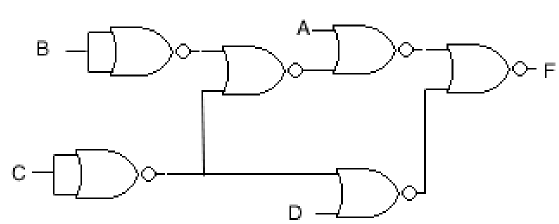
1c) $G = AC'D' + ACD + A'B'D'$
$G' = (A' + C + D)(A' + C' + D')(A + B + D)$

3i)

a) One possible solution: $F(A,B,C,D) = (A + (B C))(C' + D) = ((A'(B C)')')(C D')')'$



b) One possible solution: $F(A,B,C,D) = (A + (B C))(C' + D) = ((A + (B'+C')')' + (C' + D)')'$



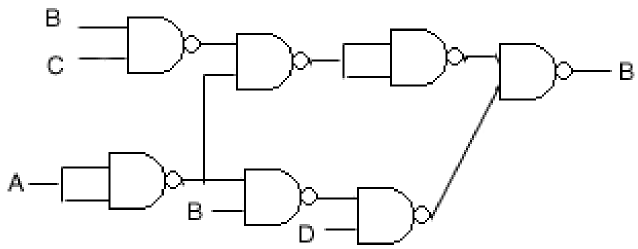
c) $F = AC' + AD + BCD$
d) $F = (A+B)(A+C)(C'+D)$

CD \ AB	00	01	11	10
00	0	0	1	1
01	0	0	1	1
11	0	1	1	1
10	0	0	0	0

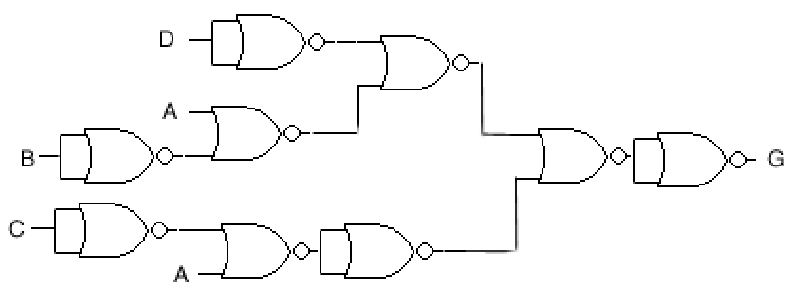
e) The simplest implementation is the NOR gate implementation. It uses the least gates of all the above implementations.

3ii)

a) One possible solution: G(A,B,C,D) =((A+B')D) + (A+(BC)) = ((A'B')D)' ((A'(BC)')')')'



b) One possible solution: G(A,B,C,D) = ((A+B')D) + (A+(BC))
= (((A+B')' + D')' + ((B' + C')' + A)'')''



c) G = A + BC + B'D

d) G = (A+B'+C)(A+B+D)

CD \ AB	00	01	11	10
00	0	0	1	1
01	1	0	1	1
11	1	1	1	1
10	0	1	1	1

e) The sum of products implementation is the simplest to implement in this case. It uses the least gates of the four implementations.

4a)

A	B	C	D	W	X	Y	Z
0	0	0	0	X	X	X	X
0	0	0	1	0	0	0	0
0	0	1	0	0	0	0	0
0	0	1	1	0	0	0	0
0	1	0	0	X	X	X	X
0	1	0	1	0	1	0	0
0	1	1	0	0	0	0	1
0	1	1	1	0	0	0	1
1	0	0	0	X	X	X	X
1	0	0	1	1	0	0	0
1	0	1	0	0	1	0	0
1	0	1	1	0	0	1	0
1	1	0	0	X	X	X	X
1	1	0	1	1	1	0	0
1	1	1	0	0	1	0	1
1	1	1	1	0	1	0	0

4b) W = AC’

CD \ AB	00	01	11	10
00	X	X	X	X
01	0	0	1	1
11	0	0	0	0
10	0	0	0	0

X = BC’ + AB + AD’

CD \ AB	00	01	11	10
00	X	X	X	X
01	0	1	1	0
11	0	0	1	0
10	0	0	1	1

Y = AB’CD

CD \ AB	00	01	11	10
00	X	X	X	X
01	0	0	0	0
11	0	0	0	1
10	0	0	0	0

Z = BD’ + A’BC

CD \ AB	00	01	11	10
00	X	X	X	X
01	0	0	0	0
11	0	1	0	0
10	0	1	1	0

4c) W = A+C’ X = (A+C’)(B+D’) Y = AB’CD Z = BC(B’+D’)

5a)

A	B	C	D	By2	By3	By6
0	0	0	0	1	1	1
0	0	0	1	0	0	0
0	0	1	0	1	0	0
0	0	1	1	0	1	0
0	1	0	0	1	0	0
0	1	0	1	0	0	0
0	1	1	0	1	1	1
0	1	1	1	0	0	0
1	0	0	0	1	0	0
1	0	0	1	0	1	0
1	0	1	0	1	0	0
1	0	1	1	0	0	0
1	1	0	0	X	X	X
1	1	0	1	X	X	X
1	1	1	0	X	X	X
1	1	1	1	X	X	X

5b) By2 = D’

CD \ AB	00	01	11	10
00	1	1	X	1
01	0	0	X	0
11	0	0	X	0
10	1	1	X	1

By3 = A’B’C’D’ + A’B’CD + AC’D + BCD’

CD \ AB	00	01	11	10
00	1	0	X	0
01	0	0	X	1
11	1	0	X	0
10	0	1	X	0

By6 = A’B’C’D’ + BCD’

CD \ AB	00	01	11	10
00	1	0	X	0
01	0	0	X	0
11	0	0	X	0
10	0	1	X	0

5c) By2 cannot be simplified any further.
By3 can be rewritten as A’B’(C’D’ + CD) + AC’D + BCD’ or A’B’(’C’D’ XOR CD) + AC’D + BCD’
By6 can be rewritten as D’(A’B’C’+BC) but is not that much simpler than the original other than the fact that the four input AND gate can be replaced by a three input AND gate.