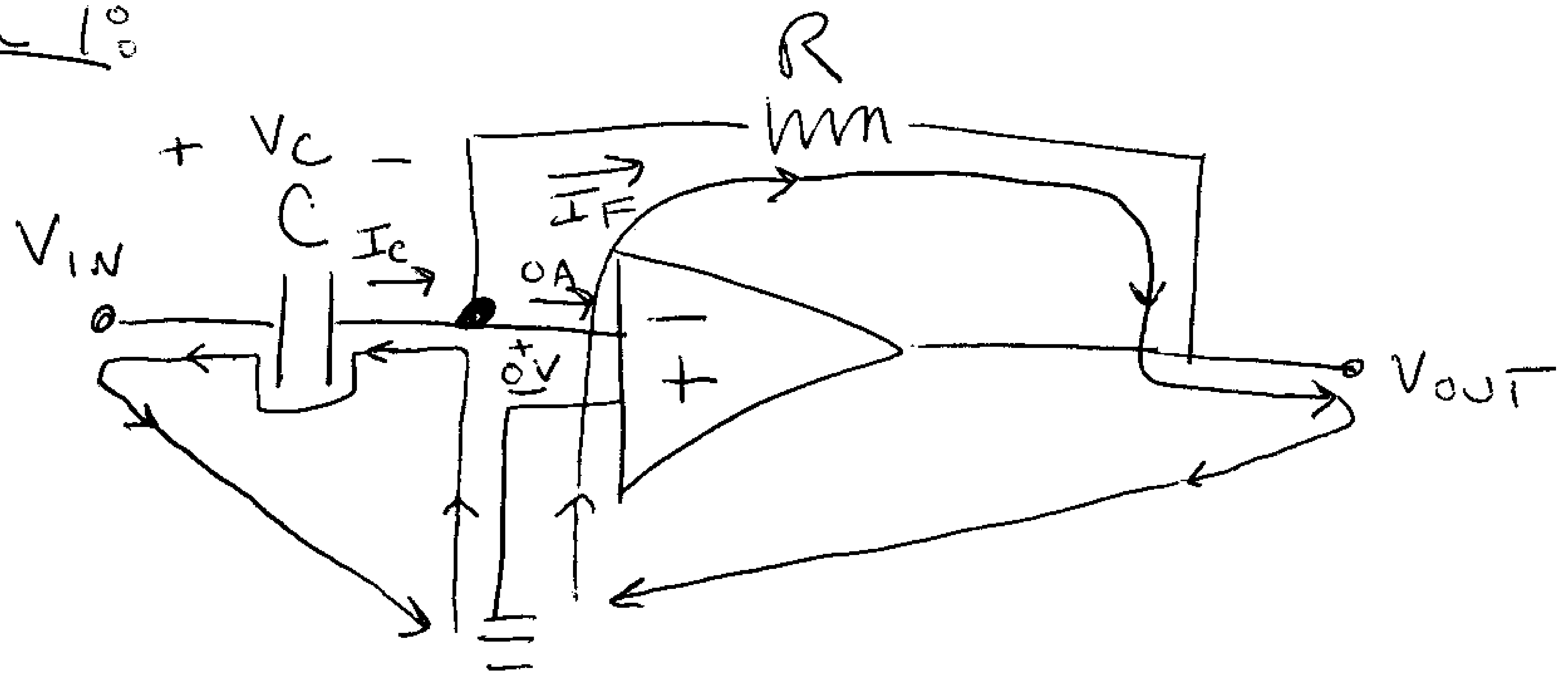
EE 42

Solutions to "Hard" MT 2 Review Problems

Problem 1:

a)



KVL around input loop: $-0V - V_C + V_{IN} = 0$

$V_C = V_{IN}$

KCL: $I_C = I_F + 0A$

$I_C = I_F$

Capacitor law: $I_C = C \frac{dV_C}{dt}$

$I_F = C \frac{dV_C}{dt} = C \frac{dV_{IN}}{dt}$

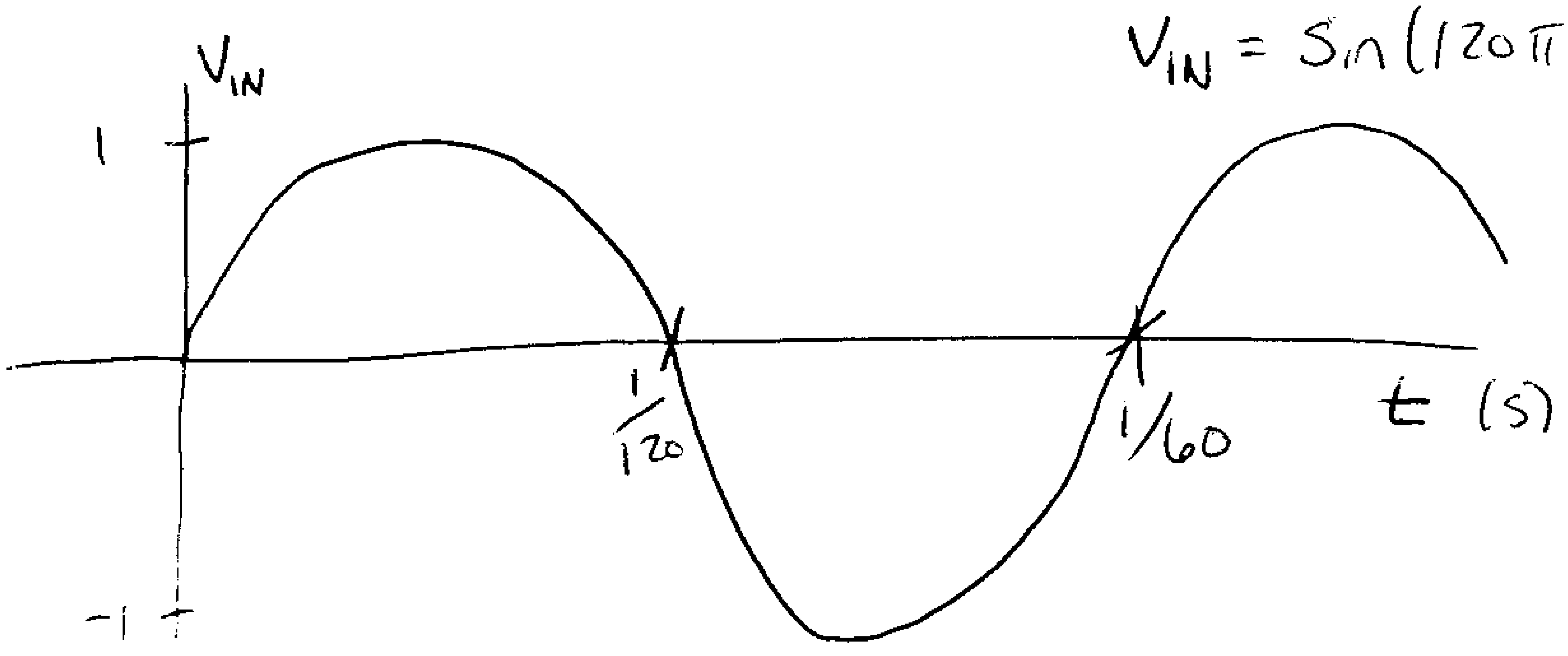
KVL around output loop:

$-0V + I_F R + V_{OUT} = 0$ $\quad V_{OUT} = -I_F R$

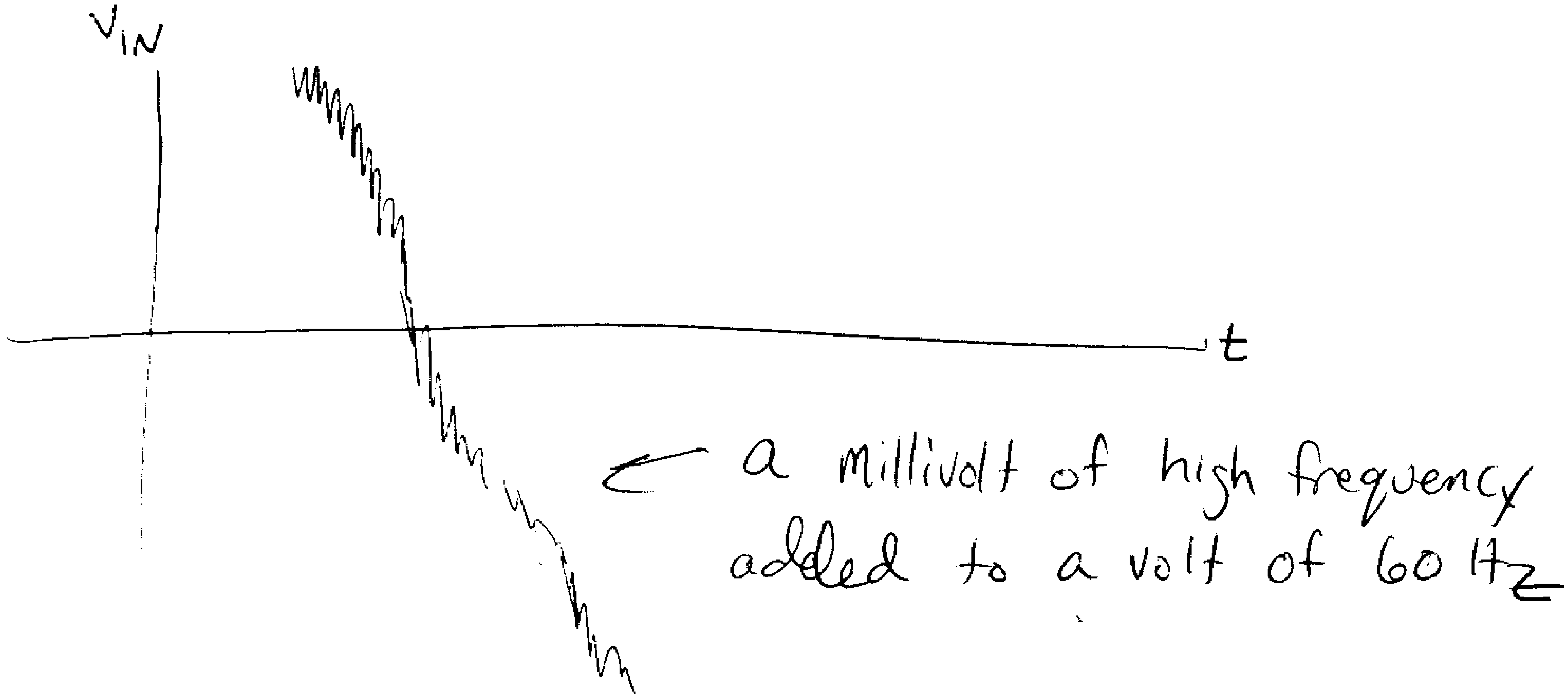
$V_{OUT} = -RC \frac{dV_{IN}}{dt}$ (Differentiator!)

b) Assume RC = 1.

$V_{IN} = \sin(120\pi t) + 10^{-3}\sin(120 \cdot 10^{6}\pi t)$



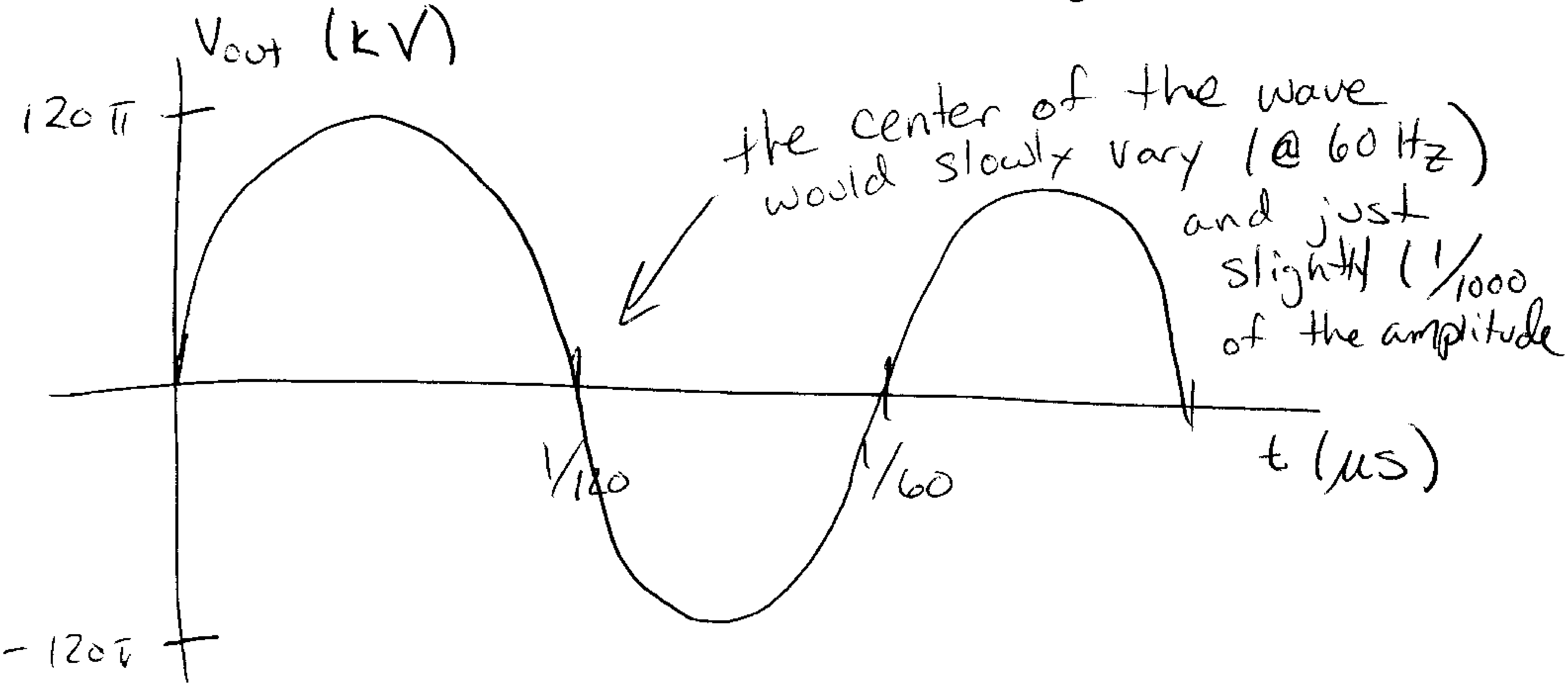
The 60 Hz signal is visible; the 60 MHz sine wave is not because it is multiplied by 10^{-3}. If you zoomed in, it would look like



$$V_{out}(t) = -RC \frac{dv_{IN}}{dt}$$

$$= -\left(120\pi \sin(120\pi t) + 120\pi \cdot 10^3 \cos(120\pi \cdot 10^6 t)\right)$$

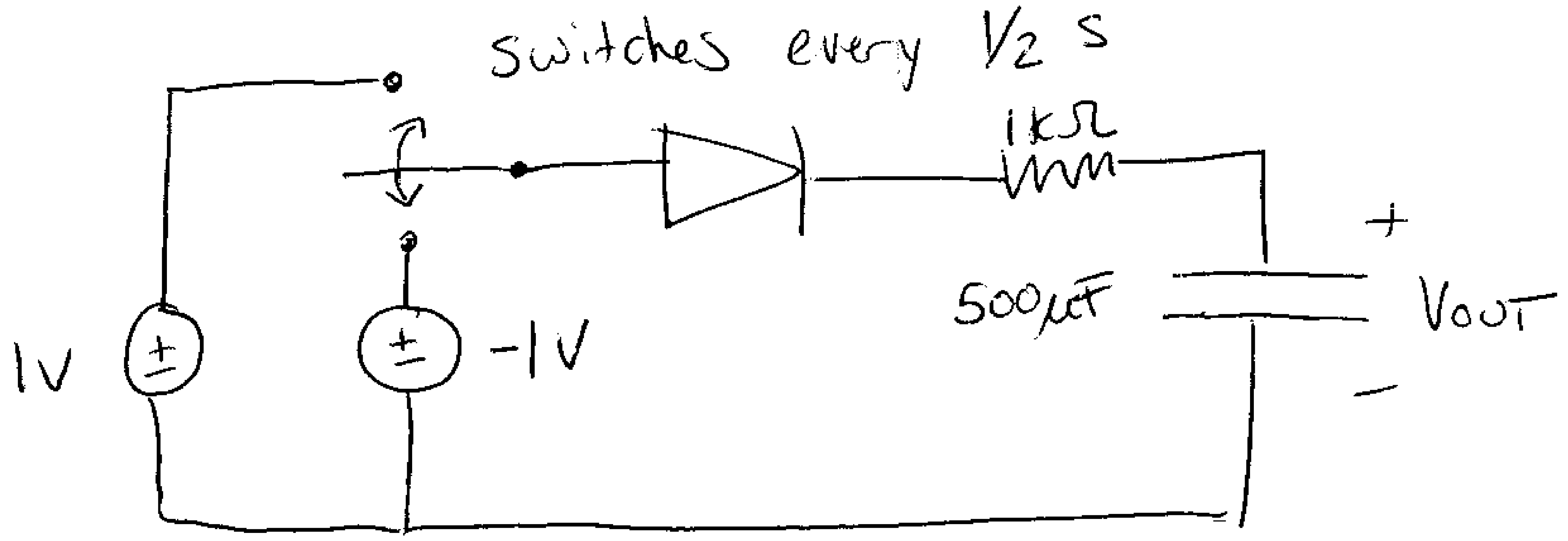
The sinusoids get amplified by their frequencies. The 60MHz noise becomes huge compared to the 60 Hz signal.



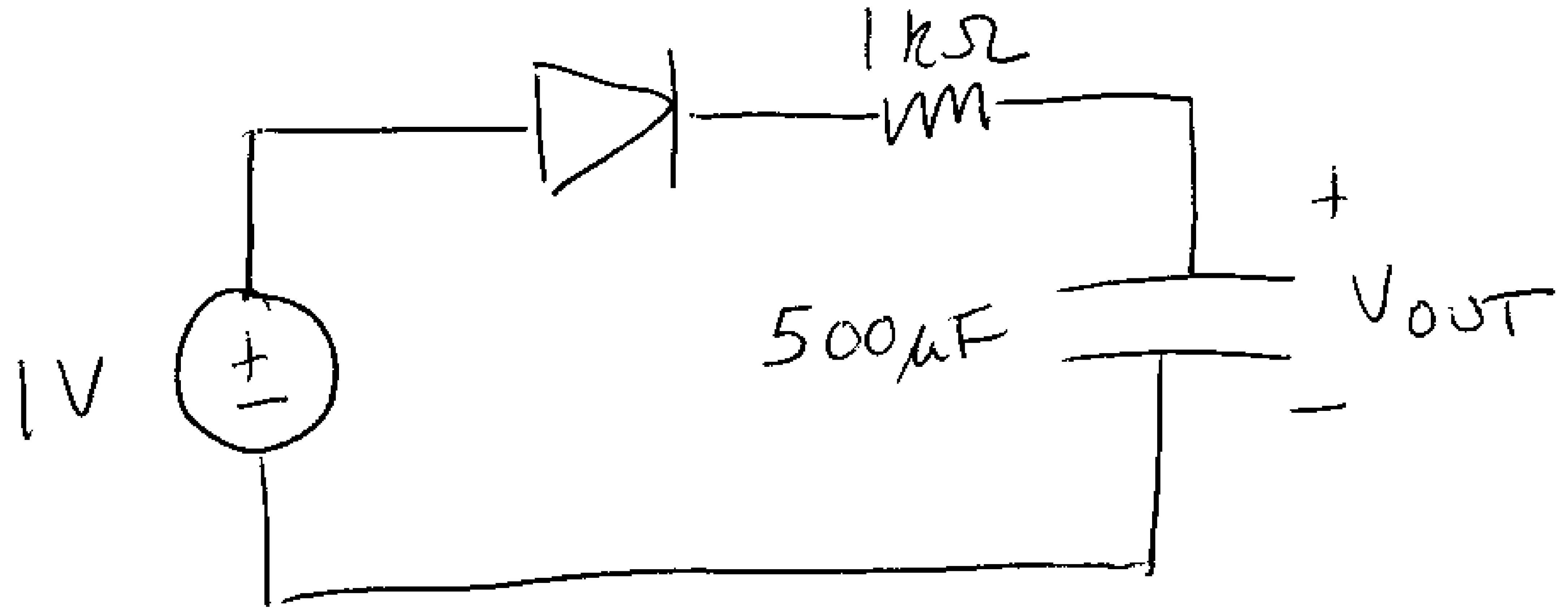
Moral: Differentiators are noisy and don't work that well.

Problem 2:

We can look at the V_s source as a switch that goes back & forth between 1V and -1V:

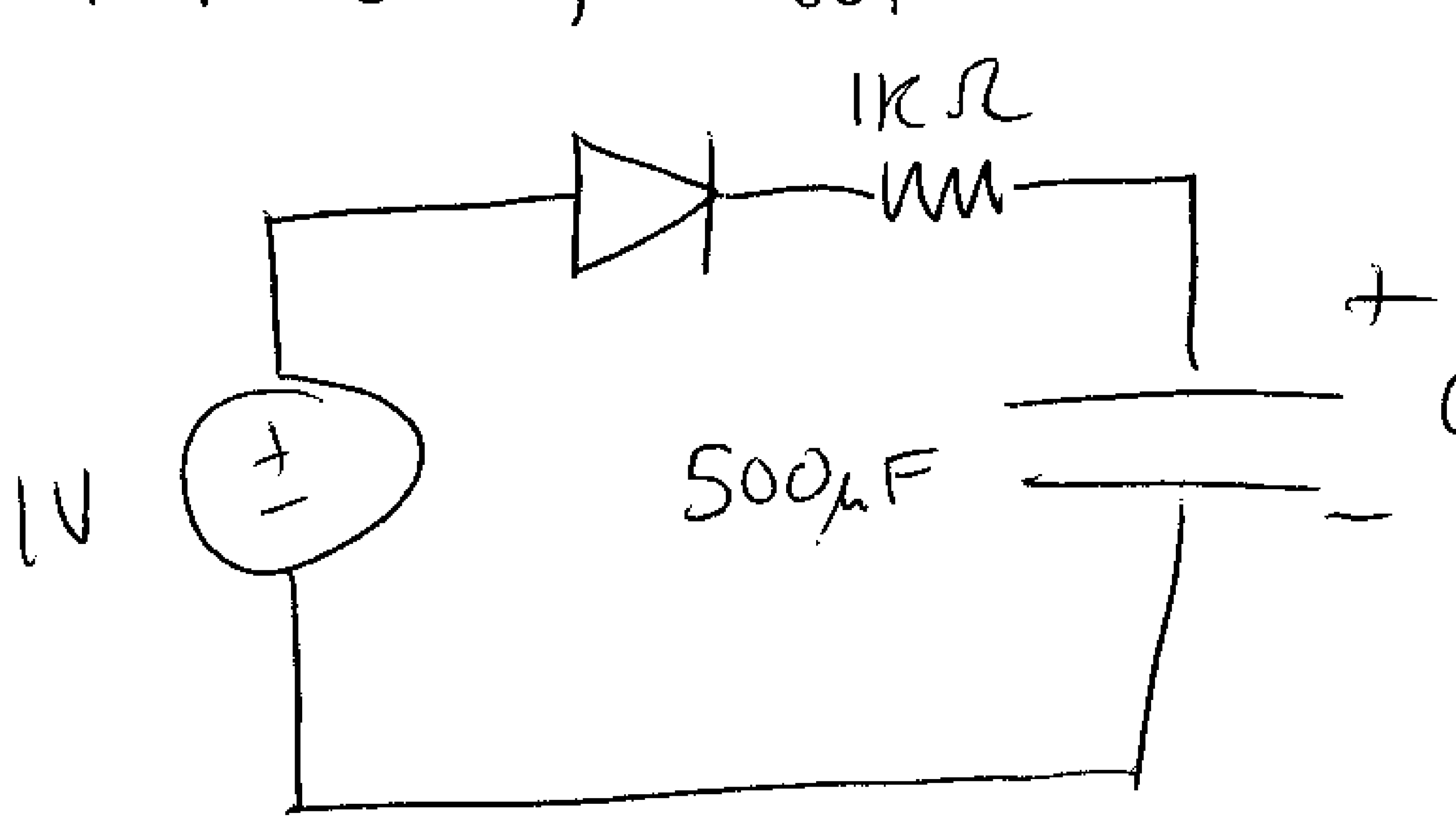


First, the V_s is at 1V.



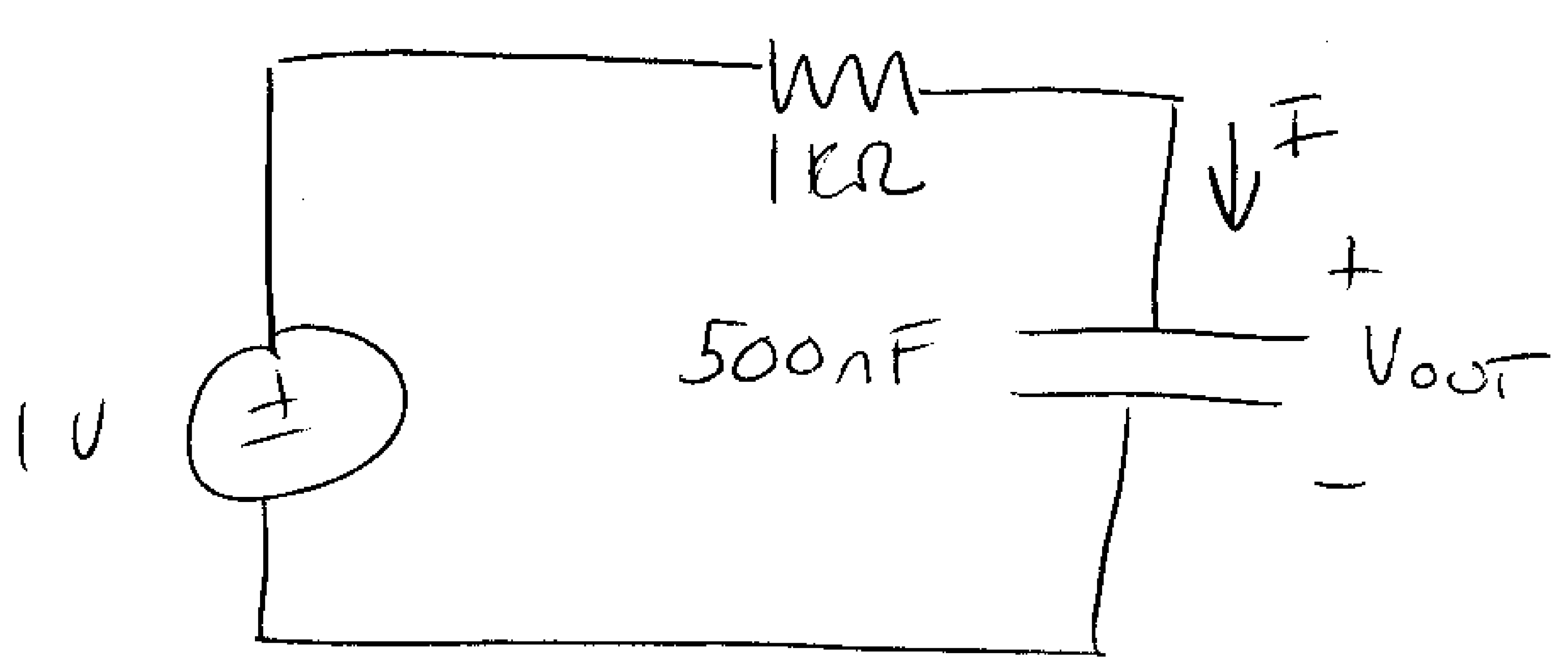
Valid for $0 < t \leq \frac{1}{2}$ s

At $t=0$, $V_{OUT} = 0V$:



Diode cannot be reverse biased — if so, 0A current, so 0V across resistor, so 1V must fall over diode. Not possible for reverse bias.

↓ forward bias = wire



$t = 0$

RC circuit!

Will the diode ever be reverse biased?
The current would have to be zero in that case.

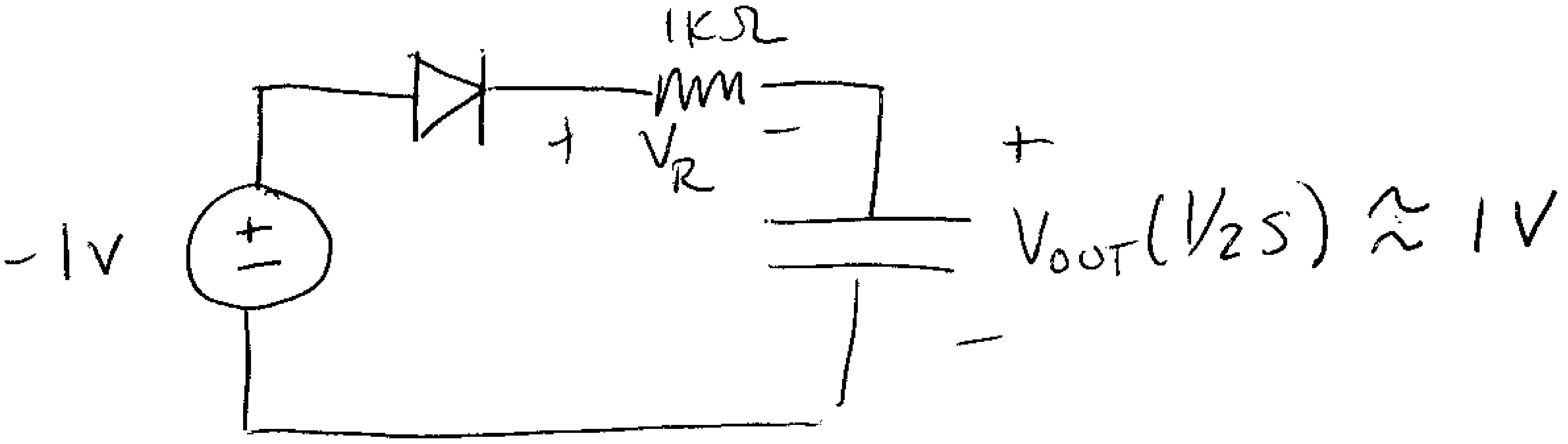
Since $V_{OUT}(t) = 1V + (0V - 1V)\, e^{-t/0.5ms}$

so $I(t) = C \dfrac{dV_{OUT}}{dt} = \dfrac{500nF}{0.5ms} e^{-t/0.5ms} = 1mA \cdot e^{-t/0.5ms}$

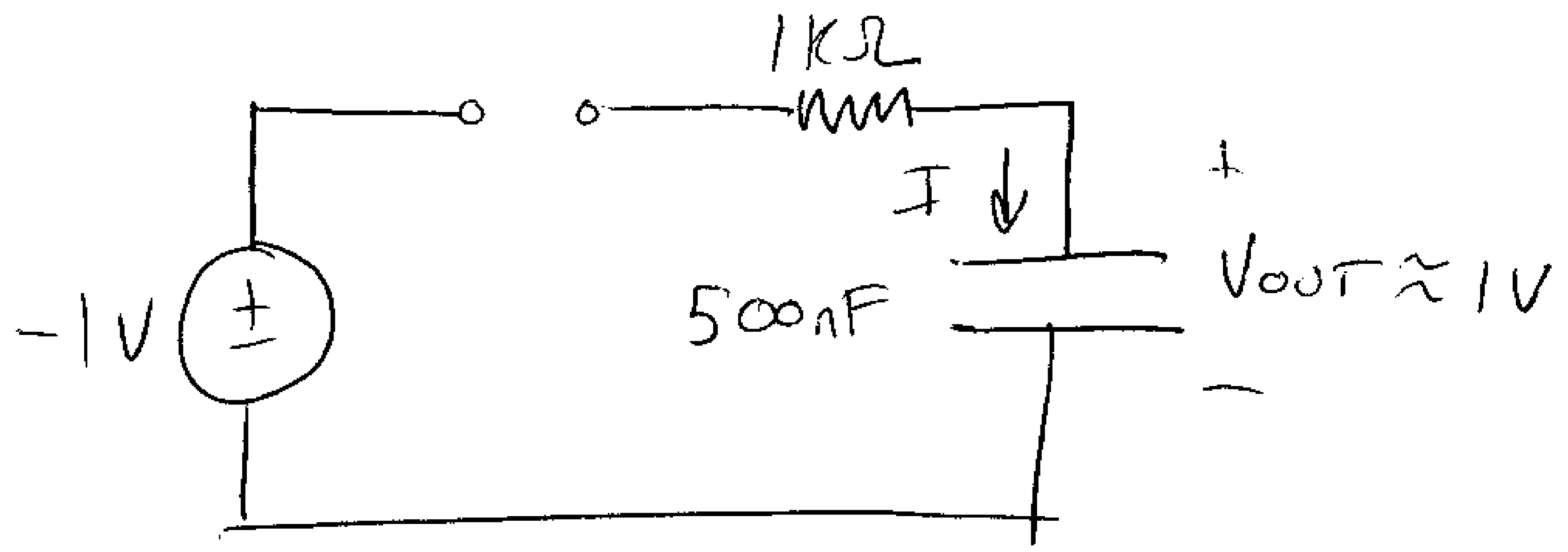
Under forward bias assumption for all t from 0 to ½ s, diode current is always positive. So we are always forward biased in this time interval, and

$V_{out}(t) = 1 - e^{-t/0.5ms}$ for $0 < t \le ½\,s$

Everything changes at $t = ½\,s$.



Diode cannot be forward biased since V_R would be $-2V \Rightarrow$ negative diode current

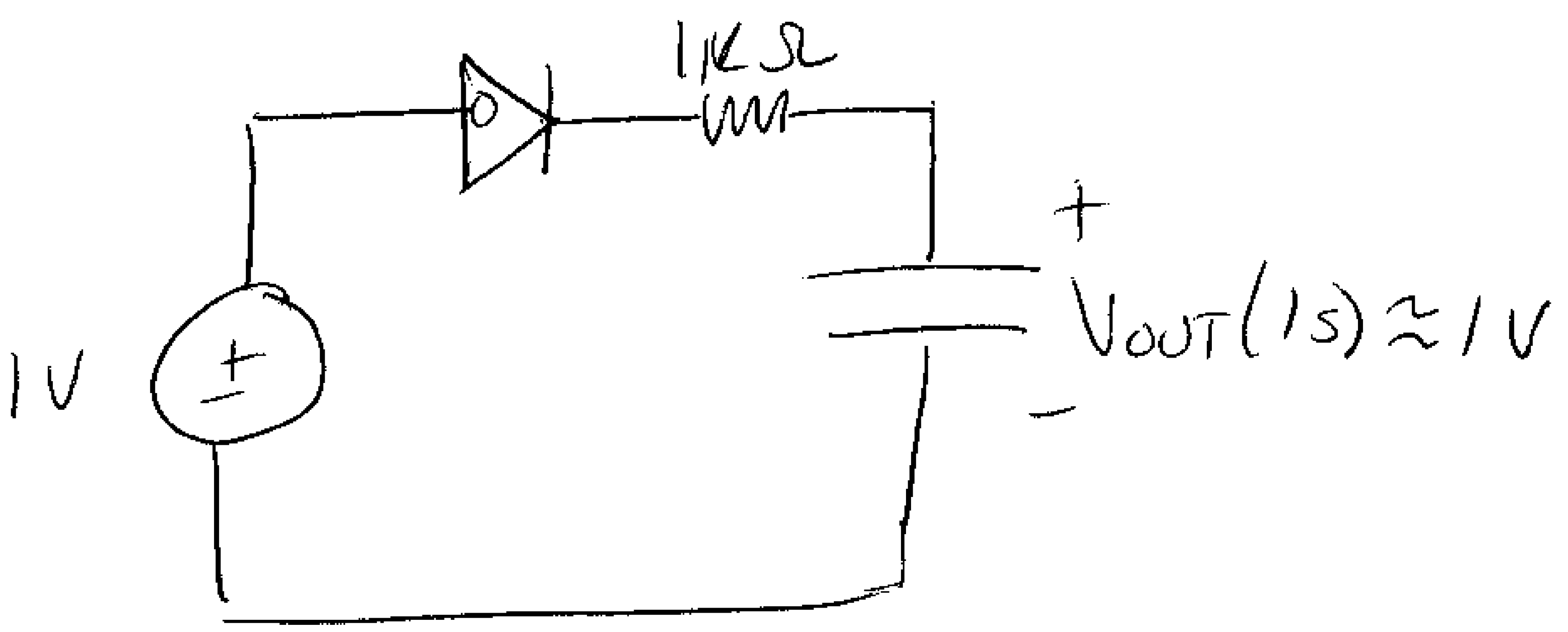


$t = 1/2$ s

I = 0, V_{OUT} does not change,

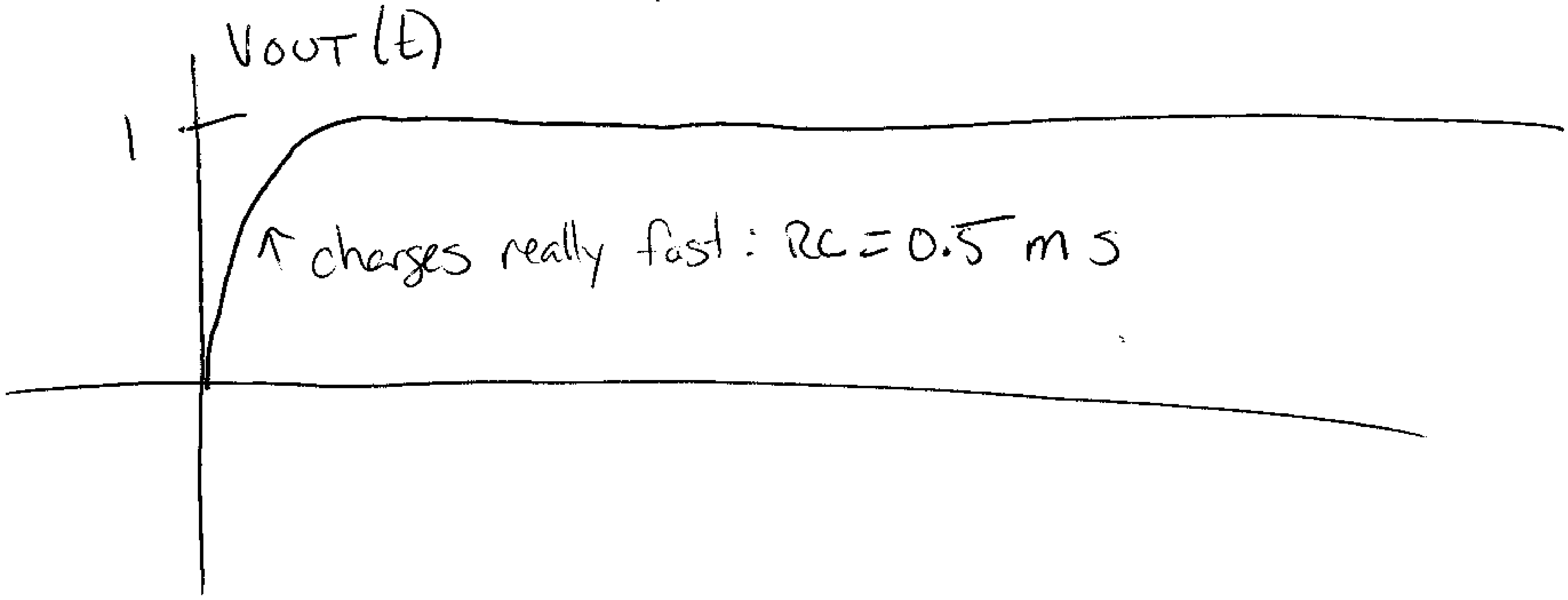
Diode situation does not change,

until circuit switches again

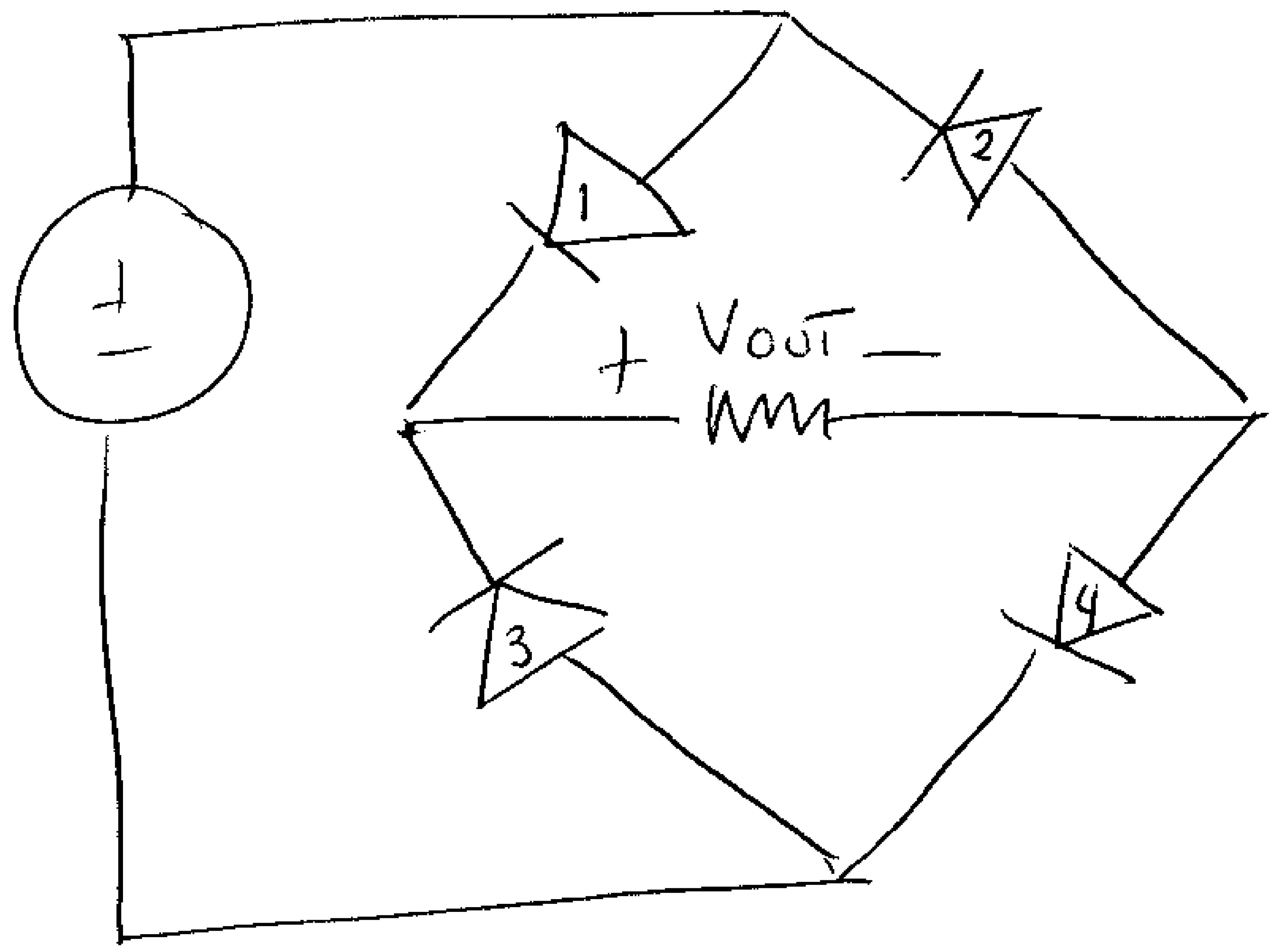


$t = 1$ s

$V_{OUT}(1s)$ is a little less than 1V from our previous calculation. Like before, diode is forward biased. Capacitor charges even closer to 1V.

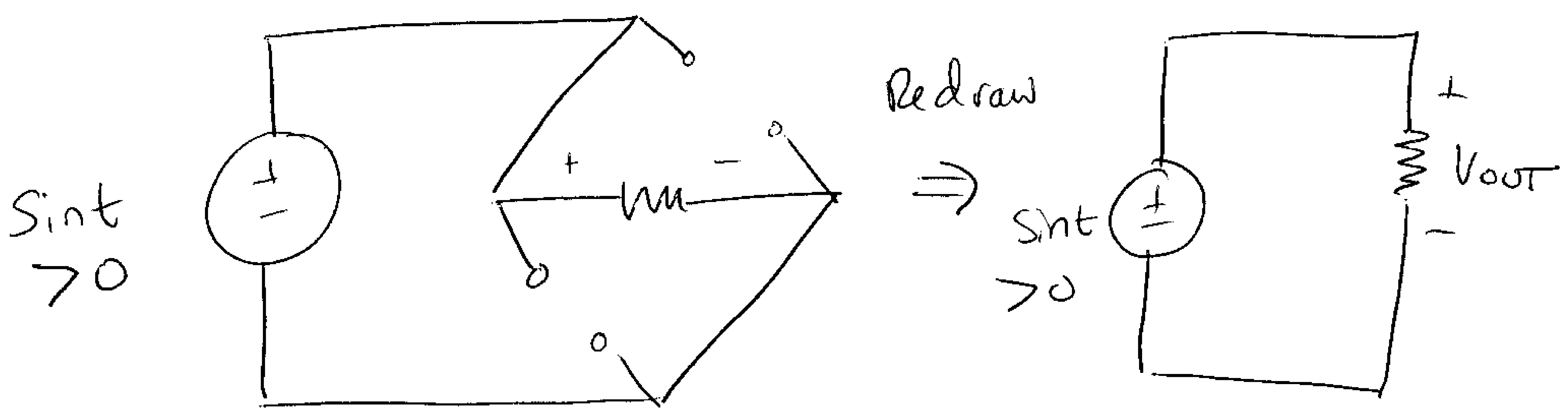


Problem 3:



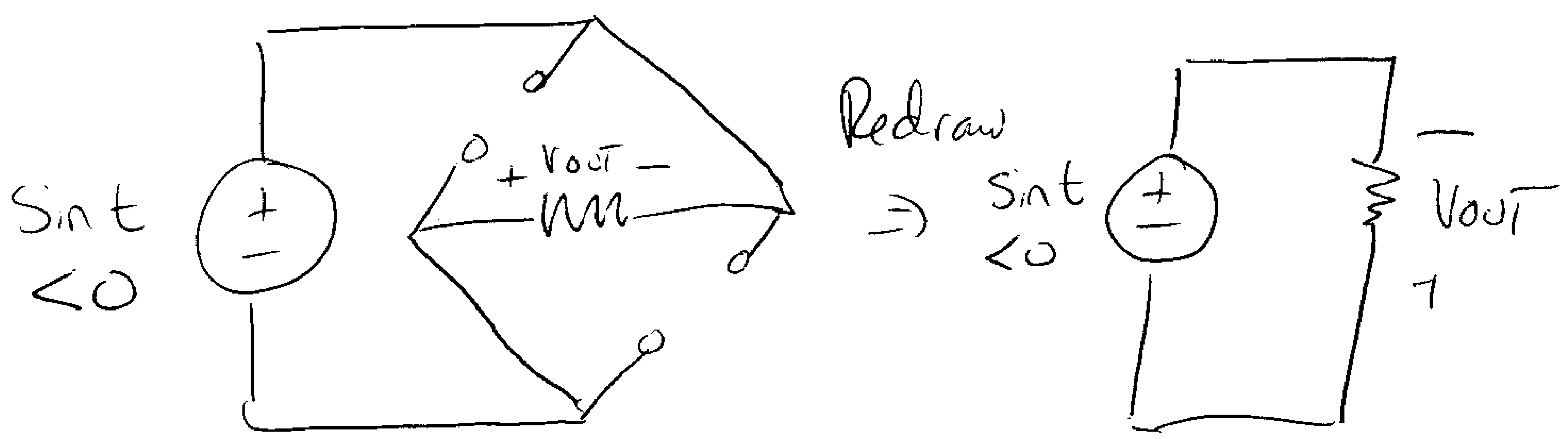
When sin t > 0, diodes 1 & 4 are "right-side-up" compared to the applied voltage; 2 & 3 are "upside-down".

Guess 1 & 4 are forward biased, 2 & 3 reverse.



$V_{OUT} = \sin t$

Roles of diodes switch when sin t < 0...

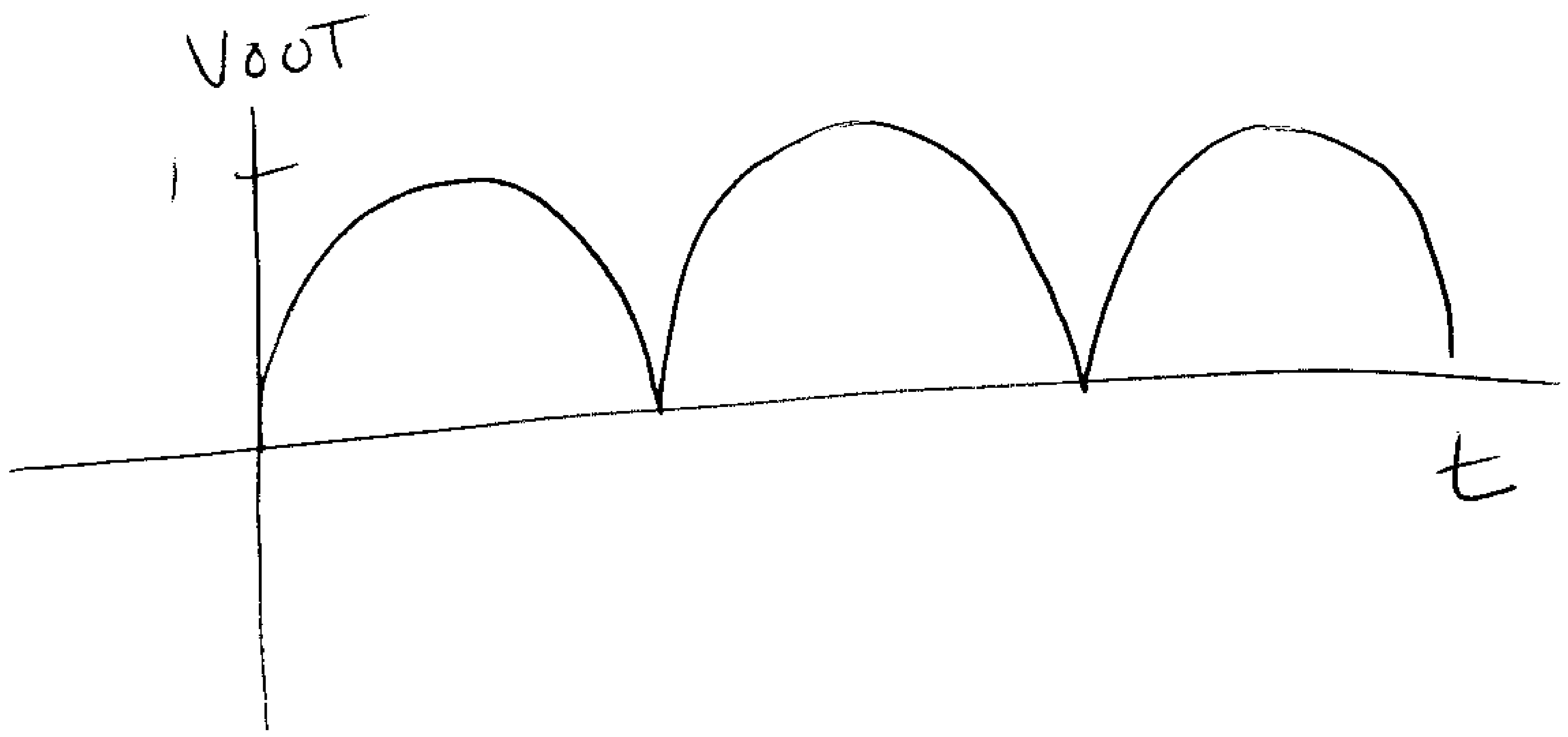


$-V_{OUT} = \sin t$

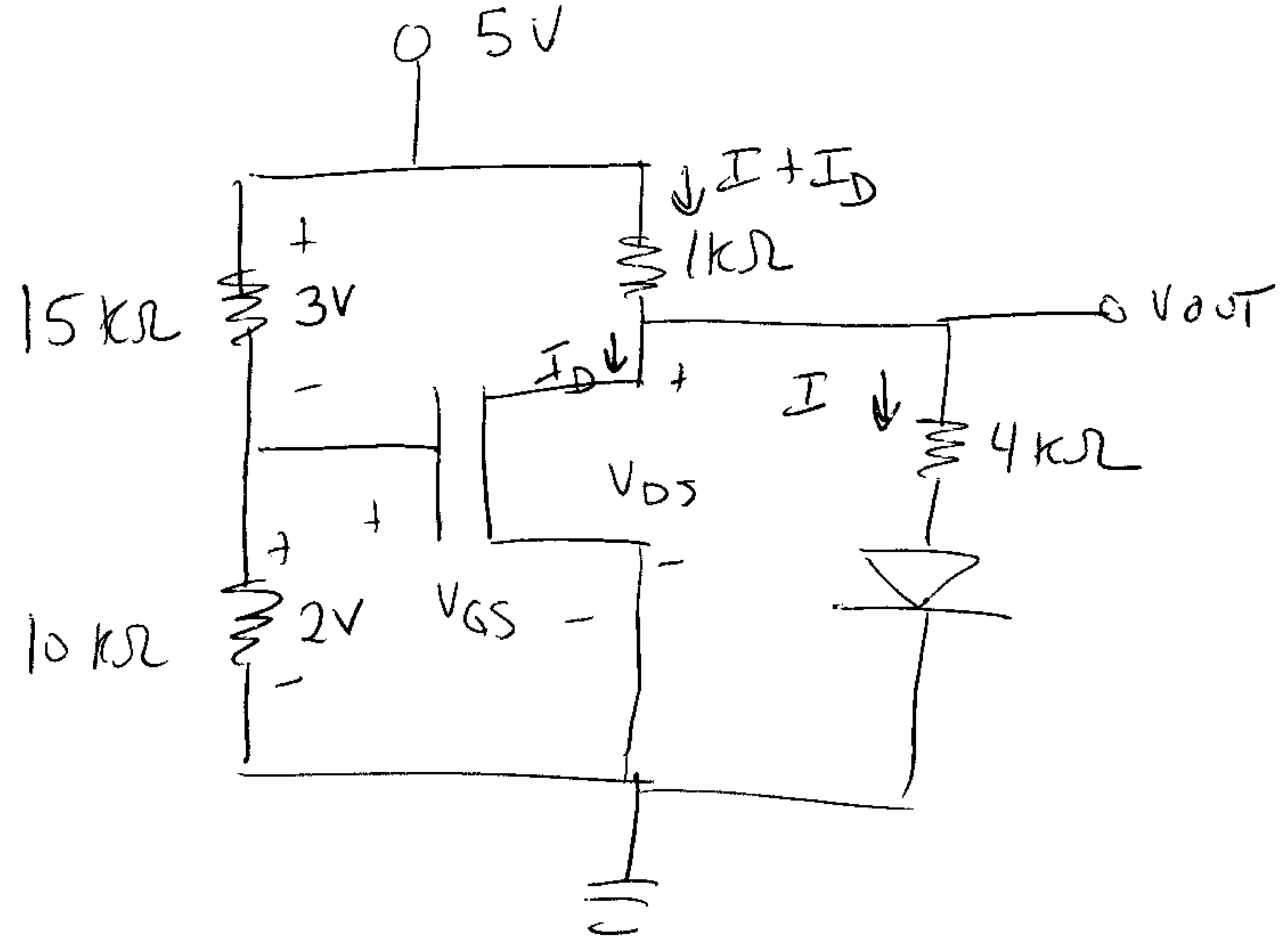
So when sin t is positive, $V_{OUT} = \sin t$.

" " " negative, $V_{out} = -\sin t$.

Hence $V_{OUT} = |\sin t|$.



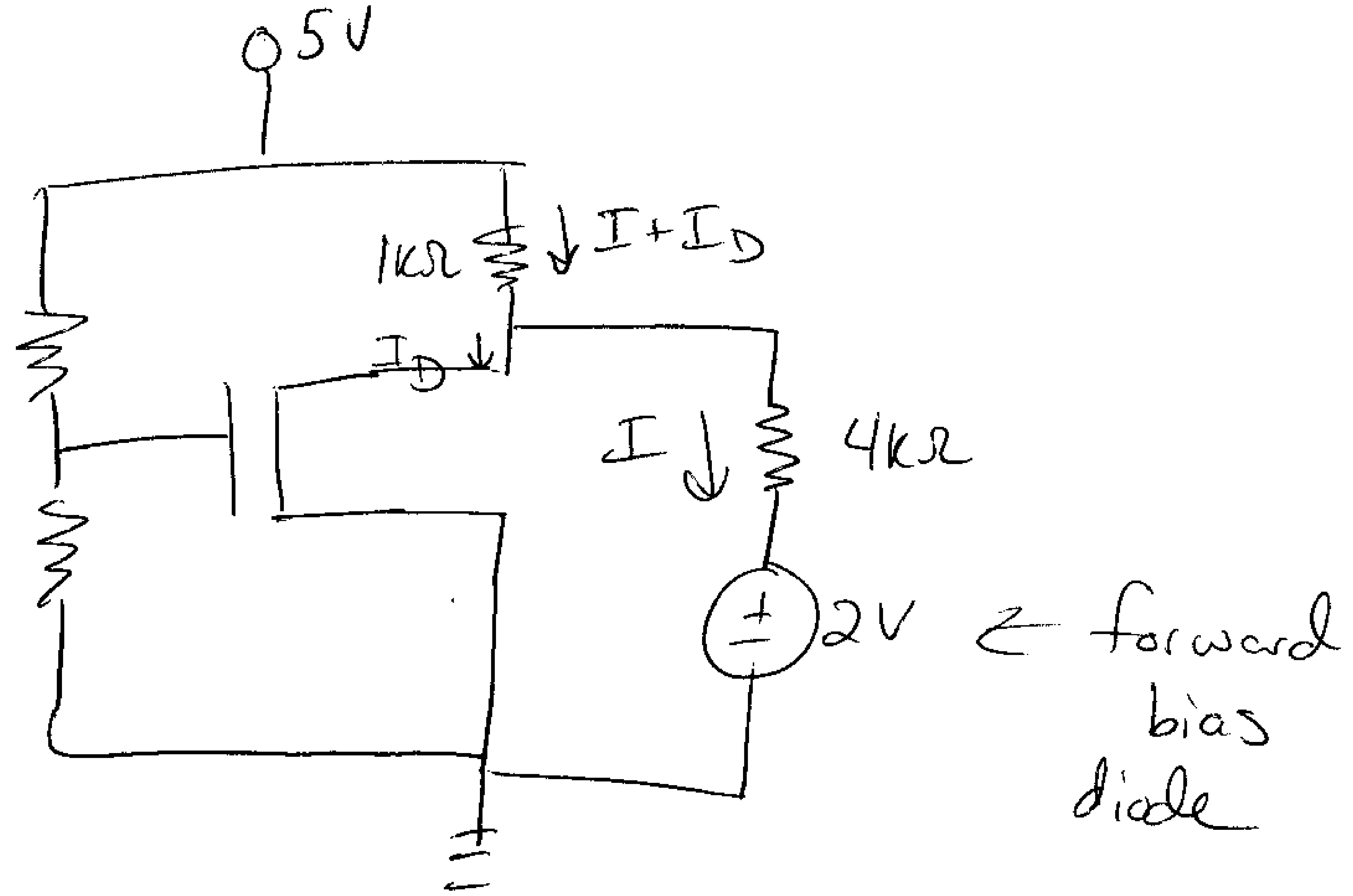
Problem 4:



Since no current goes into/out gate, 15kΩ + 10kΩ effectively in series: voltage division gives their voltages. Guess saturation (since $V_{GS} = 2V$ means not in cutoff):

$$I_D = 1mA/V^2 \cdot 1/2 \cdot (2V - 1V)^2 = 1/2 \, mA$$

Need to find V_{DS} to verify saturation.
Need to deal with voltage over 4kΩ resistor + diode.
Guess diode forward biased since V_{GS} little ⇒
Vout big. Find I to see if we are right.



KUL: $-2V - 4k\Omega I - 1k\Omega(I + I_D) + 5V = 0$
(start at ground, go up)

We said $I_D = 1/2\ mA$; substitute & solve

$I = 1/2\ mA$

$V_{DS} = I \cdot 4k\Omega + 2V = 4V$ $\quad V_{OUT} = 4V$