EE40

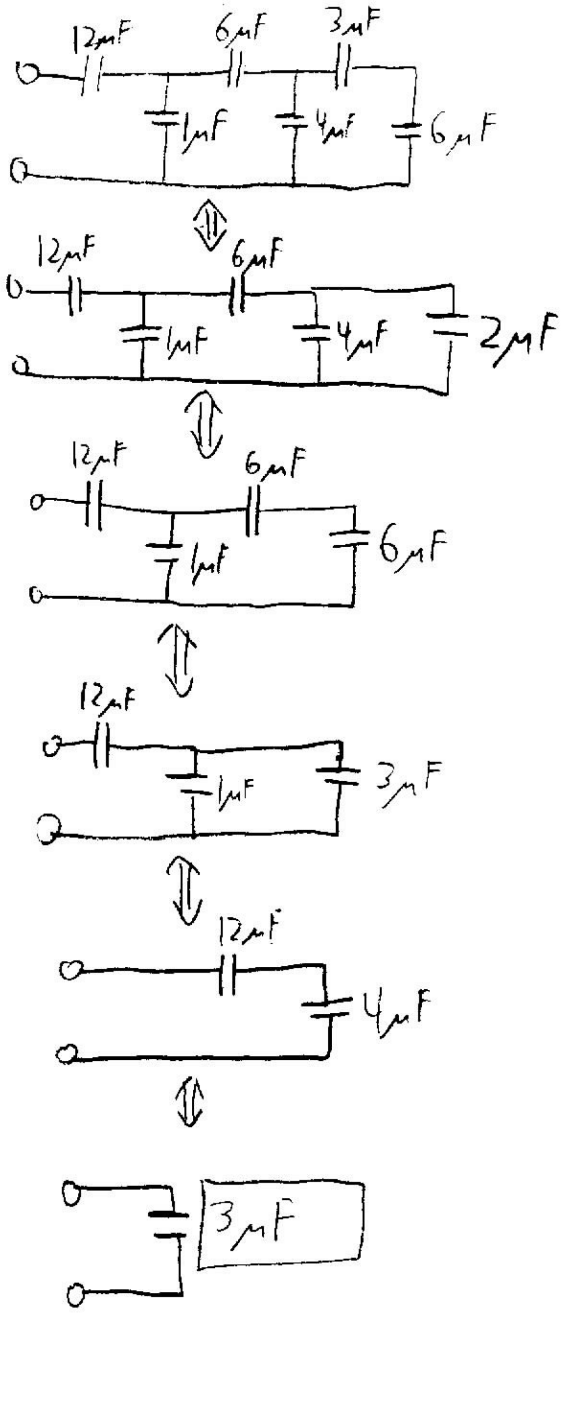
HW#5 Soln.

1) a) $C = 10\mu F \quad V = 20V$

$$Q = VC = .2\,mC$$

$$E = \frac{1}{2}CV^2 = 2\,mJ$$

b)

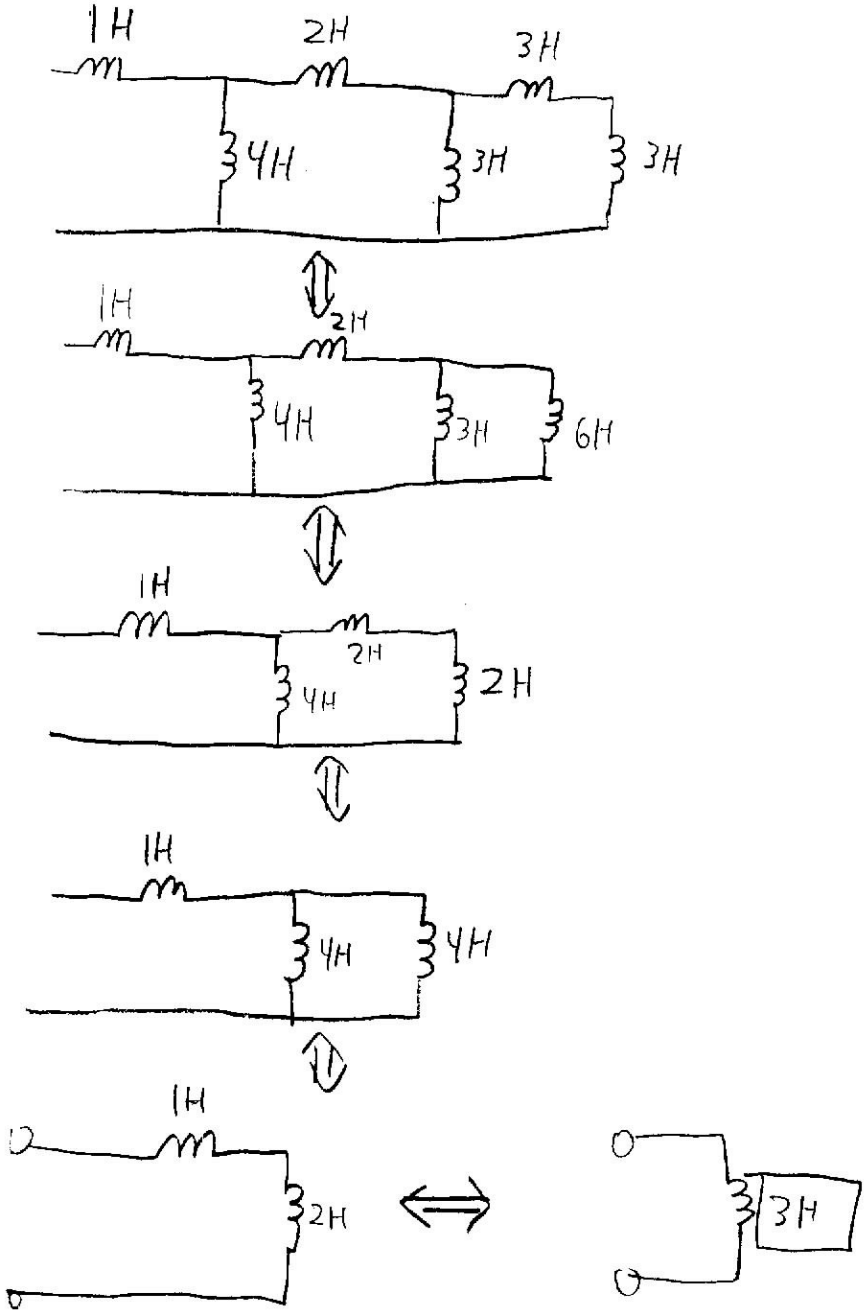


c) $L = 5\,mH \qquad i = 200\,mA$

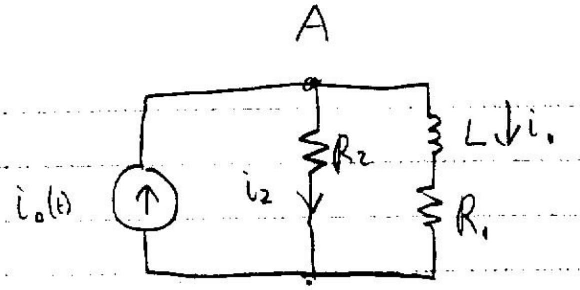
$$\lambda = Li = (5\,mH)(200\,mA) = \boxed{1\,m\,Wb}$$

$$E = \tfrac{1}{2}Li^2 = \left(\tfrac{1}{2}\right)(5 \cdot 10^{-3}\,H)\,(.2\,A)^2 = \boxed{.1\,m\,J}$$

d)



2)



at steady-state the inductor becomes a short

$i_0(t<0) = -1mA \qquad i_0(t>0) = 2mA$

$R_1 = 2k\Omega \qquad R_2 = 3k\Omega \qquad L = 20mH$

a) $i_1(0^-) = \left(\frac{R_2}{R_1+R_2}\right) i_0(0^-) = \left(\frac{3}{2+3}\right)(-1mA) = -.6mA$

$i_1(0^+) = i_1(0^-)$

$i_f = \left(\frac{R_2}{R_1+R_2}\right) i_0(\infty) = \left(\frac{3}{5}\right)(2mA) = 1.2mA$

$\tau = \frac{L}{R} = \frac{20mH}{(R_1+R_2)} = 4\mu sec \qquad t_0 = 0$

$i_1(t) = i_f + [i_1(0^-) - i_f]\exp\{-(t-t_0)/\tau\} \qquad t>0$

$i_2(t) = i_0(t) - i_1(t) = .8 + 1.8\exp\{-t/\tau) \; mA \qquad t>0$

$V_A = R_2 i_2(t) \qquad t>0$

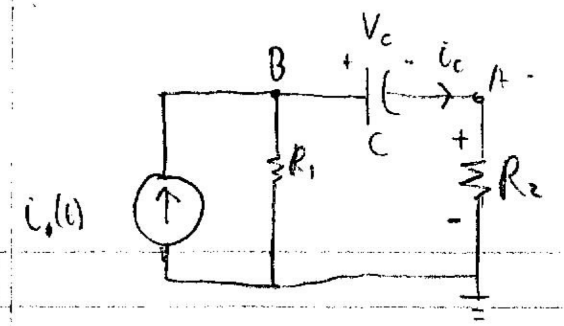
$\boxed{V_A(t) = 2.4 + 5.4\exp(-t/4\mu sec) \quad V \qquad t>0}$

b) at $t = 5\tau$ the transient is ~ .6% of its intial value

$V_A(5\tau) = 2.4 + 5.4\exp\left(\frac{-5\tau}{\tau}\right) = 2.43 \; V$

$5\tau = 20\mu sec$

c) $V_A(t<0) = (R_2)(i_0(t<0) - i_1(t<0)) = 3k \cdot (-.4mA) = -1.2V$

3) 

$i_1(t<0) = 4mA$
$i_1(t>0) = 0mA$
$R_1 = 500\,\Omega$
$R_2 = 1500\,\Omega$
$C = 1\mu F$

at steady state the cap. becomes an open

$V_c(t<0) = V_B - V_A = (4mA)(500) - 0 = 2V$

$V_c(0^-) = V_c(0^+) = 2V$

$V_c(\infty) = V_B - V_A = (0mA)(500) - 0 = 0V$

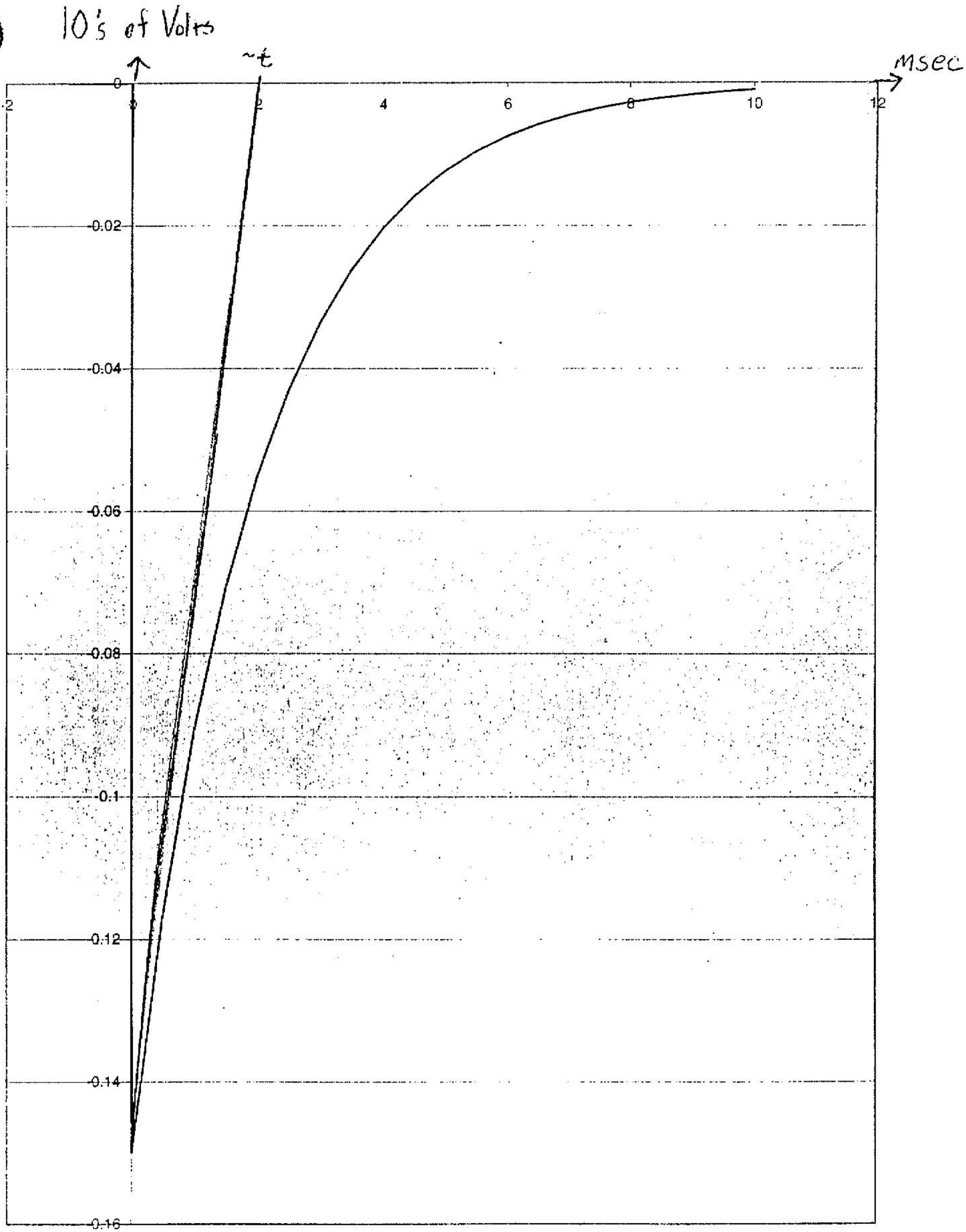
$\tau = (R_1 + R_2)C = 2msec \qquad t_0 = 0$

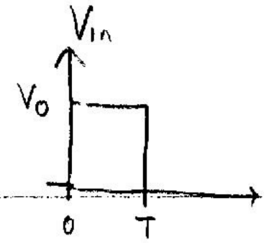
$V_c(t) = V_c(\infty) + [V_c(0^+) - V_c(\infty)]\exp(-t/\tau) \qquad t > 0$

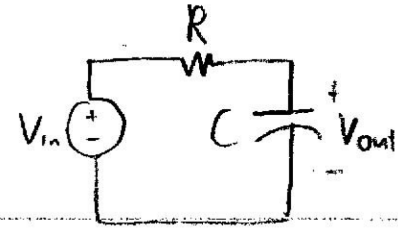
$$V_c(t) = \begin{cases} 2V & t<0 \\ 2\exp(-t/2msec)\,V & t \geq 0 \end{cases}$$

$$V_A = i_c R_2 = R_2 C \frac{dV_c}{dt} = \begin{cases} 0 & t<0 \\ -1.5 \cdot \exp\left(\frac{-t}{2msec}\right) & t \geq 0 \end{cases}$$

b) the transient dies down $\sim 5\tau = 10$ milliseconds c)







4)

a) analyze, assuming that $V_{in}(t>0)=V_0$

$V_{out}(0)=xV_0=V_{out}(0^+)$

$V_{out}(\infty)=V_0$ (the cap becomes an open)

$\tau=RC \quad t_0=0$

$V_{out}(t)=V_{out}(\infty)+[V_{out}(0^+)-V_{out}(\infty)]\exp(-(t-t_0)/\tau) \quad T>t>0$

$$\boxed{V_{out}(t)=V_0+[xV_0-V_0]\exp(-t/RC) \qquad 0<t<T}$$

b) $t_0=T \quad \tau=RC$ analyze assuming $V_{in}(t>T)=0$

$V_{out}(T)=V_{out}(T^+)=V_0[1+[x-1]\exp(-T/RC)]$

$V_{out}(\infty)=0$

$$\boxed{V_{out}(t)=V_{out}(T^+)\exp(-(t-T)/RC) \qquad t>T}$$

c) let $V_{out}(2T)=xV_0$ $\therefore$ the output will be periodic

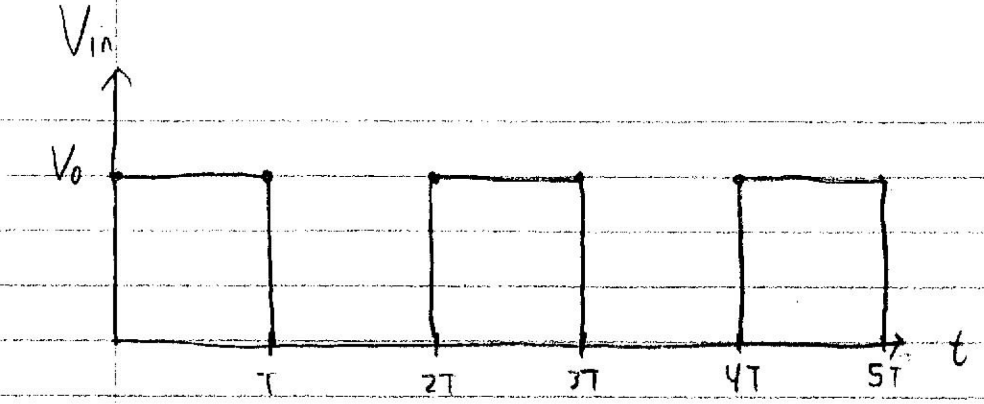
Let $V_{out}(T^+)$ be as defined above

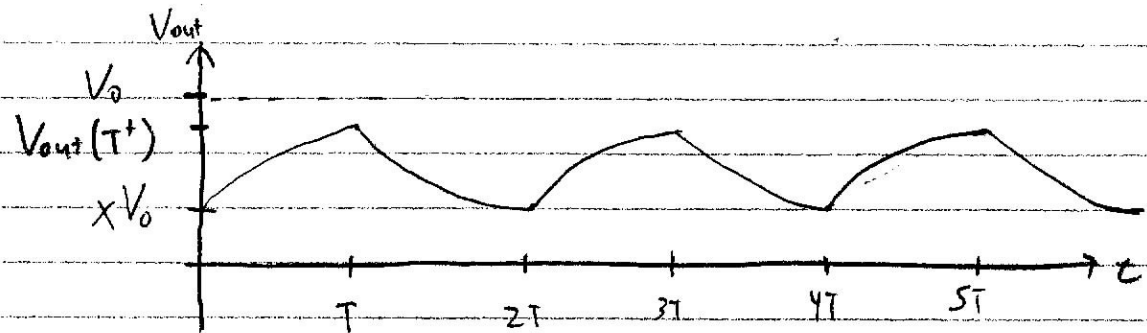
$$\therefore V^{(1)}_{out}(ht)=\begin{cases} V_0[1+(x-1)\exp(-t/RC)] & 0<t<T \\ V_{out}(T^+)\exp(-(t-T)/RC) & T<t<2T \end{cases} \qquad k\in\mathbb{N}$$

$\max(V^{(1)}_{out}(ht))=V_{out}(T^+)$

$\min(V^{(1)}_{out}(ht))=xV_0$

$$V_0>V_{out}(T^+)>xV_0>0$$





- Solving for x:

we know $V_{out}(2T) = V_{out}(0) = xV_0$

$$V_{out}(0) = V_{out}(2T)$$

$$\cancel{V_0}\left[1 + (x-1)e^{-0/RC}\right] = \cancel{V_0}\left[1 - (x-1)\exp(-T/RC)\right]\exp\left(\frac{-(2T-T)}{RC}\right)$$

let $\alpha = \exp(-T/RC)$

$$\cancel{1} + x - \cancel{1} = \alpha + \alpha^2(x-1)$$

$$x = \left(\frac{\alpha - \alpha^2}{1-\alpha^2}\right) = \left(\frac{1-\alpha}{\alpha^{-1} - \alpha}\right) = \left(\frac{1 - \exp(-T/RC)}{\exp(T/RC) - \exp(-T/RC)}\right)$$