CS3:
Introduction to Symbolic Programming

Lecture 8:
The last bit of recursion
Miniproject #2

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Schedule

7	Oct 9-13	Advanced recursion
8	Oct 16-20	Finishing recursion Miniproject #2: Number names
9	Oct 23-27	Introduction to Higher Order Procedures
10	Oct 30 -Nov 3	More HOF
11	Nov 6-10	Finish HOF Miniproject #3: Election processing
12	Nov 13-17	Lecture: *Midterm #2* Lab: Start on "Lists"

Any "notetaker" volunteers?

- A student in the course needs a note taker, which does pay a stipend. If you are taking notes anyway…
  - Come and see me after lecture if interested

The "screwed up" labs

- This is the order things should have happened:
  - First "advanced recursion" Lab: recursions with multiple arguments
    - `my-equal?`, `zipping`, `merging`
  - Second Lab
    - patterns in recursion, no-vowels, `sort` (using `insert`), `roman-sum-helper`
  - Last Lab
    - `mad-libs` quiz, `1-extra?`, `fibonacci`, `thorough-reversal`

Number Spelling Miniproject

- Read *Simply Scheme*, page 233, which has hints
- Another hint (principle): don't force "everything" into the recursion.
  - Special/border cases may be easier to handle before you send yourself into a recursion

"Tail" recursions

- **Accumulating recursions are sometimes called "tail" recursions (by TAs, me, etc).**
  - **But, not all recursions that keep track of a number are "tail" recursions.**

- **A tail recursion has no combiner, so it can end as soon as a base case is reached**
  - **Compilers can do this efficiently**
- **An embedded recursion needs to combine up all the recursive steps to form the answer**
  - **The poor compiler has to remember everything**

Tail or embedded? (1/3)

```
(define (length sent)
   (if (empty? sent)
      0
      (+ 1 (length (bf sent)))))
```

Embedded!

```
(length '(a b c d)) →
   (+ 1 (length '(b c d)))
   (+ 1 (+ 1 (length '(c d))))
   (+ 1 (+ 1 (+ 1 (length '(d)))))
   (+ 1 (+ 1 (+ 1 (+ 1 (length '()))))))
   (+ 1 (+ 1 (+ 1 (+ 1 0))))
   (+ 1 (+ 1 (+ 1 1)))
   (+ 1 (+ 1 2))
   (+ 1 3)
   4
```

Tail or embedded? (2/3)

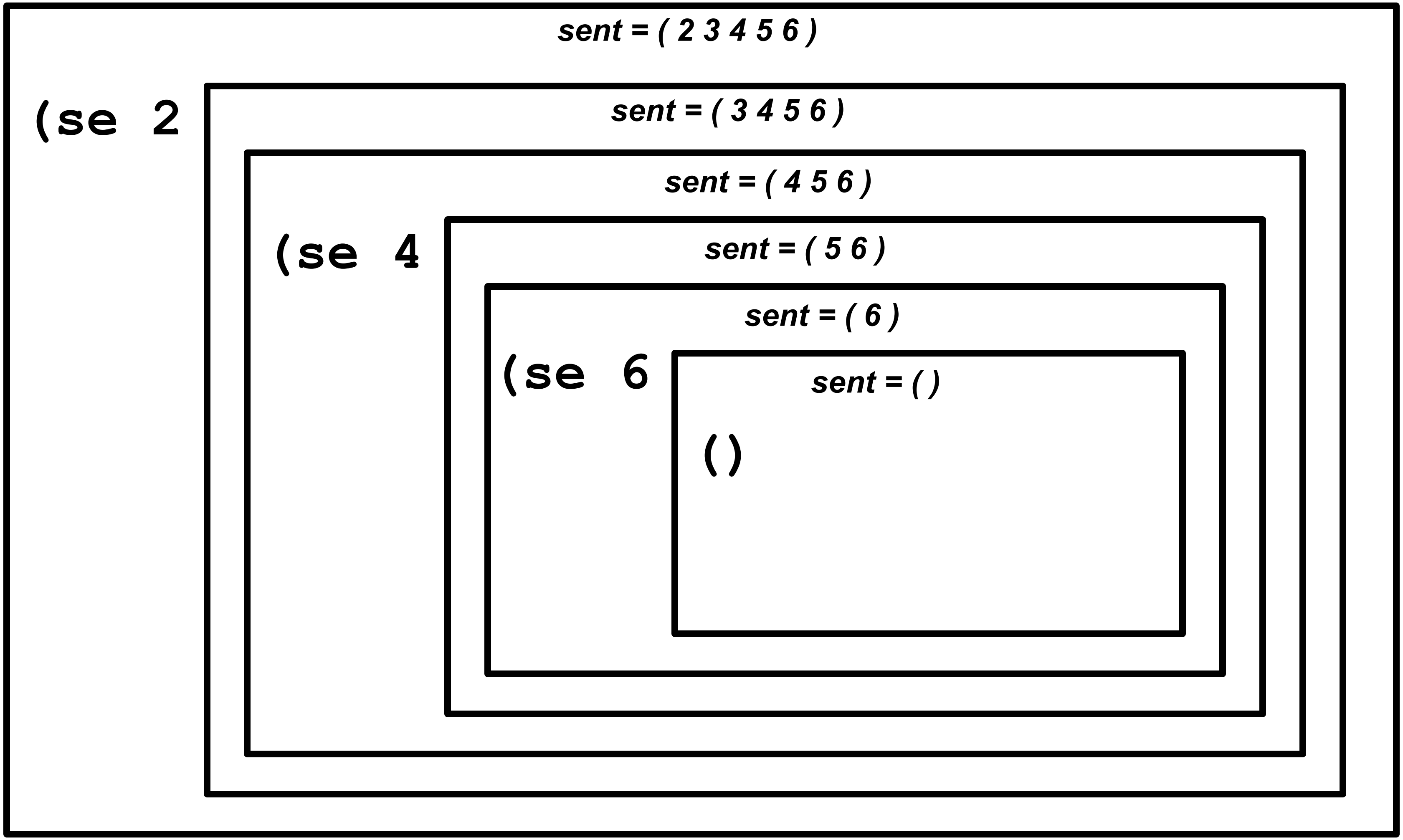
```
(define (sent-max sent)
  (if (empty? sent)
    '()
    (sent-max-helper (bf sent) (first sent))))


(define (sent-max-helper sent max-so-far)
  (if (empty? sent)
    max-so-far
    (sent-max-helper (bf sent)
                     (if (> max-so-far (first sent))
                       max-so-far
                       (first sent)))))
```

Tail or embedded? (3/3)

```
(define (find-evens sent)
  (cond ((empty? sent)              ;base case
         '()           )
        ((odd? (first sent)) ;rec case 1
         (find-evens (bf sent)) )
        (else                       ;rec case 2: even
         (se (first sent)
             (find-evens (bf sent))) )
        ))
```

```
> (find-evens '(2 3 4 5 6))
```



sent = (2 3 4 5 6)

(se 2

sent = (3 4 5 6)

sent = (4 5 6)

(se 4

sent = (5 6)

sent = (6)

(se 6

sent = ( )

()

```
➔  (se 2 (se 4 (se 6 ()))
➔  (2 4 6)
```

Tree recursion: fibonacci

- **The fibonacci sequence:**

1 1 2 3 5 8 13 21 34 55

```
(define (fib n)
   (if (<= n 2)
      1                   ;; base case
      (+ (fib (- n 1))    ;; recursive case
         (fib (- n 2)))))
```

Tree recursion: Pascals triangle

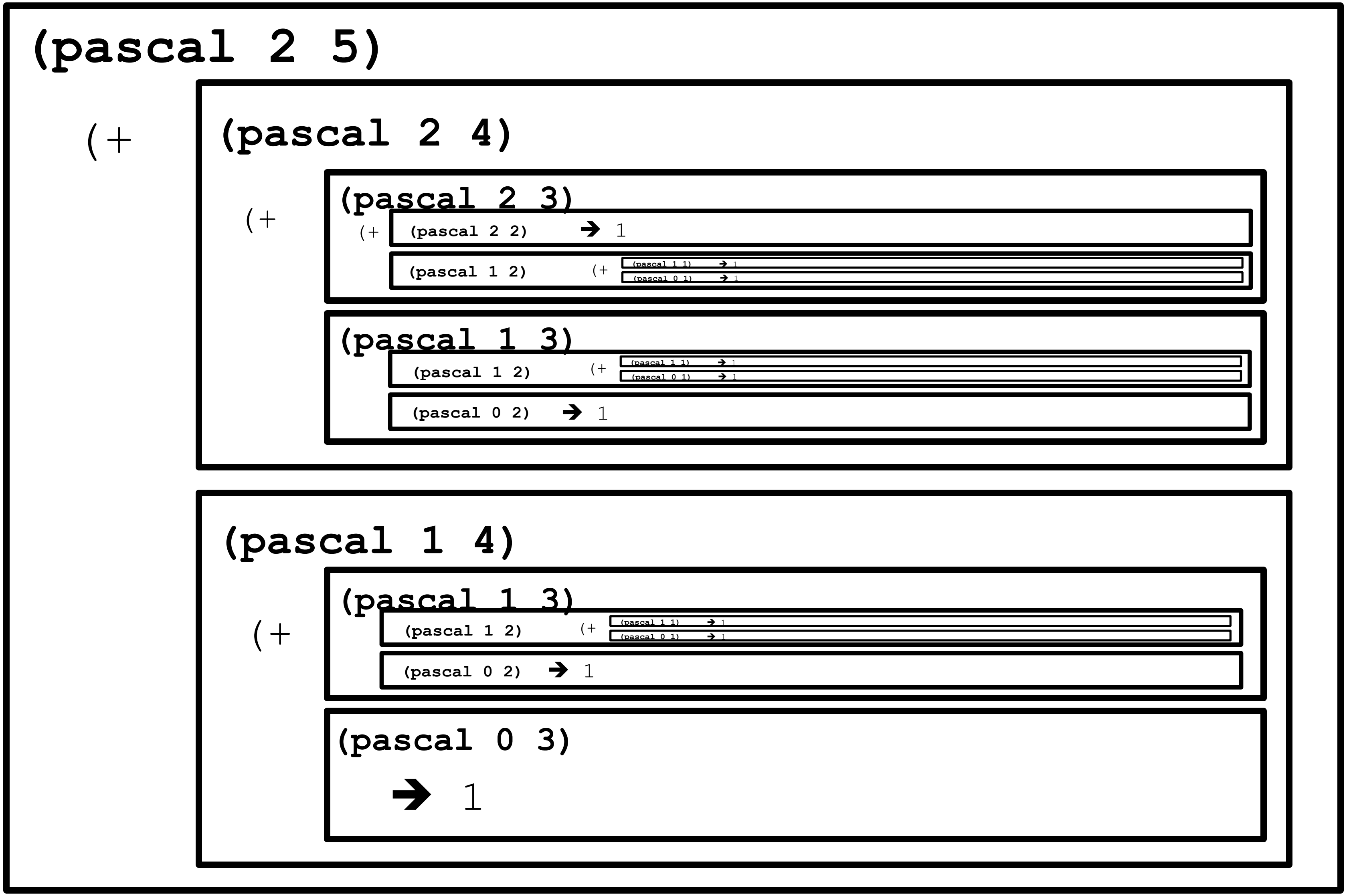
		columns (C)						
		0	1	2	3	4	5	...
rows (R)	0	1						...
	1	1	1					...
	2	1	2	1				...
	3	1	3	3	1			...
	4	1	4	6	4	1		...
	5	1	5	10	10	5	1	...
	...	...	...	...	...	...	...	...

Pascal’s Triangle

- How many ways can you choose C things from R choices?
- Coefficients of the (x+y)^R: look in row R
- etc.

```
(define (pascal C R)
  (cond
    ((= C 0) 1)   ;base case
    ((= C R) 1)   ;base case
    (else         ;tree recurse
     (+ (pascal    C    (- R 1))
        (pascal (- C 1) (- R 1)) )
     )))
```

> (pascal 2 5)



(pascal 2 5)

(+

(pascal 2 4)

(+

(pascal 2 3)

(+

(pascal 2 2) → 1

(pascal 1 2) (+

(pascal 1 1) → 1

(pascal 0 1) → 1

(pascal 1 3)

(pascal 1 2) (+

(pascal 1 1) → 1

(pascal 0 1) → 1

(pascal 0 2) → 1

(pascal 1 4)

(+

(pascal 1 3)

(pascal 1 2) (+

(pascal 1 1) → 1

(pascal 0 1) → 1

(pascal 0 2) → 1

(pascal 0 3)

→ 1

pair-all

- Write `pair-all`, which takes a sentence of `prefixes` and a sentence of `suffixes` and returns a sentence pairing all `prefixes` to all `suffixes.`
  - `(pair-all '(a b c) '(1 2 3))` ➔ `(a1 b1 c1 a2 b2 c2 a3 b3 c3)`
  - `(pair-all '(spr s k) '(ite at ing ong))` ➔ `(sprite sprat spring sprong site sat sing song kite kat king kong)`

binary

- **Write `binary`, a procedure to generate the possible binary numbers given `n` bits.**

```
(binary 1)→(0 1)
(binary 2)→(00 01 10 11)
(binary 3)→(000 001 010 011 100 101 110 111)
```

roman-sum-helper (from lab)

Write roman-sum-helper:

```
(define (roman-sum number-sent)
  (if (empty? number-sent)
      0
      (roman-sum-helper (first number-sent)
                        (bf number-sent)
                        (first number-sent)) ) )
```

Roman-sum-helper takes three arguments:

```
(define (roman-sum-helper so-far number-list most-
 recent) ... )
```

`(roman-sum '(100 10 50 1 5))` **will recurse with:**

```
(roman-sum-helper 100 '(10 50 1 5) 100)
(roman-sum-helper 110 '(50 1 5) 10)
(roman-sum-helper 140 '(1 5) 50)
(roman-sum-helper 141 '(5) 1)
(roman-sum-helper 156 '( ) 5)
```