HW #1

Due September 13 (Thursday) in class

1. We have derived the expressions for the 3-d (bulk) and 2-d (quantum well) electron density of states in class. Follow the same procedure, derive the 1-d electron density of state function for a "quantum wire" with dimension of L_x and L_y in the x and y direction, and unconfined in the z direction. Assume the electron effective mass $m_e^* = 0.067 \cdot m_0$, where m_0 is the free electron mass.
   a. Plot the energy band (E-k relation). For simplicity, assume infinite potential barrier. Note: there are more than one band. Plot the first 3 bands.
   b. Derive the 1-D electron density of state function, $\rho_{1D}(E)$.
   c. If L_x = 10 nm and L_y = 20 nm, plot $\rho_{1D}(E)$ for the first three energy states with E in eV. Please be quantitative in the plot.

2. Consider a "quantum box" (also called quantum dot or QD) with dimensions of L x L x L. Assume the electron effective mass $m_e^* = 0.067 \cdot m_0$, where m_0 is the free electron mass.
   a. Find the E-k relation for the quantum box. For simplicity, assume infinite potential barrier.
   b. Derive the 0-D electron density of state function, $\rho_{0D}(E)$.
   c. If L = 10 nm, plot $\rho_{0D}(E)$ for the first five energy states with E in eV. Please note that some energy states are degenerate (i.e., states with different quantum numbers might have the same energy). ** Hint: the density of states is discrete. Please use a delta function to represent the density of state, i.e., $\rho_0(E) = \sum_n a_n \delta(E - E_n)$, where E_n is the energy of the n-th state, while a_n is the number of degeneracy of that state. Use the height of the delta function to represent a_n.
   d. For L= 10 nm, find the number of electrons *inside* the quantum box when the Fermi energy is 50 meV above the lowest energy state. For simplicity, consider T = 0 K. Note that each state accommodates two electrons (spin up and spin down).

3. When a semiconductor is forward-biased, the Fermi level split into two quasi-Fermi levels, F_C and F_V. Refer to the diagram on the right. Show that under the condition of equal electron and hole concentration, the electron and hole quasi-Fermi levels occur at the same $|\vec{k}|$. In other words, when an electron at energy F_C recombines with a hole and emits a photon, the hole energy would be F_V. ** Hint: Use effective mass (i.e., parabolic) approximation for the energy bands.