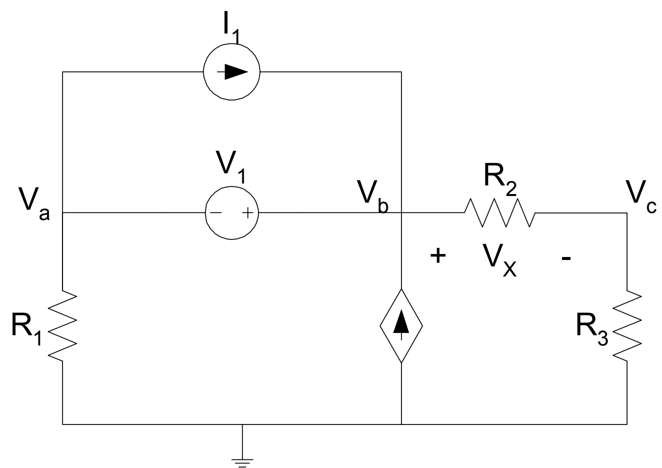
EE 42

Homework #2 Solutions

Problem 1:

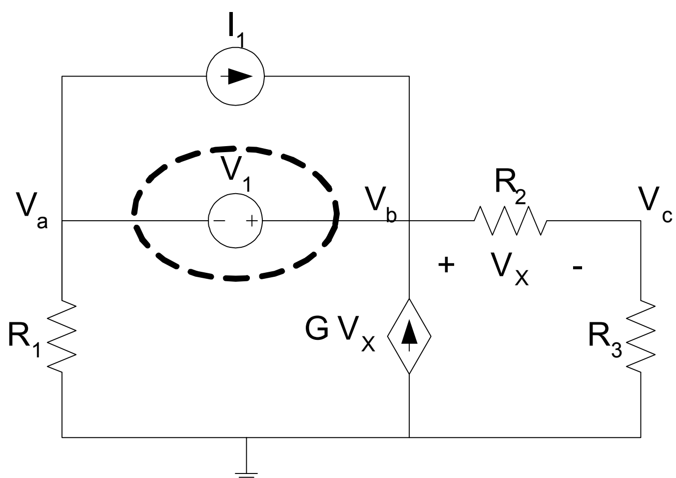
First, the nodes with unknown voltage are labeled V_a, V_b, and V_c:



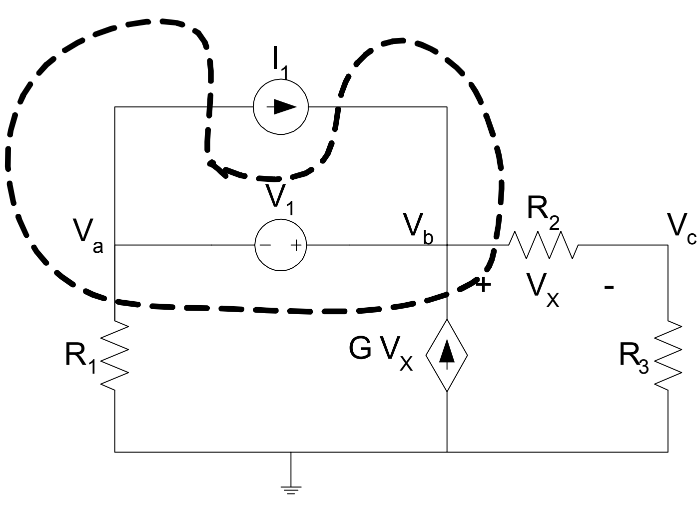
We now try to write KCL equations at each node with unknown node voltage.

The voltage source V_1 prevents us from writing KCL equations. We draw a supernode.

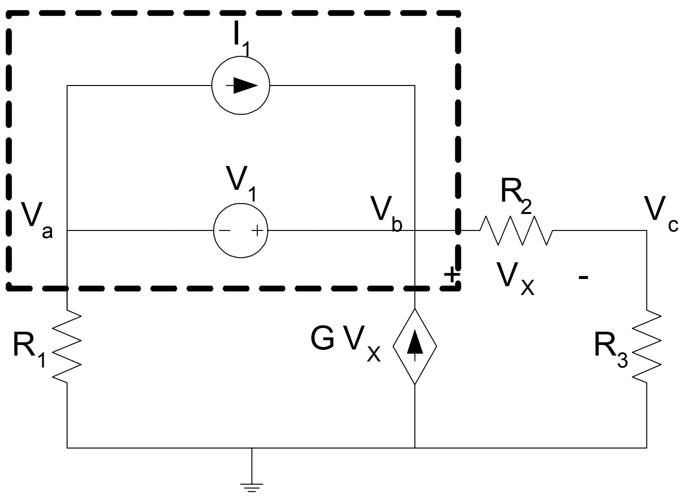
There are many valid ways to draw the supernode:



Here, one could write the currents entering the supernode as a combination of other currents.



Here, the supernode has been drawn as close as possible to neighboring elements. The currents entering the supernode can be easily identified.



Here, the supernode is drawn around the current source as well. Anything that is in parallel with the floating voltage source can be included in the supernode.

No matter what the shape of the supernode, the KCL equation is:

$$\frac{V_a}{R_1} - GV_X + \frac{V_b - V_c}{R_2} = 0$$

The floating source itself provides the equation: $V_1 = V_b - V_a$

Finally, the node with voltage V_c permits a KCL equation:

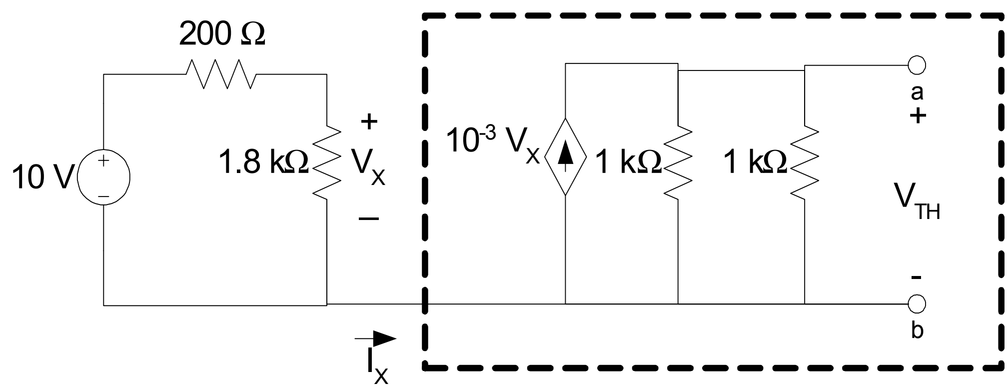
$$\frac{V_c}{R_3} + \frac{V_c - Vb}{R_2} = 0$$

The controlling voltage must be defined: $V_X = V_b - V_c$

These are all the equations needed to solve for the node voltages.

Problem 2:

First, find V_{TH} by analyzing the circuit with the terminals open.



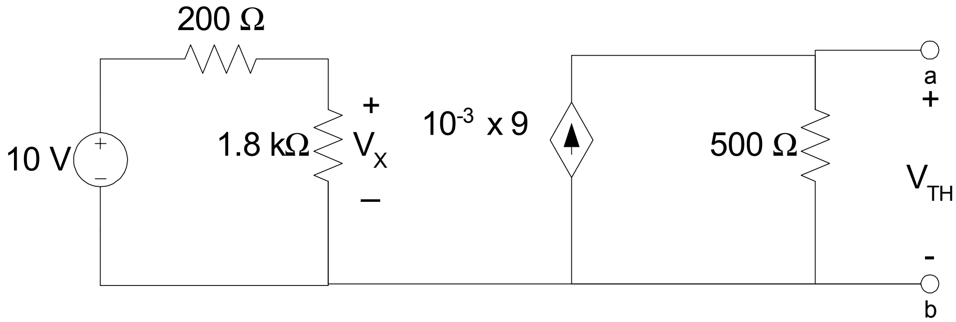
Notice that the current I_X must be zero. It is the only current going into the surface.

Also notice that the 200 Ω and 1.8 k Ω resistor are in series.

By voltage division, $V_X = 9$ V.

Since $I_X = 0$ A, the only current in the 1 k Ω resistors comes from the dependent source.

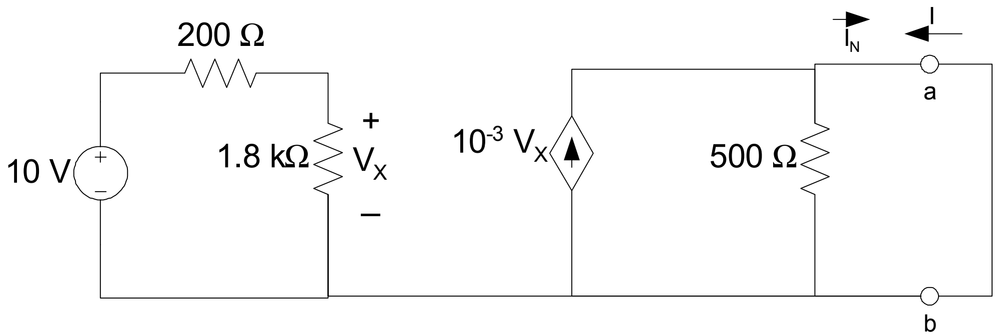
The resistors can be combined in parallel.



The 9 mA current flows down through the resistor. By Ohm's law, the voltage drop is 4.5 V.

Thus, V_{TH} = 4.5 V.

To find I_N, we short the terminals.



By shorting the terminals, we have changed the circuit. We cannot assume that the voltage and current values found in the V_{TH} calculation are still the same.

However, some things are the same. There is still no current in the bottom connecting wire, and the 200 Ω and 1.8 k Ω resistor are still in series. So by voltage division, V_X = 9 V.

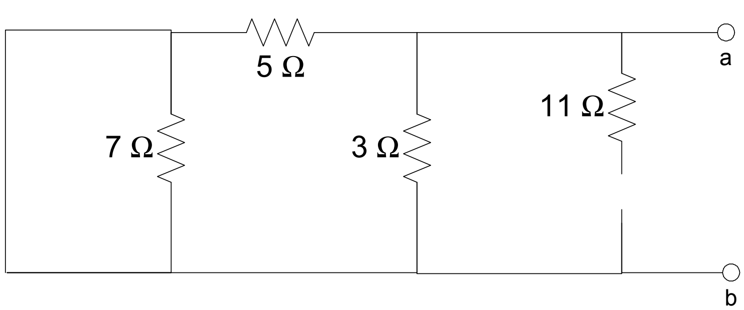
With the short in the circuit, the 500 Ω equivalent resistance can have no voltage. Thus, it can have no current. All of the current from the dependent source bypasses the resistor and flows down through the short.

Thus, $I_N = 10^{-3} V_X$ = 9 mA.

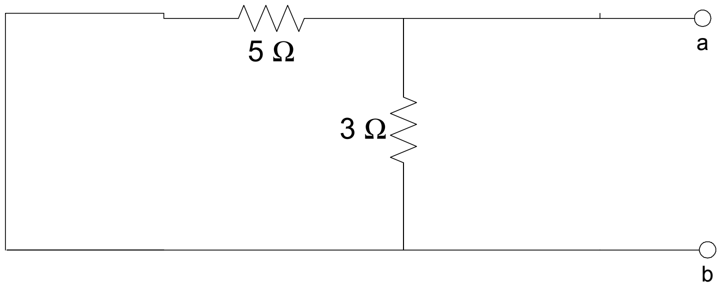
$R_{TH} = R_N = V_{TH} / I_N$ = 500 Ω.

Problem 3:

For this problem, finding R_{TH} (R_N) is straightforward. Turn off the independent sources:



The 7 Ω resistor is shorted out, so it cannot carry nonzero voltage. The 11 Ω resistor is disconnected, so it cannot carry current. Neither have an impact on the circuit.



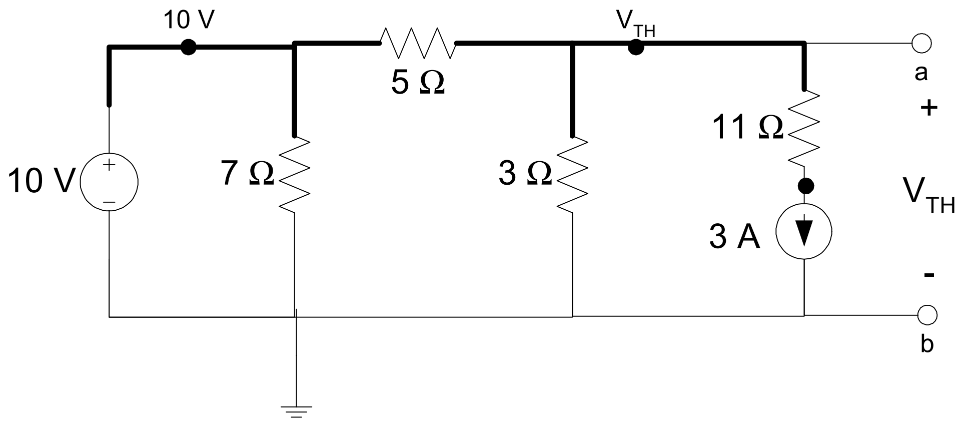
The remaining resistors are in parallel: notice they are directly connected by wire on both ends.

Therefore,

$$R_{TH} = R_N = \left(\frac{1}{5\Omega} + \frac{1}{3\Omega}\right)^{-1} = 1.875\Omega$$

Now we can either find I_N or V_{TH} and find the other through multiplication or division.

I think V_{TH} is easier to find in this case, as one nodal analysis equation will do it:



When finding V_{TH} by nodal analysis, it's a good idea to put ground at the – of V_{TH}. That way, V_{TH} is a node voltage.

To do a shortcut, we can skip the KCL equation for the node between the 11 Ω resistor and the 3 A source. We know 3 A goes in and 3 A comes out, so 3 A goes down the 11 Ω resistor.

The KCL equation is: $\frac{V_{TH} - 10}{5\Omega} + \frac{V_{TH}}{3\Omega} + 3A = 0$

Solving, we get: V_{TH} = 1.875 V

Lastly, $I_N = V_{TH} / R_{TH}$ = 1 A.

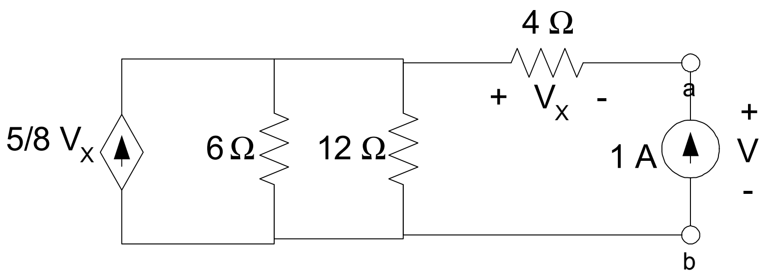
Problem 4:

In this problem, there are no independent sources. So we know right away that:

V_{TH} = 0 V $\quad$ I_N = 0 A.

This means that the I-V graph goes through the origin. It also means that the circuit acts like a pure resistance. To determine this resistance, apply a test voltage (or current) and find the resulting current (or voltage).

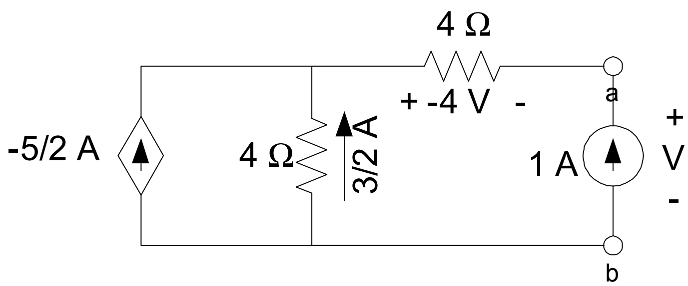
In this circuit, if we apply a test current, it will flow through the 4 Ω resistor from right to left. Then we know V_X right away by Ohm's law, which makes things easier.



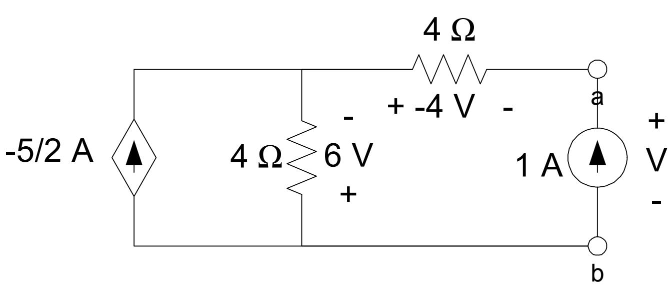
Here, a 1 A test current has been applied. We need to find V.

According to Ohm's law, V_X = -4 V. The negative sign occurs since the 1 A current is flowing from – to + .

Now we know V_X, and the circuit can be simplified even further by combining resistors:



By KCL, the current flowing up the equivalent resistor must be 3/2 A.



By Ohm's law, there is a 6 V drop as shown. V can now be determined using KVL (clockwise, starting at a):

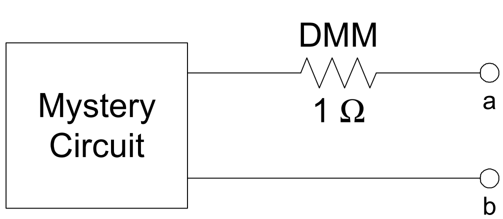
V + 6 V + -4 V = 0

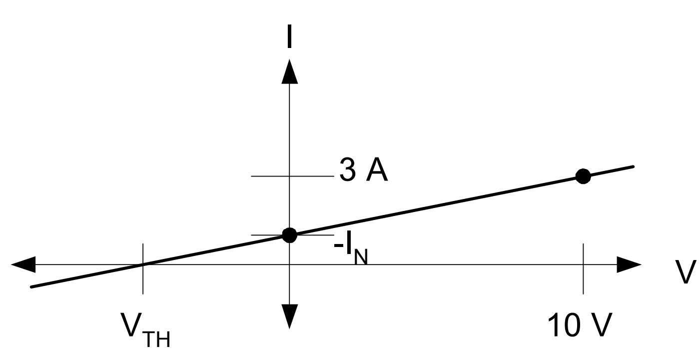
V = -2 V

This means that R_{TH} = -2 V / 1 A = -2 Ω . That seems strange, but the negative resistance was caused by the dependent source. This couldn't happen in real life!

Problem 5:

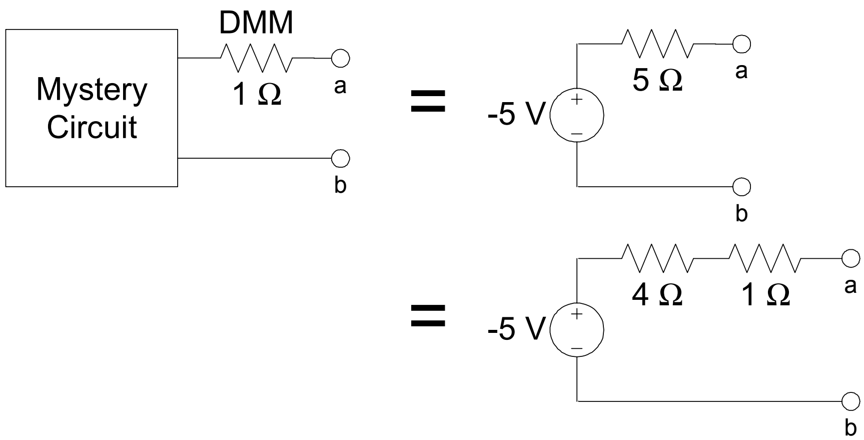
In this problem, we have two points on the I-V graph for the circuit:





Using basic algebra, we can see that for the circuit with the DMM in it,

V_{TH} = -5 V , I_N = -1 A, and R_{TH} = R_N = 5 Ω.



We can see that the mystery circuit will have the same V_{TH}.

The R_{TH} for the mystery circuit is 1 Ω less, or 4 Ω.

Thus, I_N for the mystery circuit is -5/4 A.

You could also solve this problem by replacing the mystery circuit with its Thevenin equivalent, with V_{TH} and R_{TH} left as unknowns. Writing KVL for the circuit with the source at 0 V, then again with the source at 10 V, will leave two equations in two unknowns and you can solve for V_{TH}, R_{TH}, and I_N.