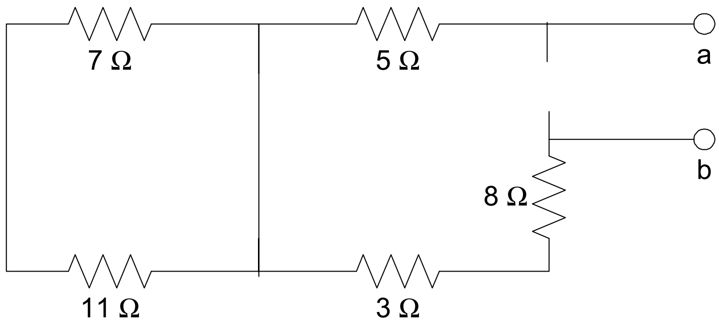
EE 42

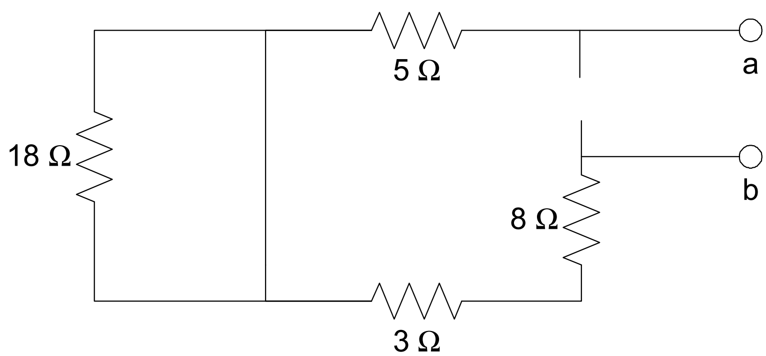
Midterm #1 Review Solutions

Problem 1:

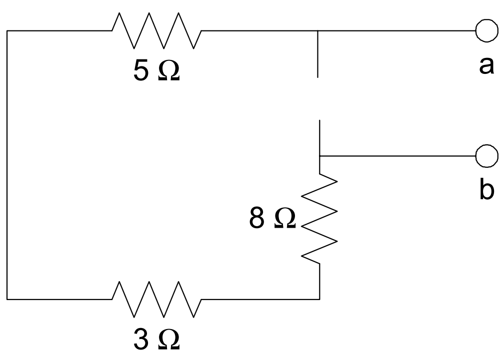
To find R_{TH} (R_N), we can turn off the independent sources:



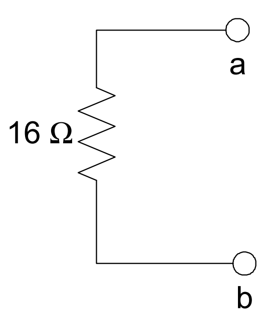
The 7 Ω and 11 Ω resistor are in series. They can be combined into a 7 Ω + 11 Ω = 18 Ω resistor.



Now, the 18 W resistor is in parallel with a wire. The resistor must have 0 V, or KVL would be violated. So we can ignore this resistor, since it has 0 V and therefore 0 A.



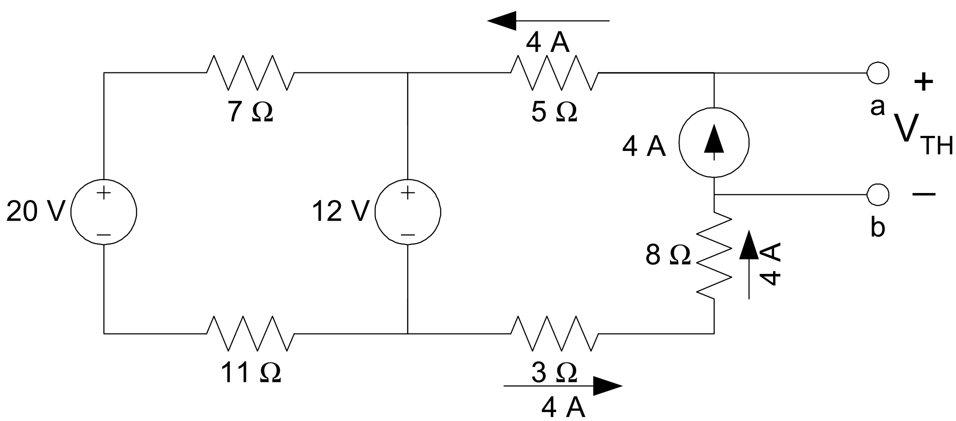
Now, the 5 Ω, 3 Ω, and 8 Ω resistors are in series since they have the same current: no current can go through the hole where the current source used to be. They combine into a 5 Ω + 3 Ω + 8 Ω = 16 Ω resistor.



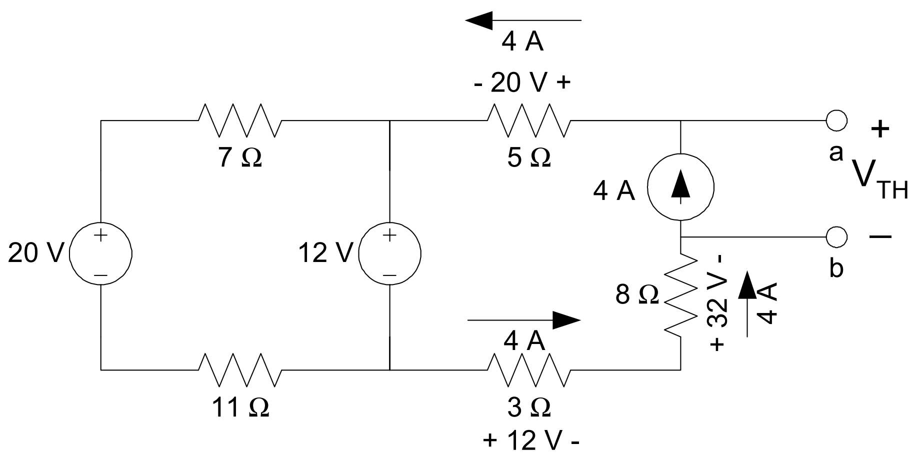
So, $R_{TH} = R_N = 16\ \Omega$.

To find V_{TH}, we find the voltage drop from a to b with the terminals left open.

With the terminals left open, the 5 Ω, 3 Ω, and 8 Ω resistors, and the current source, all share the same current. There is nowhere for the current to escape within that branch. The current source dictates that the current is 4 A as shown:



The voltage over these resistors can be found using Ohm's law:



V_{TH} can now be found using a KVL loop:

Starting at point a and going counter-clockwise,

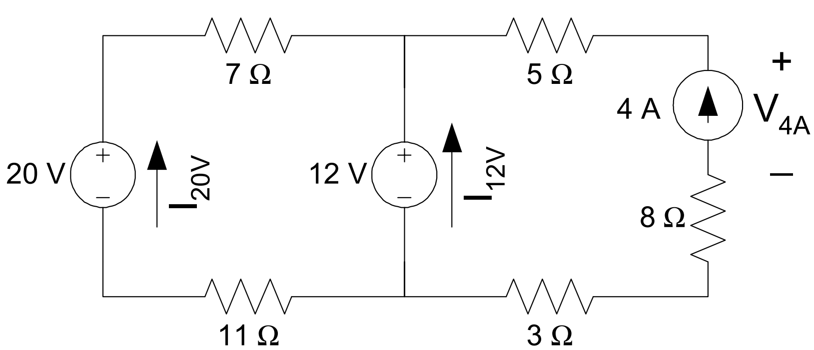
$20\text{ V} + 12\text{ V} + 12\text{ V} + 32\text{ V} - V_{TH} = 0$

$V_{TH} = 76\text{ V}$

$I_N = V_{TH} / R_{TH} = 19/4\text{ A}$

Problem 2:

To find power generated, we need to find the current going from – to + over the device voltage and multiply these values together.



Notice that the the 5 Ω, 3 Ω, and 8 Ω resistors, and the current source, all share the same current. There is nowhere else for the current to go in that path.

The same is true for the 7 Ω and 11 Ω resistors, and the 20 V voltage source.

By KVL, counter-clockwise in the right hand loop,

$(4\text{ A})(5\ \Omega) + 12\text{ V} + (4\text{ A})(3\ \Omega) + (4\text{ A})(8\ \Omega) - V_{4A} = 0$ $\qquad V_{4A} = 76\text{ V}$

By KVL, clockwise in the left hand loop,

$(I_{20V})(7\ \Omega) + 12\ V + (I_{20V})(11\ \Omega) - 20\ V = 0$ $\qquad I_{20V} = 4/9\ A$

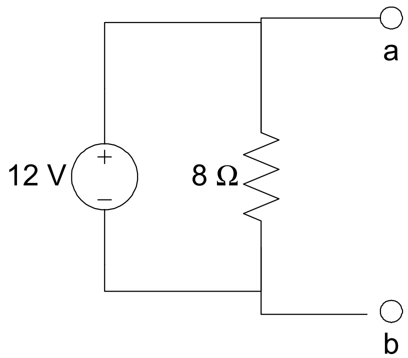
By KCL at the top-middle node,

$I_{20V} + I_{12V} + 4\ A = 0$ $\qquad I_{12V} = -\ 40/9\ A$

Now, we have the quantities we need to calculate power generated:

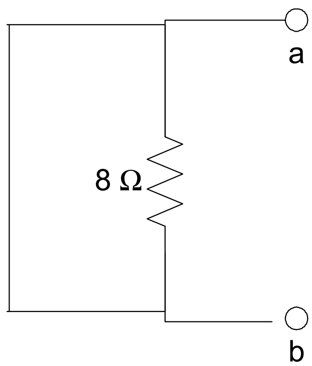
4 A source: P = (4 A)(76 V) = 304 W
20 V source: P = (4/9 A)(20 V) = 80/9 W
12 V source: P = (-40/9 A)(12 V) = -160/3 W

Problem 3:



When the terminals are left open, the voltage drop from a to b is 12 V (the source is directly attached from a to b).

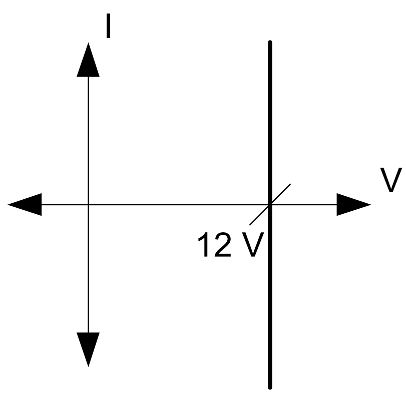
$V_{TH} = 12\ V$



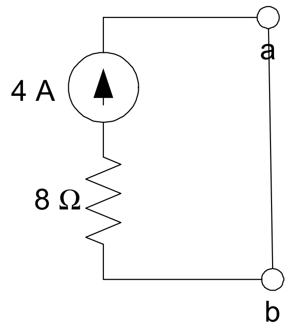
If we find R_{TH} by replacing the voltage source with a wire, we end up with 8 Ω in parallel with a wire. Now the resistor must have 0 V and therefore no current, so it can be removed. This leaves only the wire from a to b.

$R_{TH} = 0\ \Omega$ (wire)

This means the Thevenin equivalent is just the 12 V source, with no resistor.

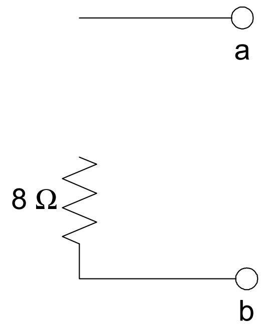


The I-V graph for this circuit is a vertical line. There is no y-intercept, so I_N, and the Norton equivalent circuit, do not exist for this example.



The current flowing from a to b when the terminals are shorted is 4 A, so

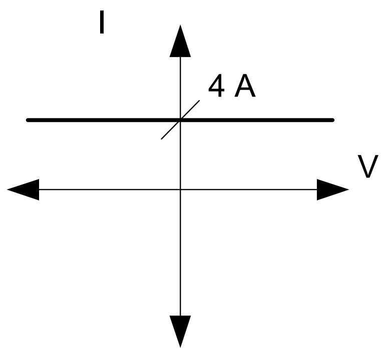
I_N = 4 A



If we find R_{TH} by replacing the curent source with air, we end up with 8 Ω in series with air. Now the resistor must have zero current, so it can be removed. This leaves only air from a to b.

$R_N = \infty\ \Omega$ (air)

This means the Thevenin equivalent is just the 4 A source, with no resistor in parallel.

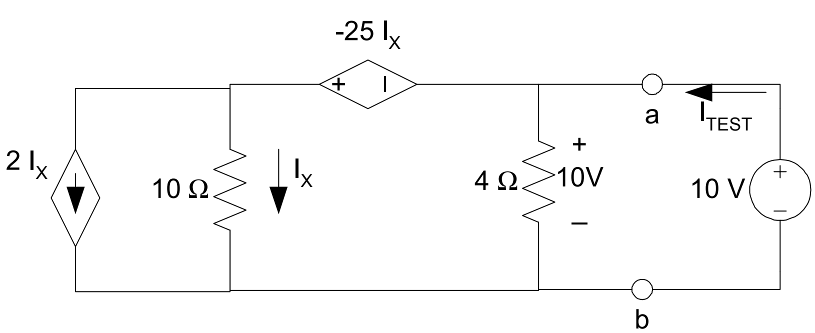


The I-V graph for this circuit is a horizontal line. There is no x-intercept, so V_{TH}, and the Thevenin equivalent circuit, do not exist for this example.

Problem 4:

We have no independent sources in this circuit. This means that V_{TH} = 0 V and I_N = 0 A.

So, the circuit is simply a resistor. To find the resistance, R_{TH}, we apply a test voltage (any voltage we want; 10 V for example) and measure the current going through the circuit:

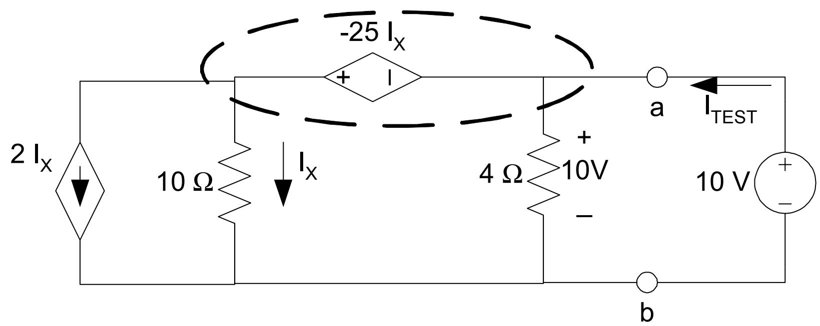


By KVL on the right-hand loop, we see that the 4 Ω resistor has 10 V as shown.

KVL on the center loop counter-clockwise:

$-10\text{ V} - -25\, I_X + (10\ \Omega)\, I_X = 0$

$I_X = 2/7$ A



By KCL on the surface shown,

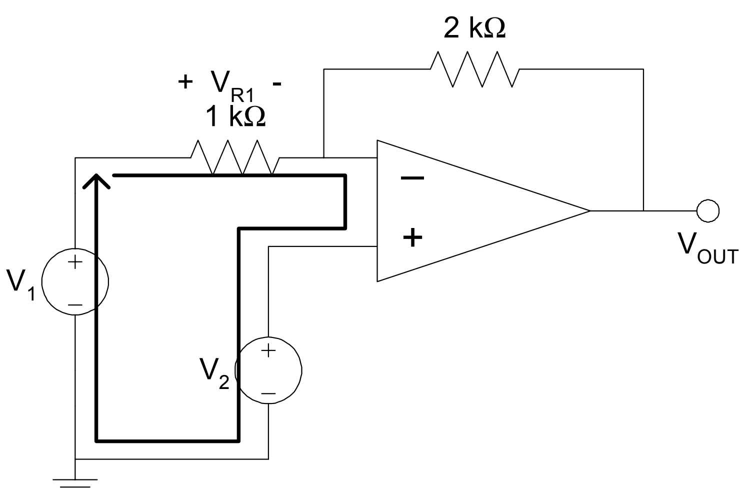
$2\ I_X + I_X + (10\ V / 4\ \Omega) - I_{TEST} = 0$

$I_{TEST} = 37/14\ A$

$R_{TH} = V_{TEST} / I_{TEST} = 10\ V / (37/14\ A)$

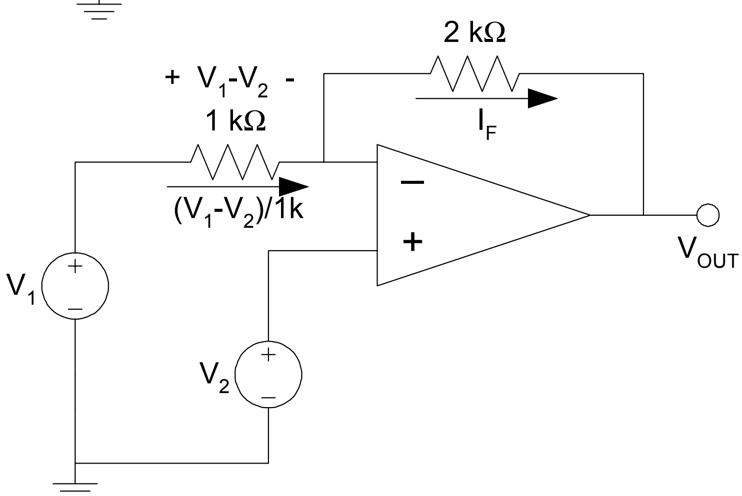
$R_{TH} = 140/37\ \Omega$

Problem 5:



By KVL around the input loop shown,

$V_{R1} + V_2 - V_1 = 0$ so $V_{R1} = V_1 - V_2$



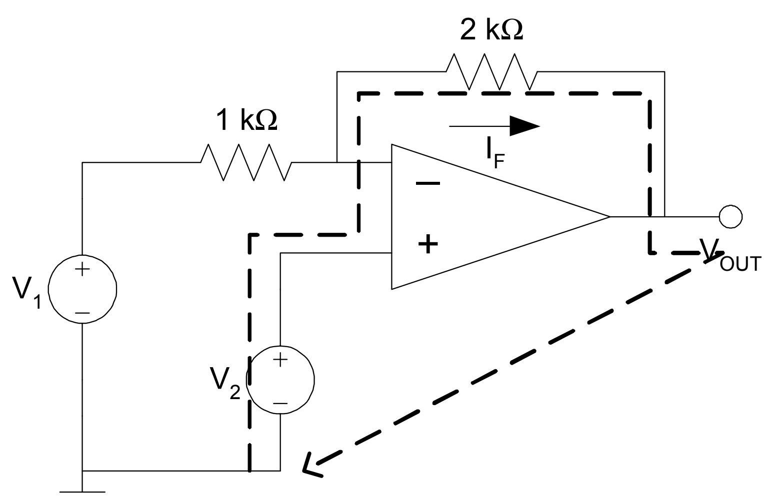
By KCL, $I_F - (V_1 - V_2)/1k = 0$, since the current into the op-amp input is 0 A.

$I_F = (V_1 - V_2)/1k$

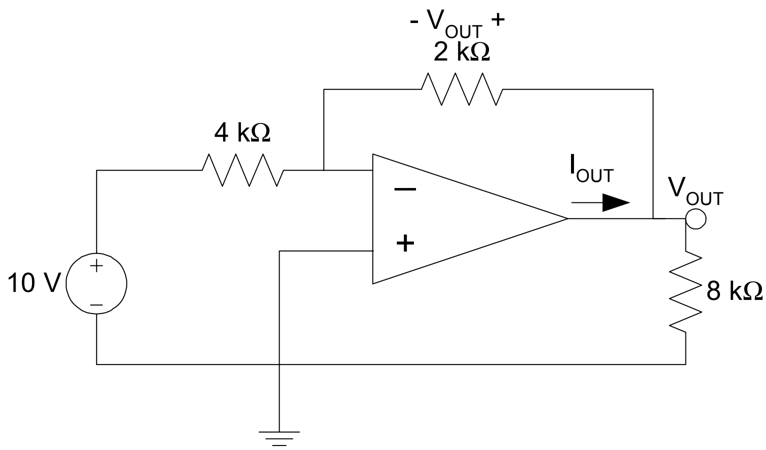
By KVL around the loop shown,

$-V_2 + 0\ V + (2\ k\Omega)I_F + V_{OUT} = 0$

$V_{OUT} = 3\ V_2 - 2\ V_1$



Problem 6:



Since this is an inverting amplifier, we know immediately that

$V_{OUT} = -(2\ k\Omega\ /\ 4\ k\Omega)\ 10\ V$

$V_{OUT} = -5\ V$

Also, note that the right side of the 2 kΩ resistor is at voltage V_{OUT}, and the left side is at ground. (Trace from the left side to ground; you only go over the op-amp inputs, a drop of 0 V).

KCL equation at the V_{OUT} node: $V_{OUT}\ /\ 8\ k\Omega + V_{OUT}\ /\ 2\ k\Omega - I_{OUT} = 0$ $I_{OUT} = -25/8\ A$

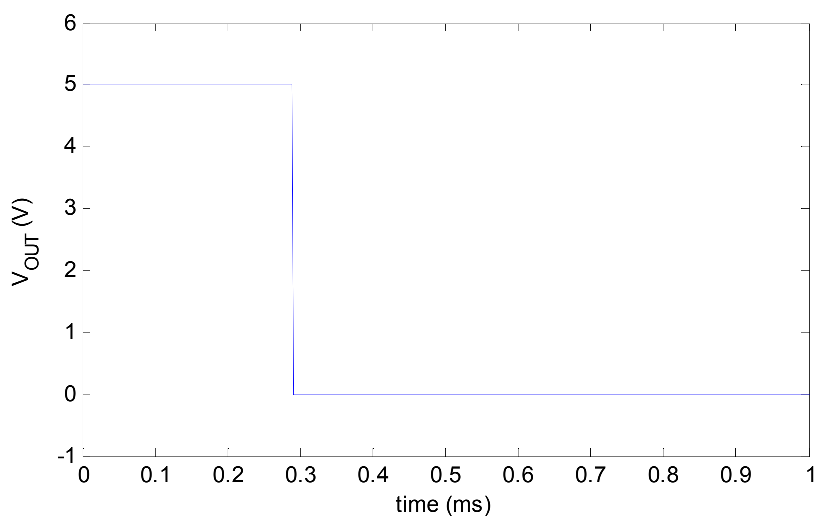
Problem 7:

The input is 5 V at t = 0, and decays exponentially to 0 V. The threshold voltage is 2 V, so the output will be at the high rail until the input gets down to around 2 V, then the output will quickly transition down to the low rail.

The output will be at a rail unless the output voltage given by the linear region formula is between the rails. So the amplifier will be in the linear region when:

$V_{OUT}(t) = A(V_+ - V_-) = 1000(V_{IN}(t) - 2\ V)$ is between 0 V and 5 V

The linear region is entered when this quantity is exactly 5 V:



$5\ V = 1000(5e^{-4000t}\ V - 2\ V)$

$t = -1/4000\ \ln(2.005\ /\ 5)$

$t = 228.45\ \mu s$

The linear region is over when this quantity is exactly 0 V:

$0\ V = 1000(5e^{-4000t}\ V - 2\ V)$

$t = -1/4000\ \ln(2\ /\ 5)$

$t = 229.07\ \mu s$

The comparator is in the linear region for 620 ns.