

Homework #8 Solutions

Problem 1

- a) $S = n(kT/q)(\ln 10)$; assume room temperature. We know that $(kT/q)(\ln 10) = 60\text{mV}$ at room temperature. Then $S = 1.15(60) \text{ mv/decade} = 69 \text{ mV/decade}$
- b) Part a) tells us that for each decade rise in drain current, we need to increase V_{GS} by 69mV . The normalized drain current at the threshold voltage is given to be 100nA and for a device where $W=L$, I_D at V_T is given to be $100\text{nA}/1 = 100\text{nA}$. We cannot have more than 100pA of current when V_{GS} is exactly 0. Then from $V_{GS} = 0\text{V}$ to $V_{GS}=V_T$, we can have a minimum $100\text{nA}/100\text{pA} = 1000 = 3$ decades rise in drain current. This implies that the minimum V_T we can have for the device is $V_{T\text{min}} = 3 \times 69\text{mV} = 207\text{mV}$ which limits the drain current to a maximum of 100pA at $V_{GS} = 0$.
- c) For a leakage current requirement of 0.1pA when $V_{GS} = 0\text{V}$, we need to increase the threshold voltage to a much higher value than the one calculated in part b) since $100\text{nA}/0.1\text{pA} = 1 \times 10^6 = 6$ decades; this gives $V_{T\text{min}} = 414\text{mV}$. The saturation current is given by the following equation:

$$I_{DSAT} = \frac{K'_n W}{2 L} (V_{GS} - V_T)^2$$

In this case, increasing to a larger threshold voltage reduces the saturation current, I_{DSAT} as compared to part b).

Problem 2

- a) In the on-state, the transistor is saturated because the gate-to-source voltage is very high (e.g. V_{DD}). Using the following equation for current when the transistor is saturated, we can obtain the saturation current as shown below.

$$I_{DSAT} = \frac{K'_n W}{2 L} (V_{GS} - V_T)^2$$

$$I_{DSAT} = 50 \mu\text{A}/\text{V}^2 \times \frac{1}{2} \times 100\mu\text{m}/10\mu\text{m} \times (5 - 0.7)^2 = 4.62\text{mA}$$

$$\text{Then } R_{eq} = \frac{3}{4} \times 5\text{V}/4.62\text{mA} \times (1 - 5/6(0)5\text{V}) = 811.2 \Omega$$

- b) Approaches to lowering R_{eq} are:
1. Reducing the threshold voltage will increase I_{DSAT} which will reduce R_{eq} .
 2. Increasing the width of the transistor will reduce R_{eq} as the resistance is inversely proportional to width. This increases the saturation current for the transistor.
 3. We can also reduce the length of the transistor to reduce the resistance since resistance is directly proportional to length, so a smaller length transistor will have a higher saturation current.
- c) For a very short-channel MOSFET, the saturation current and saturation voltage are given by the following two equations:

$$I_{DSAT} = WC_{ox} \left[V_{GS} - V_T - \frac{V_{DSAT}}{2} \right] v_{sat}$$

$$V_{DSAT} = \frac{L}{\mu_n} v_{sat}$$

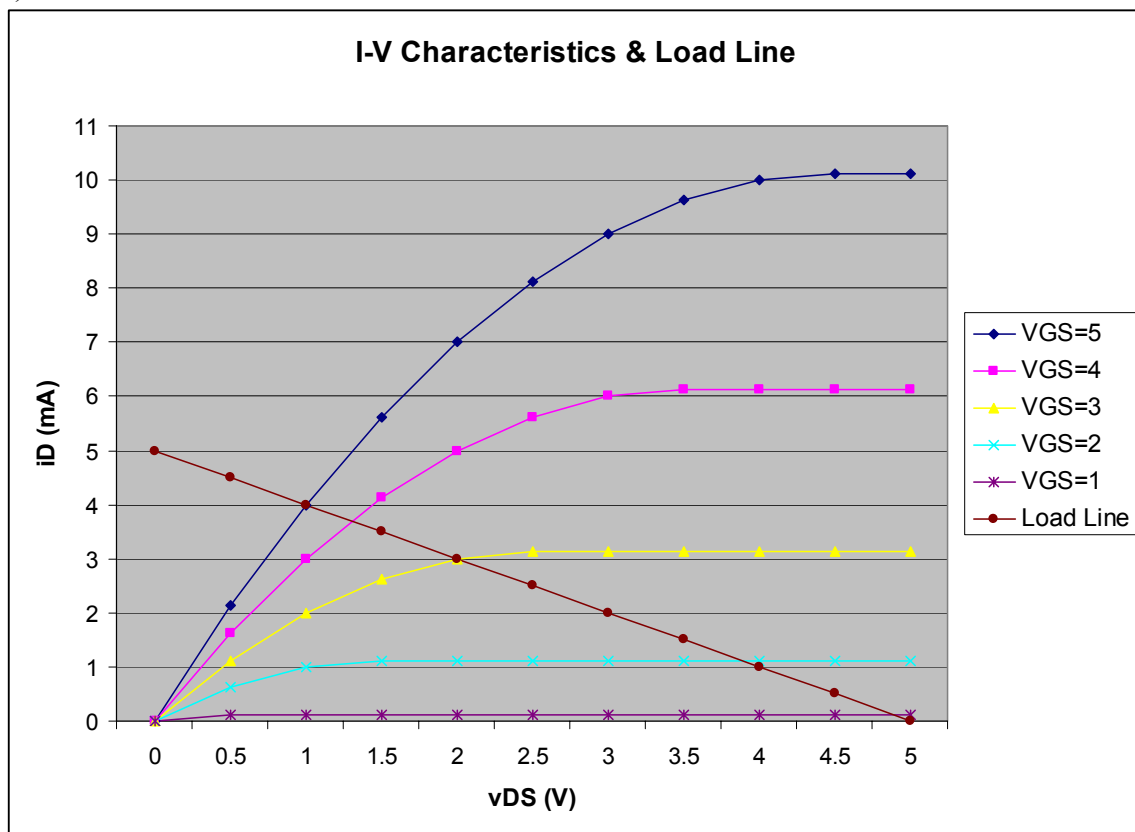
$$V_{DSAT} = 0.1 \times 10^{-4} \text{ cm} / 300 \text{ cm}^2 / \text{Vs} \times 10^7 \text{ cm/s} = 0.33 \text{ V and}$$

$$I_{DSAT} = 1 \times 10^{-4} \text{ cm} \times 2 \text{ uF/cm}^2 [1 - 0.4 - 0.33/2] [V] 10^7 \text{ cm/s} = 0.87 \text{ mA.}$$

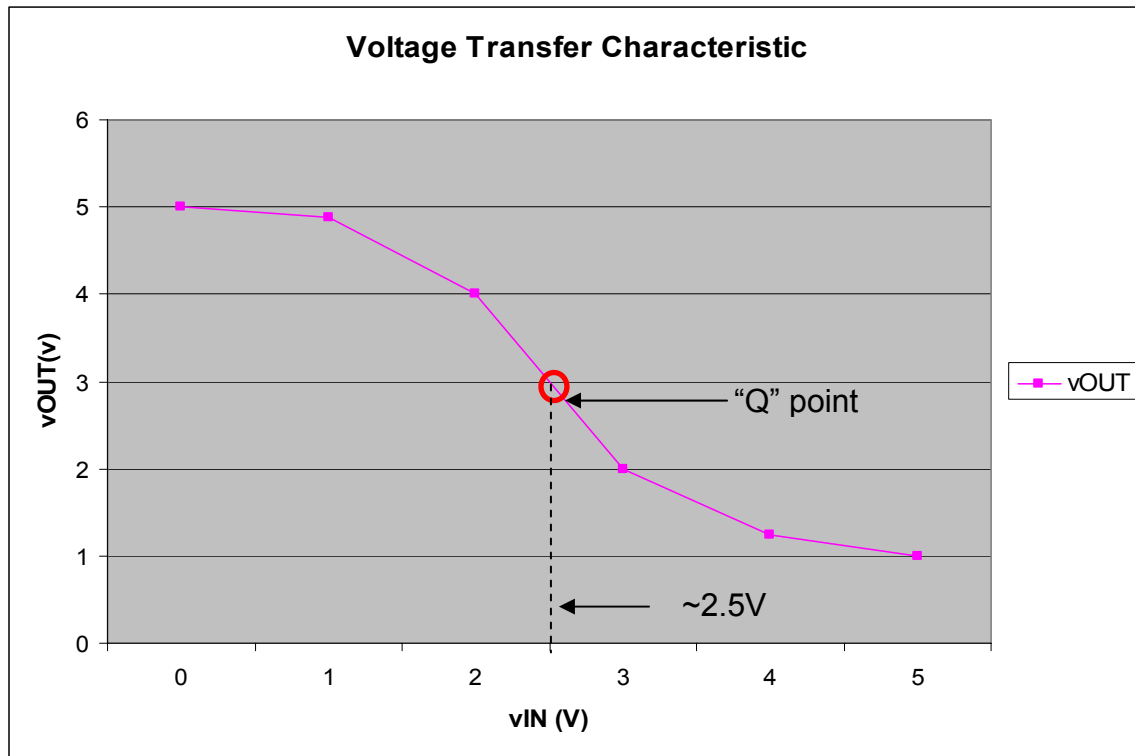
$$R_{eq} = \frac{3}{4} (1 \text{ V} / 0.87 \text{ mA}) (1 - 5/6 (0.1)) = 790 \ \Omega$$

Problem 3

a)

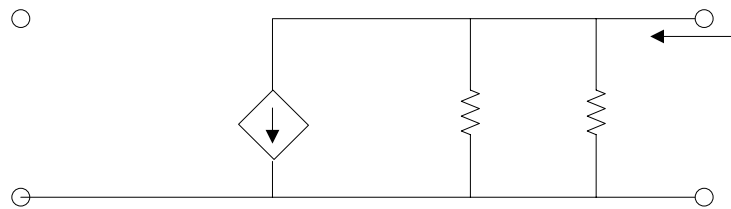


b)



c) From the voltage characteristic, the optimal operating point, or “Q” point occurs where the gain of the circuit is the highest or when the slope of the VTC is the steepest. We find that the “Q” point is approximately 2.5V as determined in the above graphical analysis. Thus, $V_{BIAS} \approx 2.5V$.

The open-circuit voltage gain is given as $A_v = v_{out}/v_{in}$. We must use the small signal model for the NMOS and draw the small-signal equivalent circuit as shown below.



Then $v_{out} = -(g_m)(v_s)(r_o || R_D)$ and $v_{in} = v_s$. Thus $A_v = -(g_m)(r_o || R_D)$. We can calculate g_m and r_o to be the following: (note $V_{GS} = V_{BIAS}$ for largest gain)
 $g_m = (W/L)(k')(V_{GS} - V_T) = 1000\mu A/V^2 \times (2.5 - 0.5) = 0.002$ siemens
 $r_o = 1/g_o = 1/\lambda I_D = \infty$. Then $A_v = (-0.002 \text{ siemens})(1k\Omega) = -2$.