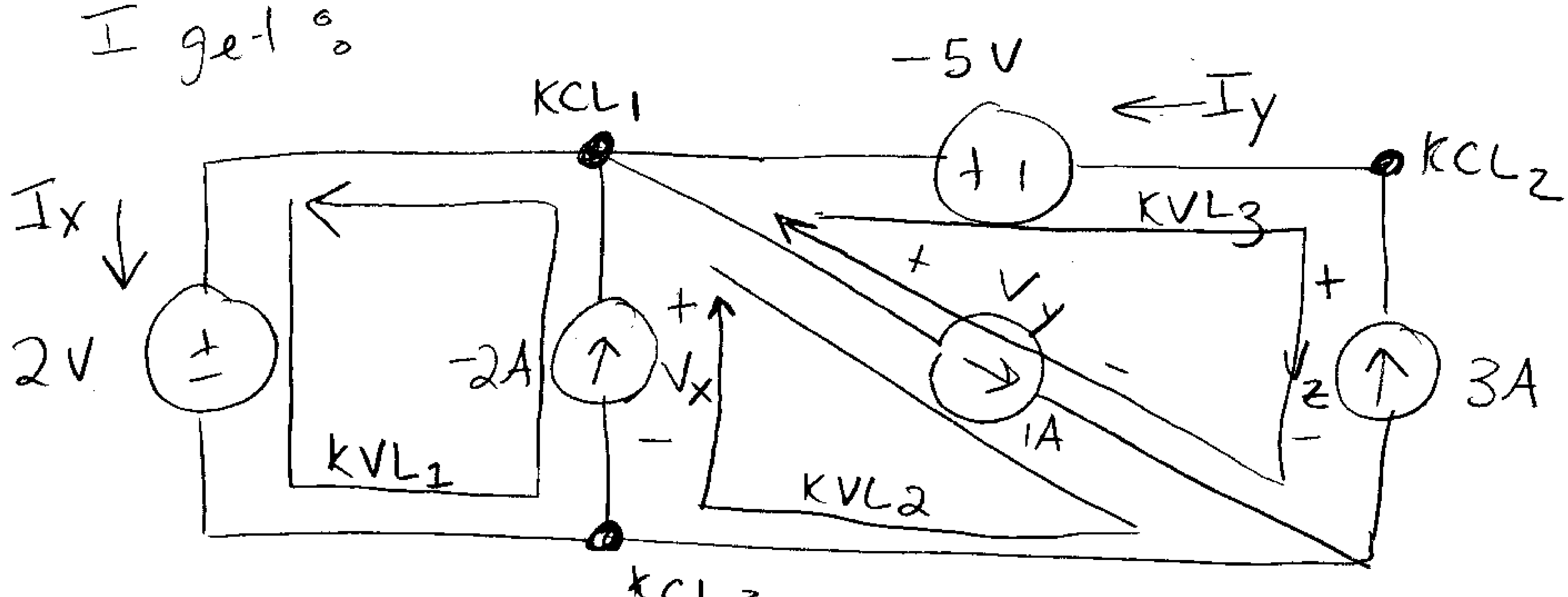
EE 42

Homework Solutions #1

Problem 1:

Writing KVL equations (assigning my own variable names to things I don't know), I get:



KVL$_1$: $2V + V_x = 0$

KVL$_2$: $V_y - V_x = 0$

KVL$_3$: $-5V + V_z - V_y = 0$

There are no contradictions in these equations.

Writing KCL, assigning variables to unknown currents,

KCL$_1$: $I_x - -2A - I_y + 1 = 0$

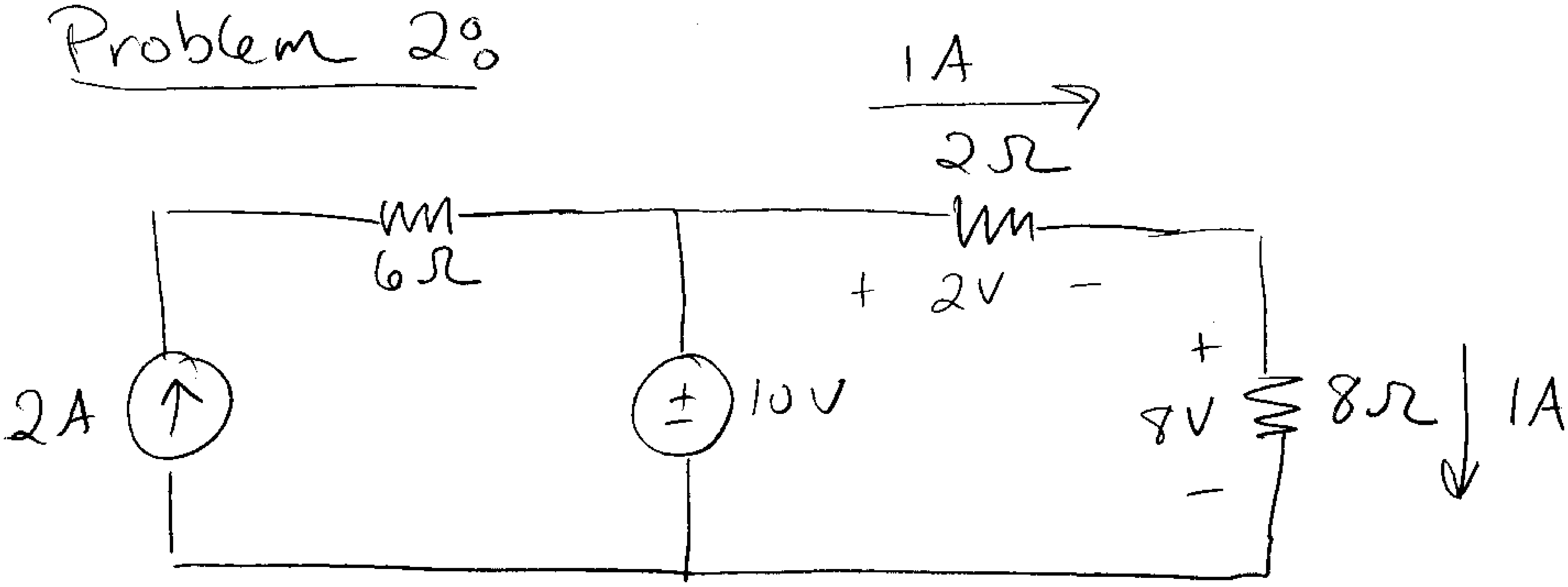
KCL$_2$: $I_y - 3A = 0$

KCL$_3$: $-I_x - 2A - 1A + 3A = 0$

There are no contradictions in these equations either.

Neither KVL nor KCL are violated.

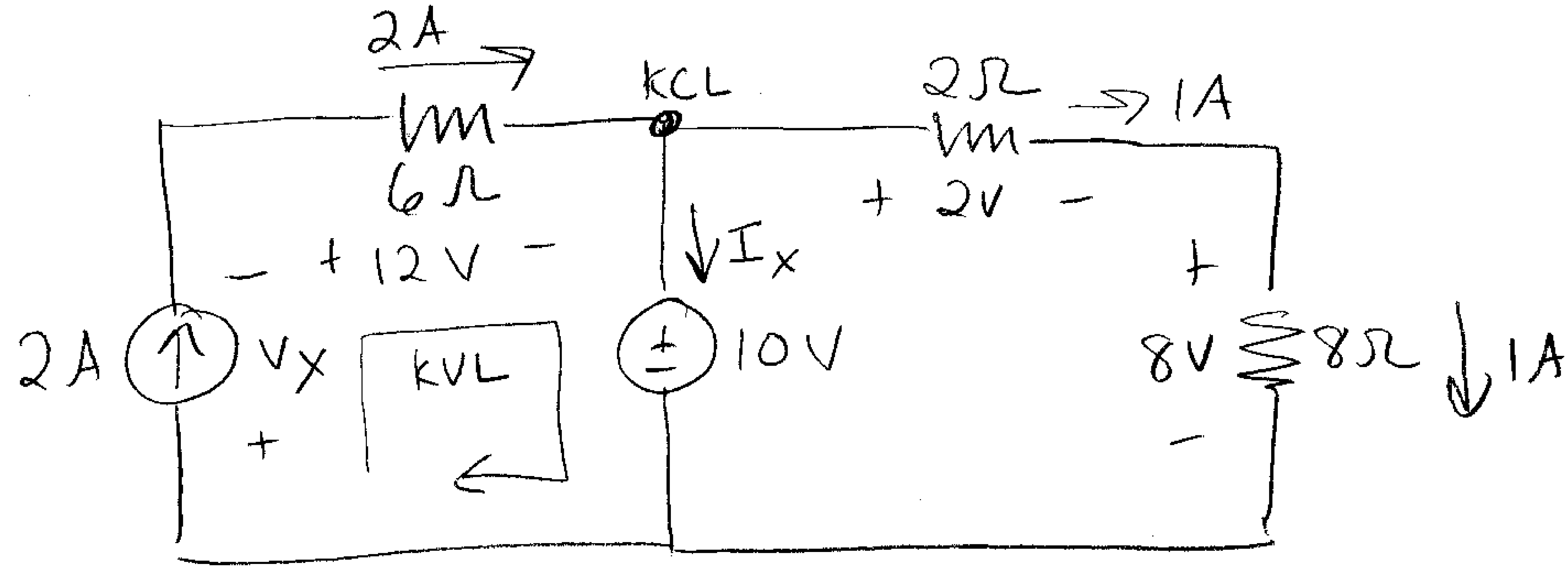
Problem 2:



I have 10 V over a pair of series resistors (2 Ω & 8 Ω). Using voltage division, the voltage over the 2 Ω resistor is

$$10\,V \cdot \frac{2\,\Omega}{2\,\Omega + 8\,\Omega} = 2\,V.$$

The 8 Ω resistor takes 8 V. The polarity is determined by the 10 V source, so the resistor voltage drops cancel the 10 V source. Using Ohm's law, the current through each resistor is 1 A, from + to −. (You could also determine this by combining the series resistors.)



Notice that the entire 2A of current from the current source must go through the 6Ω resistor (it has nowhere else to go, and current doesn't disappear). By Ohm's law, 12 V falls over the resistor with polarity shown.

Now I have all the V and I values for resistors. Still need I for 10V source and V for 2A source. I will give names to these unknowns, and give them the polarity that will result in power absorbed (current flows + to −). By KCL,

$$-2A + I_x + 1A = 0 \quad \Rightarrow \quad I_x = 1A$$

By KVL,

$$V_x + 12V + 10V = 0 \qquad V_x = -22V$$

Find power absorbed:

$$P_{2A} = (2A)(-22V) = -44\ W$$

$$P_{6\Omega} = (2A)(12V) = 24\ W$$

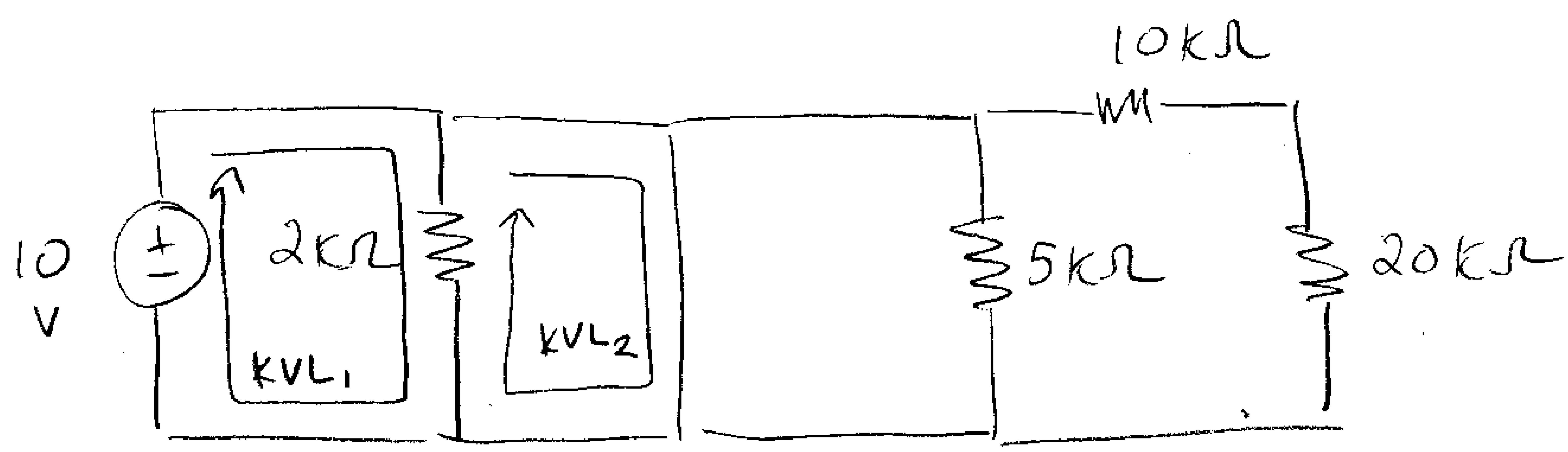
$$P_{10V} = (1A)(10V) = 10\ W$$

$$P_{2\Omega} = (1A)(2V) = 2\ W$$

$$P_{8\Omega} = (1A)(8V) = 8\ W$$

As expected, all resistors absorb (positive) power. The 10 V source also absorbs positive power! The 2 A source absorbs negative power, which means it generates 44 W of power.

Problem 3:



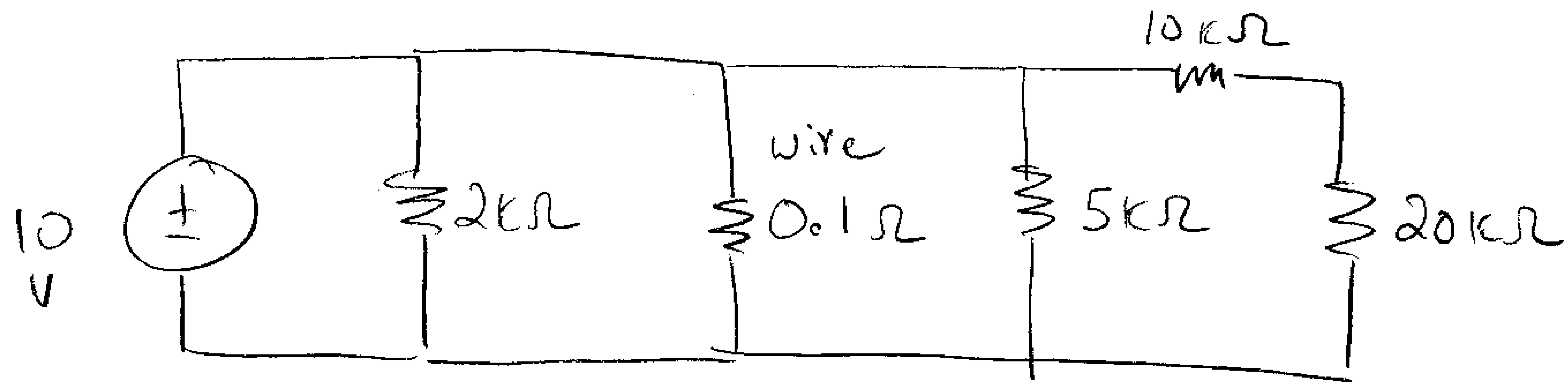
KVL_1 says the 2KΩ resistor should carry 10 V.

KVL_2 says the 2KΩ resistor should carry 0 V.

This is a contradiction: KVL is violated.

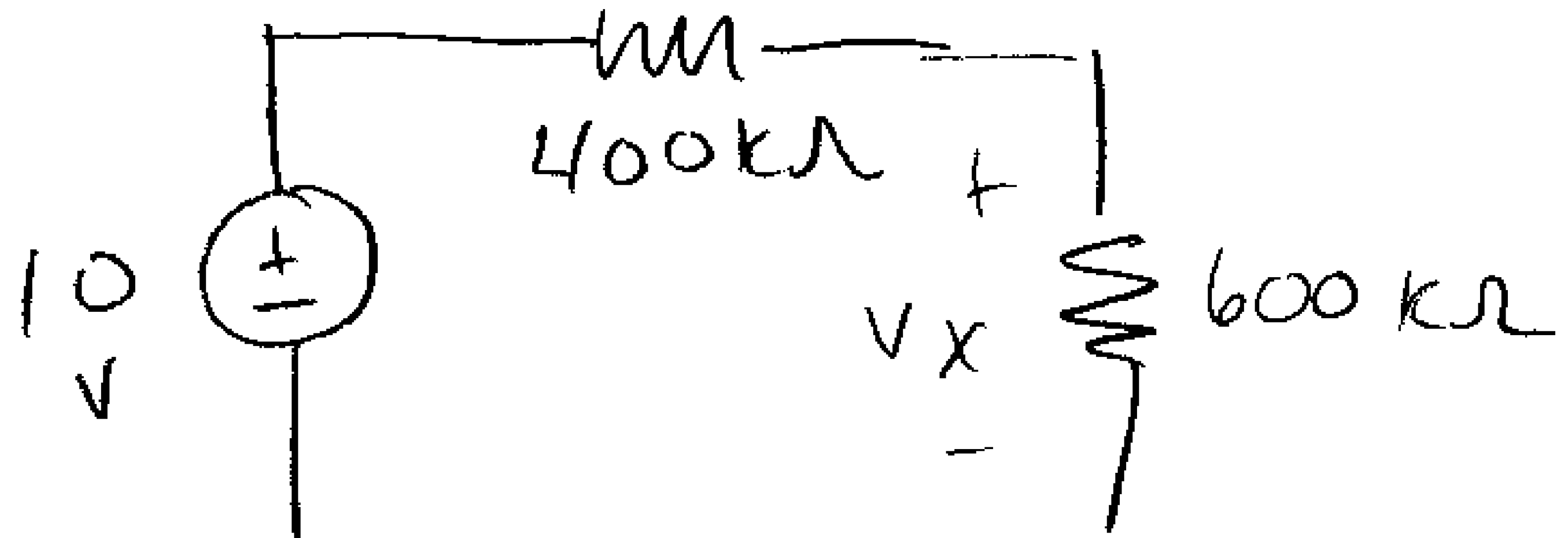
In real life, KVL is never violated.

The wire acts like a very small resistance:



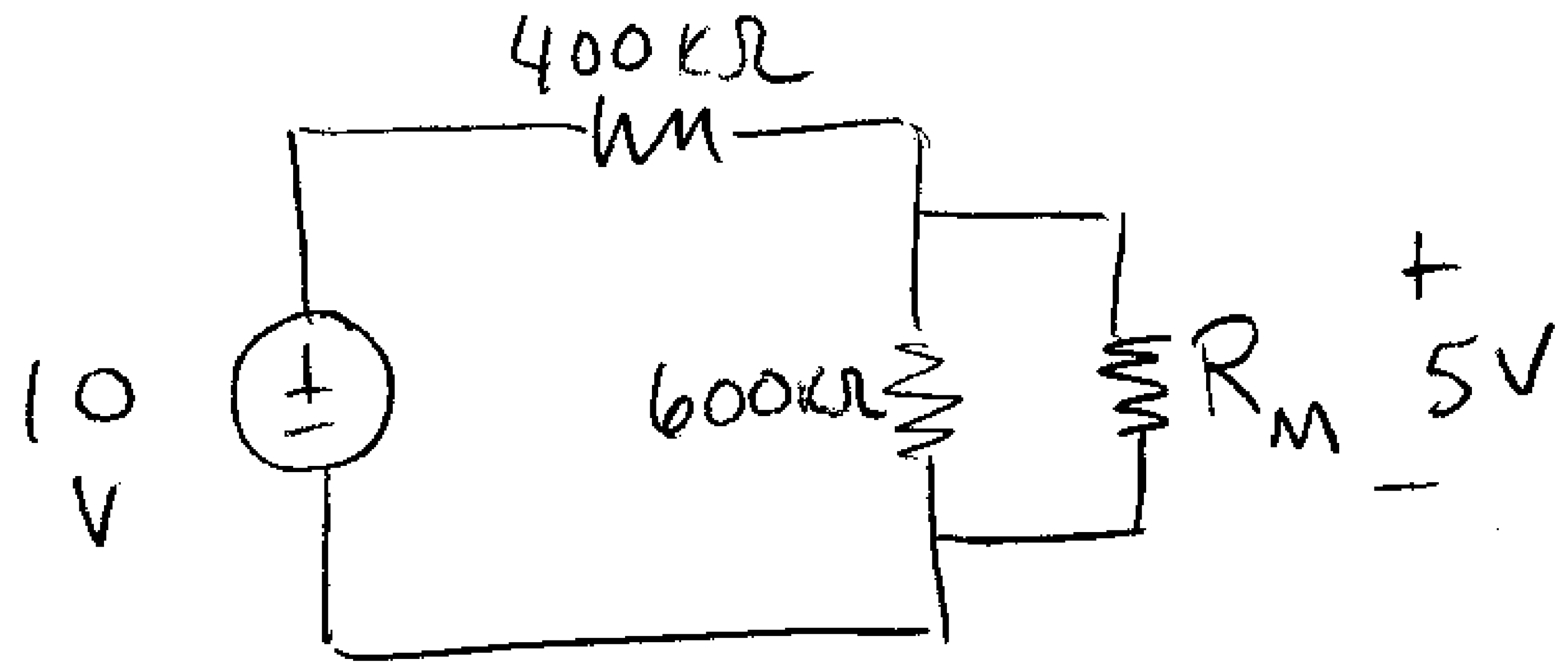
According to Ohm's law, the wire should get 100 A. This wouldn't likely happen — either the wire would melt, the 10 V source would blow a fuse, or the 10 V source would reduce its voltage so that its max current is not exceeded.

Problem 4:



By voltage division,

$$V_x = 10\,V \cdot \frac{600k\Omega}{400k\Omega + 600k\Omega} = 6\,V$$

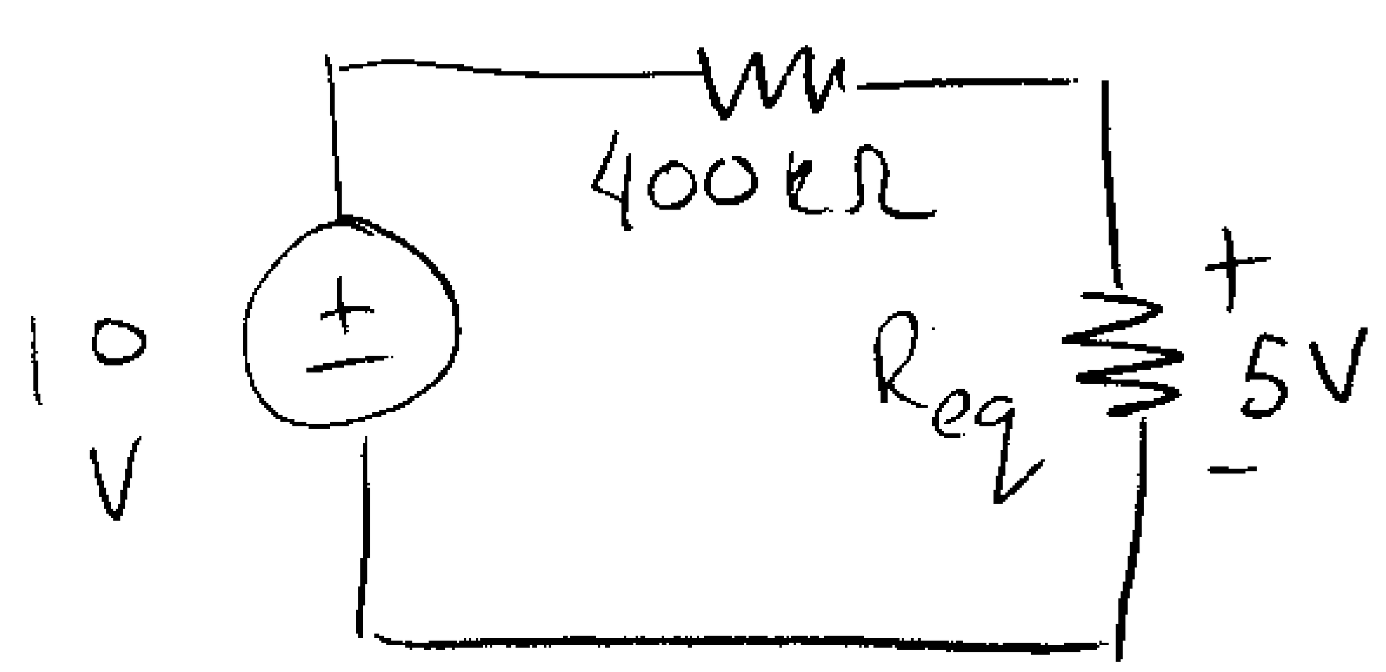


With the voltmeter in, all voltages and currents get changed. Now, the voltage across the 600 kΩ is 5V.

We could combine the 600kΩ + R_m in parallel:

$$R_{eq} = \left(\frac{1}{600k\Omega} + \frac{1}{R_M}\right)^{-1} = \frac{R_m \cdot 600k\Omega}{R_m + 600k\Omega}$$

That R_{eq} has the same voltage as the original parallel resistors:



By voltage division,

$$5V = 10V \cdot \frac{R_{eq}}{400k\Omega + R_{eq}}$$

So $R_{eq} = 400\,k\Omega$.

Now we know

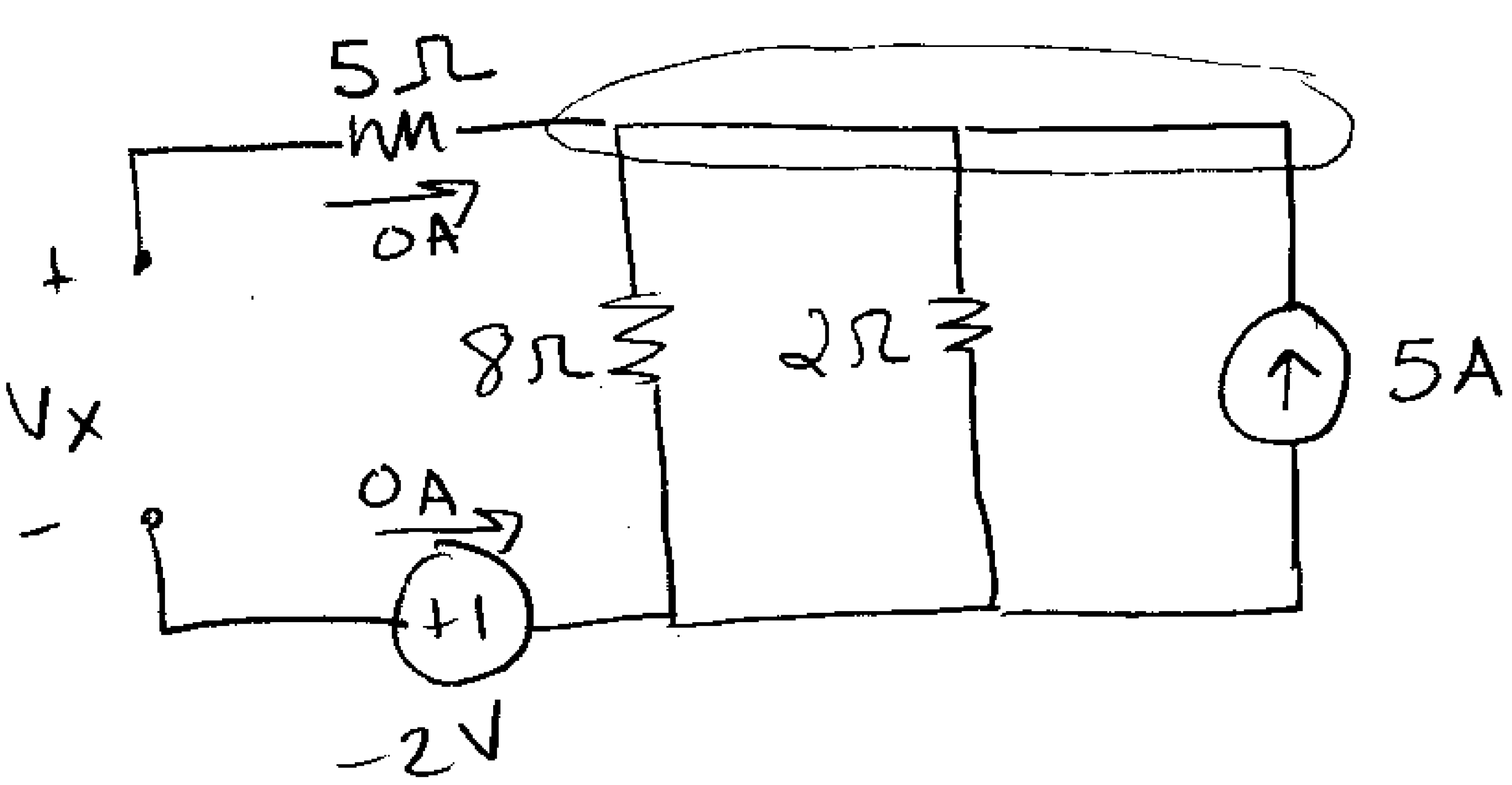
$$R_{eq} = 400\,k\Omega = \frac{R_m \cdot 600k\Omega}{R_m + 600k\Omega}$$ so $R_m = 1.2\,M\Omega$

Problem 5:

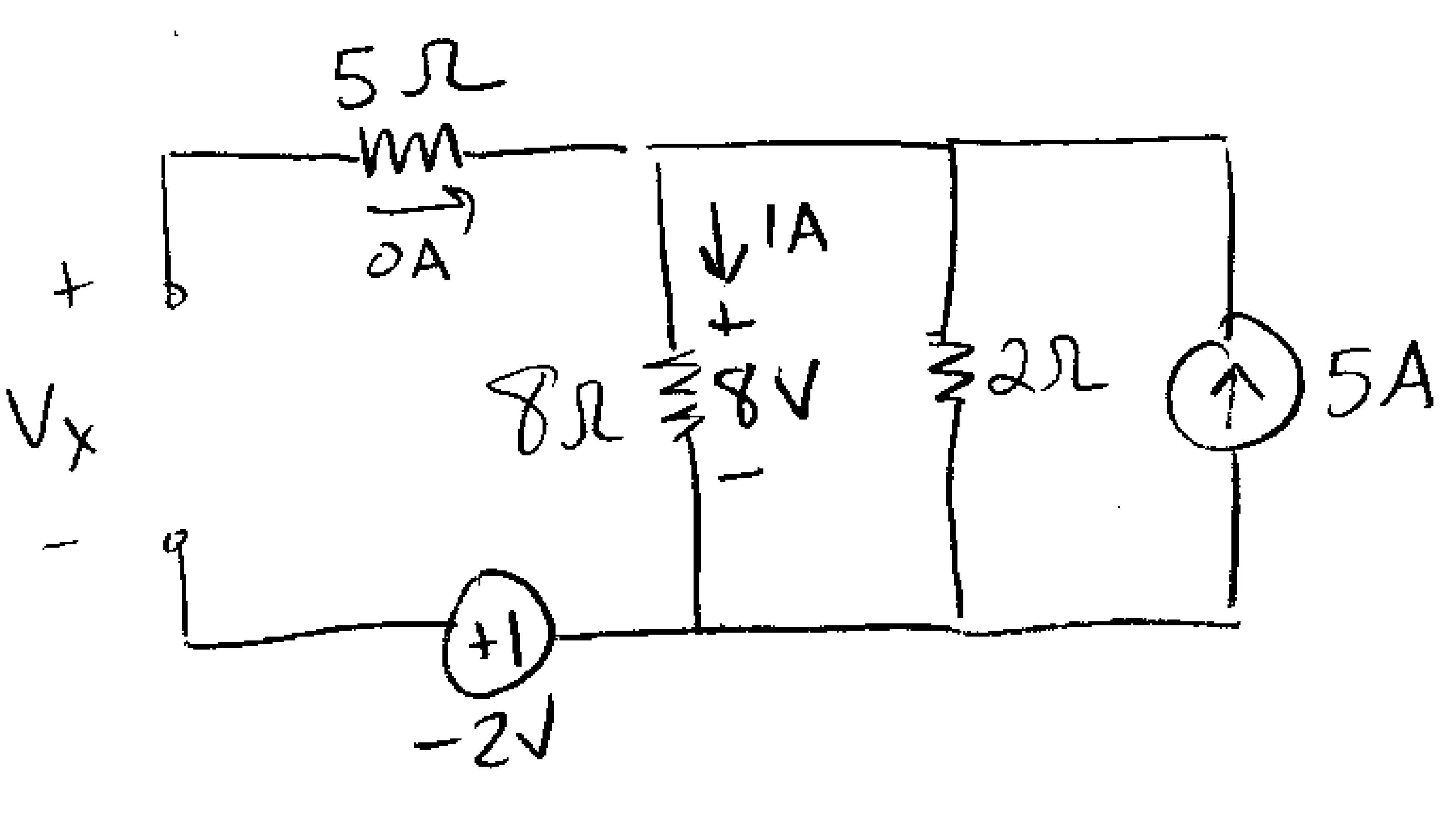
One could use nodal analysis.

I prefer to piece together clues:

- There can be no current through the hole in the circuit. So there is no current in the 5Ω resistor or in the -2V source.



- By KCL at the circled node, 5A goes in, and no current can enter the 5Ω resistor. So all 5A is divided between the 8Ω and 2Ω resistors.



- By current division, the 8Ω resistor gets 1A ($5A \cdot \frac{2\Omega}{8\Omega + 2\Omega} = 1A$). Therefore, the 8Ω resistor gets 8V.

(You could also combine the resistors in parallel to reach this conclusion.)

- By KVL, $V_x + -2V - 8V - (0V) = 0$ — the 5Ω resistor has 0V, by ohm's law

so $V_x = 10V$.